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ABSTRACT

Unified Technical Concepts (UTC) is a modular system for teaching applied physics in two-year postsecondary technician programs. This UTC laboratory textbook, the first of two volumes, consists of 56 learning modules dealing with basic concepts of physics. Addressed in the individual chapters of the guide are the following topics: force, work, rate, momentum, resistance, power, and potential and kinetic energy. Each chapter contains a series of modules that include an introductory statement, objectives, instructional text, a summary, a laboratory experiment, and a self-check. (MN)

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PREFACE

The textbooks entitled UNIFIED TECHNICAL CONCEPTS and UTC APPLICATION MODULES form an integral set of teaching materials for the presentation of UTC Physics. UNIFIED TECHNICAL CONCEPTS is the "classroom text" designed for teaching the basic technical concepts in physics and demonstrating their wide application to the world of the technician. UTC APPLICATION MODULES--in two volumes--is the "laboratory text" which complements the subject matter found in the classroom text and provides the technician with valuable practice and application of physics concepts in the laboratory. The rationale for UTC Physics, the identification of its essential parts, and its importance in the education and training of modern technicians are covered in the Preface of UNIFIED TECHNICAL CONCEPTS--and are not repeated here. Suffice it to say that a successful presentation of UTC Physics requires an integrated presentation of the unified technical concepts in the classroom and meaningful experiments and measurements with modern-day devices and instruments in the laboratory.

In combination, Volumes I and II of UTC APPLICATION MODULES contain a total of 111 modules--carefully selected from the original 170 application modules (laboratory exercises) developed for UTC Physics. Volume I contains 56 application modules that illustrate and apply the concepts of FORCE, WORK, RATE, MOMENTUM, RESISTANCE, POWER and ENERGY to practical, real-world problems. Volume II contains 55 application modules that illustrate and apply the concepts of FORCE TRANSFORMERS, ENERGY CONVERTORS, TRANSDUCERS, VIBRATIONS AND WAVES, TIME CONSTANTS, and RADIATION to similar problems. In both Volumes I and II--as an aid to organization and presentation--all of the application modules associated with one particular technical concept are grouped together in a major section of each text and are identified as such at the beginning of each section dealing with that particular concept. For example, in Volume I, under the technical concept entitled FORCE, there are 12 application modules included to illustrate and apply the concept of a "force" to meaningful laboratory experiments and measurements in four important easily recognized energy systems--mechanical, fluidal, electrical, and thermal.

The notation we have adopted for identifying and keeping track of the many application modules deserves a word of explanation. We have used a three-symbol code--such as 1M1, 2F2, 3E1, or 5T2. The number on the left--varying from 1 to 13--refers to the particular concept which the application module supports. In this numbering system, FORCE → 1, WORK → 2, RATE → 3, etc. The letter in the middle refers to the particular energy system--mechanical (M), fluidal (F), electrical (E) and thermal (T). The number on the right refers to the particular module in the series written for a given concept (left number) and given energy system (middle letter). Thus the code 5T2 signifies that this application module is the second in a series written in support of concept 5 (RESISTANCE) and dealing with a thermal (T) energy system. As the table of contents indicates, 5T2 is entitled "Heat Flow through Insulation"--clearly a topic involved with thermal resistance.

For the most part the application modules contained in Volumes I and II of UTC APPLICATION MODULES were chosen to provide the student with important hands-on practice and application of technical concepts to meaningful, modern devices and instruments--and provide, at the same time, a broad experience in the mechanical, electrical, fluidal and thermal energy systems found in modern electromechanical technologies. A small group of application modules contained herein were chosen because their content appropriately supports and further strengthens the thirteen principal concepts covered in the classroom text. In some instances this group of modules introduces new ideas not found in the classroom text. These application modules, referred to appropriately as CONCEPT SUPPORT MODULES, are identified with an asterisk (*) in the table of contents for each of the two volumes and also in the list of application modules that precedes each of the major thirteen concepts, e.g., FORCE, WORK, etc.

As alluded to earlier, 170 application modules were developed for UTC Physics originally and all are available. The 111 modules we have included in the two volumes here represent our choice of the best of the 170. However, it has been our experience with different schools and faculties that some of the 59 not included in the two volumes are at times desirable. Therefore, we have listed in an appendix at the end of each volume, a title and brief description of each module not contained in either of these two volumes. One or all of these may be ordered from CORD as needed.

Finally, as teachers talking with teachers, we would strongly recommend that the implementation of UTC Physics not include more than 90 application modules in any full-year program--3 quarters or 2 semesters. In general, we advise that one begin with 60-80 modules in the first year. Then after the logistics in the laboratory have been worked out and teaching experience has been acquired, an increase to 90 (or beyond) can be considered. For planning and scheduling purposes, one should count on an average of about two hours per laboratory application module.

It goes without saying that a successful presentation of UTC Physics requires integrated teaching (and learning) in the classroom and the laboratory. We consider that portion of student time spent in the laboratory on meaningful, hands-on, technician-oriented exercises to be of vital importance. Overall success of UTC Physics requires a dedicated, enthusiastic teacher. It is not an easy course to teach--but it is exciting and the rewards are great.

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* Identifies concept support modules that introduce new physics.

ACKNOWLEDGMENTS

Unified Technical Concepts (UTC)--Physics for Technicians is a major innovation in technical education for the 1980s. UTC Physics is innovative because it provides a broad, interdisciplinary base for teaching technical principles, combined with the practical illustration of these principles in the work clothes of modern industrial equipment.

The evolution of this delivery system is due largely to the contribution of several outstanding technical educators: First, to Dr. Maurice W. Roney, for his conceptualization of unified physics while developing an electromechanical technology curriculum in the late 1960s. Next, to Dr. Walter J. Brooking for his encouragement to persist in the development and dissemination of the UTC curriculum. Finally, to Dr. Leno S. Pedrotti the chief editor, technical consultant and designer of the instructional format. Significant contributions to UTC were also made by CORD staff members Johnny Jones and Carl Lansford, and by project editor Frank L. Pedrotti, S.J.

Daniel M. Hull, UTC Project Director
President, Center for Occupational
Research and Development
June 1983

FORCE

APPLICATION MODULES

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* Identifies concept support modules that introduce new physics.

1M1

GRAVITY

Gravitational force is exerted by each object in the universe upon all other objects. The force of gravity upon an object is weight — not to be confused with mass, which is the resistance of an object to a change in its motion. Gravity influences man and machinery upon earth and in space. The relationship between gravitational force and acceleration of free-falling objects is discussed in this module. In the laboratory, the student will determine both the weight and the mass of several objects.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define Newton's "law of universal gravitation."
- Explain the difference between weight and mass.
- Calculate the gravitational attraction of one object upon another, given their mass and separation distance.
- Describe the inverse-square law.
- Calculate the velocity of an object in free fall, given the time of the fall.
- Given the appropriate equipment, measure the weight of several objects by two methods; and calculate their mass.

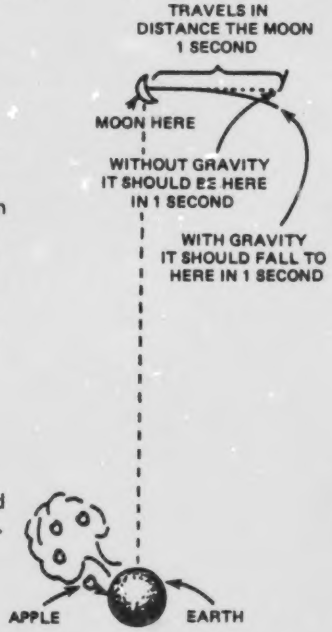
DISCUSSION

THE LAW OF UNIVERSAL GRAVITATION

Newton's "law of universal gravitation" states, "Every object in the universe attracts every other object with a force directly proportional to each of their masses and inversely proportional to the square of the distance separating them."

If a ball is thrown upward, it is pulled back to earth by the gravitational force exerted by the earth upon the ball. Also, the earth itself moves around the sun in an elliptical orbit because of the gravitational force which the sun exerts upon it; otherwise, the earth would move away in a straight line.

Newton was the first person to conclude that the same force that causes the moon to move about the earth also causes an apple to fall from a tree to the earth. The moon also is falling toward the earth; otherwise, it would move away in a straight line. It falls just enough each second to remain in its orbit around the earth, as illustrated in Figure 1. From this original insight, Newton was able to explain the motion of the planets about the sun and of the moon about the earth.



TRAVELS IN DISTANCE THE MOON 1 SECOND
MOON HERE
WITHOUT GRAVITY IT SHOULD BE HERE IN 1 SECOND
WITH GRAVITY IT SHOULD FALL TO HERE IN 1 SECOND
APPLE
EARTH

FIG. 1

WEIGHT AND MASS

An important distinction exists between weight and mass. Weight is "the force due to gravity upon a body" and depends upon its location relative to an astronomical body. An object would weigh more upon earth than upon the moon, for example. Mass, however, is the same for an object everywhere in the universe and can be defined as "the quantity of matter in a body, or the property of a body's resistance to change of motion." Mass is thus a measure of the inertia, or sluggishness, of an object in stopping or changing its motion where there is no friction. An object weighs less upon the moon than upon the earth, yet its mass is the same.

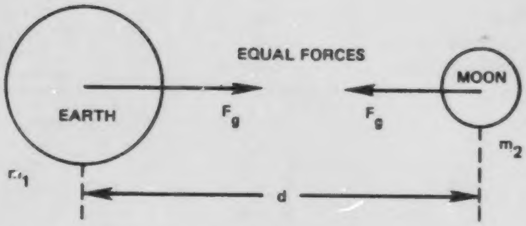
FORCE OF GRAVITY

Newton's law of gravitation, or gravitational force, is expressed in terms of the masses of the objects involved and their separation distance, indicated by Equation 1.

$$F_g = \frac{Gm_1m_2}{d^2} \qquad (1)$$

where

F_g = force of gravity.
G = universal gravitational proportional constant, $6.67 \times 10^{-11}\ m^3/kg - s^2$.
m_1 = mass of object 1.
m_2 = mass of object 2.
d = distance between objects.



(d MEASURED FROM CENTER OF BOTH BODIES)

FIG. 2

Equation 1 can be used to explain the motion of all celestial bodies in our solar system. Figure 2 illustrates the gravitational force between two (approximately) spherical bodies, the earth and the moon. The distance between them is measured from their centers. The force of gravitational attraction upon one sphere is equal and opposite to that upon the other. Equation 1 is used in solving a problem in Example A.

EXAMPLE A : GRAVITATIONAL FORCE

Given: A ball with a mass of 50 kg is located at the surface of the earth (which has a radius of $d = 6.4 \times 10^6$ meters and a mass of $m_e = 6.02 \times 10^{24}$ kg).

Find: The force the earth exerts upon the ball and the force the ball exerts upon the earth.

Solution:

$$F_g = \frac{Gm_1m_2}{d^2}$$

$$= \frac{(6.67 \times 10^{11}\ m^3/kg\ s^2)(6.02 \times 10^{24}\ kg)(50\ kg)}{(6.4 \times 10^6\ m)^2}$$

$$F_g = 490 \text{ Newtons}$$

The force that the earth exerts upon the ball is equal to the force the ball exerts upon the earth; they are oppositely directed.

Figures 3 and 4 illustrate how gravitational force changes with separation distance. Within a planet of uniform composition and density, gravitational force is directly proportional to the radial distance from the center and is maximum at its surface. Because of the distance squared factor in the denominator, however, the force from the surface outward decreases rapidly as the distance increases (Figure 3). This relationship is called the "inverse-square law," and means that the earth's gravitational pull upon an outside object varies inversely with the square of its distance from the center of the earth. For example, an object that weighs one pound at the surface of the earth will weigh 1/4 pound when it is twice as far from the center of the earth (Figure 4).

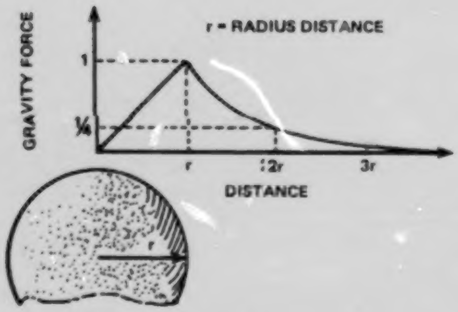


FIG. 3

Gravitational force is a relatively weak force: A mass the size of the earth exerts a force of only 150 lb upon a person weighing 150 lb. Two people cannot detect or measure their mutual gravitational attraction, no matter how close they might get. Other forces in nature, such as the electrical force, are much stronger than the gravitational force.

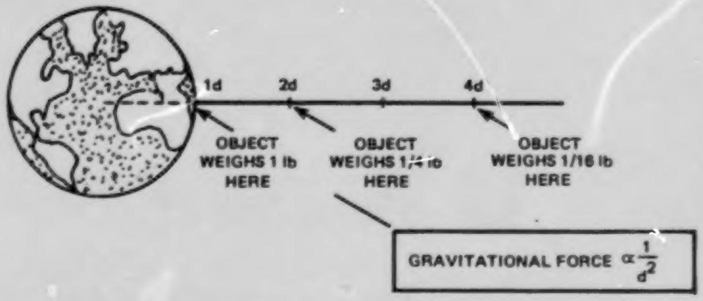

1d
2d
3d
4d
OBJECT WEIGHS 1 lb HERE
OBJECT WEIGHS 1/4 lb HERE
OBJECT WEIGHS 1/16 lb HERE
GRAVITATIONAL FORCE $\propto \frac{1}{d^2}$

FIG. 4

The gravitational force equation (Equation 1) for objects on the surface of the earth can be rearranged as in Equation 2.

$$F_g = mg \qquad (2)$$

where

F_g = force of gravity.

m = mass of object on surface.

$g = \frac{Gm_e}{d^2}$, or acceleration due to gravity; where m_e is mass of the earth, and d is radius of the earth (4,000 miles).

That is,

$$F_g = m\left(\frac{Gm_e}{d^2}\right) = mg$$

The term g (acceleration due to gravity) is almost a constant for positions close to the surface of the earth, as shown by the following calculated values:

$g = 9.800\ m/s^2$ for $d = 4000$ mi (at the surface of the earth).
$g = 9.795\ m/s^2$ for $d = 4001$ mi (one mile above the surface).
$g = 9.751\ m/s^2$ for $d = 4010$ mi (ten miles above the surface).

The accepted value of g at the earth's surface is $9.8\ m/s^2$ in the International system of units and $32\ ft/s^2$ in the British system of units. An object would have to be high above the earth's surface for the g value to be noticeably different.

VELOCITY

If a parachutist falls at constant velocity, the downward gravitational force is balanced by the upward drag force of the parachute. Since the net force is zero, he falls at a constant velocity, or distance traveled divided by time, as indicated by Equation 3.

$$v = \frac{d}{t} \tag{3}$$

where
v = velocity.
d = distance in feet.
t = time in seconds.

Equation 3 is utilized to solve a problem for constant velocity in Example B.

EXAMPLE B

Given: A parachutist falls 180 ft during each second.

Find: Velocity of the parachutist as he falls through the air.

Solution:

$$v = \frac{d}{t}$$
$$= \frac{180\ ft}{1\ s}$$
$$v = 180\ ft/s$$

Notice that this example illustrates constant velocity, a velocity that does not change from second to second.

ACCELERATION

If there were no drag force acting upon a falling object, its velocity would increase each second (Figure 4). An object dropped from rest, with no resistive force acting upon it, will be traveling with a velocity of 32 ft/s after the first second, 64 ft/s after the next second, 96 ft/s after the third second, etc. In other words, its velocity changes by 32 ft/s each second it falls. The acceleration of an object is defined by Equation 4.

$$a = \frac{\Delta v}{\Delta t} \qquad (4)$$

where

a = acceleration.
Δv = change in velocity.
Δt = time elapsed in seconds.

For a freely-falling object, this equation becomes $a = \frac{(32\ \text{ft/s})}{(1\ \text{s})}$

$$a = 32\ \text{ft/s}^2$$

Equation 4 can be arranged for velocity, as indicated by Equation 5 and Example C.

$$v = at \qquad (5)$$

where

v = velocity of free-falling body.
a = acceleration.
t = time in seconds.

EXAMPLE C

Given: An object falls for five seconds with no drag force acting upon it.

Find: The velocity of the object at the end of the five-second fall.

Solution: $v = at$

$$v = (32\ \text{ft/s}^2)(5\ \text{s})$$
$$v = 160\ \text{ft/s}$$

Since velocity increases during each second of fall for a freely-falling object, the distance of fall during each second increases. A free-falling object starting from rest has a velocity of 32 ft/s at the end of the first second, and thus an average velocity of 16 ft/s during the first second; therefore, it travels a distance of 16 ft during the first second. Figure 5 shows the distance an object falls during the first few seconds when it starts from rest. (Velocity is discussed in more detail in later modules.)

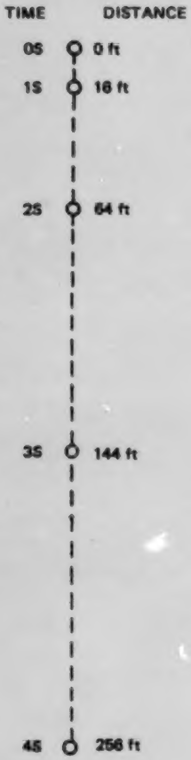


FIG. 5

FORCE ACTING UPON AN OBJECT

If the net force acting upon an object is zero, the object will have no acceleration; however, if a net force acts upon an object, the object will experience acceleration. Acceleration is proportional to force, as indicated by Equation 6.

$$F = ma \tag{6}$$

where
F = net force acting upon an object.

When the force acting upon an object is known, together with its resulting acceleration, its mass can be calculated from Equation 7.

$$m = \frac{F}{a} \tag{7}$$

The mass of an object is calculated in Example D.

EXAMPLE D

Given: An object experiences (a) an acceleration of 5 m/s^2 when a force of 10 Newtons acts upon it, and (b) an acceleration of 15 m/s^2 when a force of 30 Newtons acts upon it.

Find: Mass of the object for these conditions.

Solution: (a) $m = \frac{F}{a}$ (Recall that $\frac{1 N \cdot s^2}{m} = 1$ kg.)

$$= \frac{(10\ N)}{(5\ m/s^2)}$$

$$m = 2\ kg$$

(b) $m = \frac{F}{a}$

$$= \frac{(30\ N)}{(15\ m/s^2)}$$

$$m = 2\ kg$$

Notice that the results are the same for both sets of conditions, since the same mass is involved.

Equation 2 demonstrated that the force acting upon an object due to gravity is $F_g = m\,(\frac{Gm_e}{d^2}) = mg$. This force is related to acceleration through Equation 6, F = ma. In these equations, acceleration, a, must exactly equal g; that is, $a = g = 32\ ft/s^2$, or $9.8\ m/s^2$, which is the acceleration due to gravity.

This equality means that the value measured for the acceleration of a free-falling object ($9.8\ m/s^2$, or $32\ ft/s^2$) is the same as the value of g calculated from the relationship $g = \frac{Gm_e}{d^2}$. Thus, each second an object falls, its velocity increases by 32 ft/s. Newton was able to show, using this result, that his law of gravity (Equation 1) gave the same value for g, the acceleration due to gravity, as the experimentally-determined value. The calculation is simply as follows:

$$g = \frac{Gm_e}{d^2}$$

$$= \frac{(6.67 \times 10^{-11}\ m^3/kg - s^2)(6.02 \times 10^{24}\ kg)}{(6.4 \times 10^6\ m)^2}$$

$$g = 9.8\ m/s^2$$

When Newton first calculated a value of g using this relationship, he obtained a result different from the experimentally-measured value; and because of this discrepancy, he filed away his papers. About twenty years later, a new measurement was made for the mass of the earth. This new value differed considerably from the previous value that Newton had used. He recalculated the value of g with the improved value of the mass of the earth and obtained results very close to the experimental value. Only then did he publish his papers on gravity and mechanics.

WEIGHT

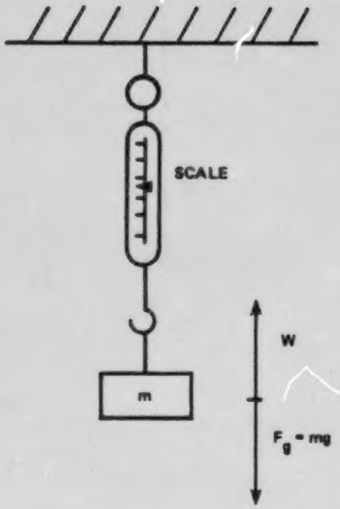


FIG. 6

The gravitational force that the earth exerts upon an object of mass m can be measured by use of a spring scale (Figure 6). Gravitational force is mg, and the upward force which the scale exerts upon the mass is W. Since the mass is not moving, the upward force minus downward gravitational force equals zero, or W - mg = 0. Rearranged, this equation becomes Equation 8.

$$W = mg \tag{8}$$

where

W = weight.
m = mass.
g = acceleration due to gravity, $9.8\ m/s^2$.

The upward scale force, which exactly balances the downward force due to gravity, is defined as the "weight" of an object, and is the product of an object's mass m times g. Example E shows how to determine the weight of an object at the earth's surface.

EXAMPLE E

Given: An object has a mass of 25 kg.

Find: Weight of the object.

Solution:
$$\begin{aligned} W &= mg \\ &= (25\ kg)(9.8\ m/s^2) \\ W &= 245\ N \end{aligned}$$

As an object moves away from the surface of the earth, the value of g decreases, which causes the object's weight to decrease. If an object is placed upon another planet or moon, the acceleration due to gravity g can be calculated from the mass and radius of that planet or moon. This calculation would give a different acceleration due to gravity and, therefore, a different weight for the object. The same object upon the moon, for example, weighs about 1/5 as much as it does upon the earth.

SUMMARY

Each object in the universe exerts a gravitational force upon all others. This force is referred to as the "weight" of an object. The resistance of an object to a change in its motion is known as its mass. The influence of gravitational force upon the motion of bodies is important and can he calculated with use of the appropriate relationships.

MEASUREMENT OF WEIGHT AND MASS

MATERIALS

Triple-beam balance.
Spring balance (pound and Newton calibrated) with appropriate scale range.
Assorted weights.

PROCEDURE

In this experiment, the student will determine the weight and mass of several objects, using both a triple-beam balance and a spring balance.

1. Place first object on triple-beam balance, and adjust sliding weights until scale is balanced. Record mass of object (in grams) in Data Table. Notice that the beam balance determines mass and not weight.

2. Find weight of object by using Equation 8, converting mass to kilograms and multiplying by g ($9.8 \, m/s^2$). Record value in Data Table.

3. Find the weight of object in both pounds and Newtons, using spring balance, as in Figure 6. Record values in Data Table. Notice that the spring balance determines weight and not mass.

4. Calculate percent difference between weight of object found using the triple beam balance and its weight found using the spring balance; record this value.

5. Repeat Steps 1 through 4 for three additional objects. Record values in Data Table.

DATA TABLE							
	Triple Beam Balance			Spring Balance			
	Measured mass in grams	Converted value in Newtons	Converted value in pounds	Measured weight in Newtons	Measured weight in pounds	Percent Difference	
						N	lbs
Object 1							
Object 2							
Object 3							
Object 4							

SELF CHECK

1. With the help of Newton's "law of universal gravitation," explain how each object in the universe affects all other objects.

2. How do weight and mass differ? Is the mass of an object the same upon earth as upon any other planet? Can the same be said of weight?

3. Calculate the gravitational force exerted by the earth upon the moon. (The moon's mass is 7.34×10^{22} kg, and it is 3.86×10^{8} m from earth.)

4. Describe how force varies with distance in the inverse-square law.

5. Calculate the velocity of a free-falling object dropped from rest and falling for ten seconds. (Assume that no resistive forces act upon the object.)

6. Determine the mass of an object under the following conditions: The object has an acceleration of 5 m/s^2 when a force of 100 Newtons acts upon it and has an acceleration of 8 m/s^2 when a force of 160 Newtons acts upon it.

7. A toy rocket experiences an upward thrust of 100 N. What is the net acceleration of the rocket if the mass of the rocket is 2 kg. Do not neglect the force due to gravity.

8. If an object weighs 50 N on the surface of the earth, how much will it weigh on the moon? The mass of the moon is 7.34×10^{22} kg and the radius of the moon is 1738 km.

ADDITION OF MECHANICAL FORCES

Force is the quality which changes, produces or stops the motion of a body; that is, it is the push-or-pull quality exerted upon an object. Two or more forces can be exerted at right angles, at other than right angles or along the same line (linear). The direction and magnitude that results when two or more forces are exerted upon an object is called the "resultant." This module presents graphical and mathematical methods for finding resultants. The methods explained in this module are valid for any vector qualities. Although most of the examples given deal with forces, the same resolution and composition procedures can be used for any other type of vector qualities, such as velocities, accelerations and momenta. Resultants are extremely important in a wide range of technical applications. In the laboratory, the student will determine several resultants experimentally, graphically and mathematically.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:

  Contact forces.
  Action-at-a-distance forces.
  Concurrent forces.
  Coplanar forces.
  Resultant.

- Determine the resultant of concurrent forces, graphically, mathematically or experimentally for the following four cases:

Two forces at right angles.
Two forces not at right angles.
Both forces in the same line.
More than two coplanar forces.

- Given the appropriate equipment, determine the resultants of forces in several directions.

DISCUSSION

Forces are exerted by a hammer hitting a nail, a brace holding up a load, a stretched spring on the bodies to which it is attached, a tractor pulling a plow, a rope lifting a bale of hay. In all these examples, force is in contact with the body upon which it is exerted, and is known as "contact" force. There are also forces that act without making contact. These forces are called "action-at-a-distance" forces; for example, the gravitational attraction, or weight, exerted upon a body by the earth. All known forces are in one of these two classes.

If a number of forces are applied at a common point, they are "concurrent." If all of the forces lie in the same plane, they are called "coplanar." If all forces acting upon a system are concurrent, to find a simple force with the proper magnitude and direction that produces the same effect is possible. This force, called the "resultant," will be determined for four different cases in this module.

The four cases of concurrent, coplanar forces considered in this module are studies in sequence. Graphical and mathematical determinations of resultants will be compared with experimental results.

TWO FORCES AT RIGHT ANGLES

Two forces $\vec{A}$ and $\vec{B}$ are applied simultaneously at the point 0, as shown in Figure 1. Arrows ordinarily are used over the letters to indicate that A and B are vector quantities.

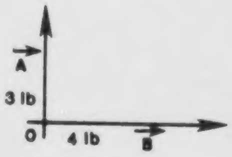


FIG. 1

GRAPHICAL SOLUTION

There are two methods of determining the resultant force graphically. As discussed in Concept Module -0, resultant force is that force that replaces the others, in this case A and B.

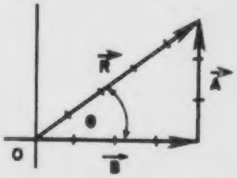


FIG. 2

1. Triangle method. Force $\vec{A}$ is laid off with its tail at the head of force $\vec{B}$, as in Figure 2, with some convenient scale.

   The closing side of the triangle, $\vec{R}$, represents the resultant. Magnitude of the resultant, R, can be measured with an appropriate scale, and the angle can be measured with a protractor. The same resultant would be obtained if force $\vec{B}$ were added to force $\vec{A}$, as in Figure 3, which indicates that the order of adding forces is not important, since the same resultant will always be obtained.

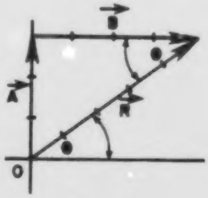


FIG. 3

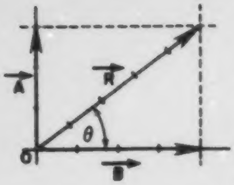


FIG. 4

2. Parallelogram method. To find the resultant force using this method, the given forces $\vec{A}$ and $\vec{B}$ are laid off to scale horizontal and vertical construction lines are drawn parallel to $\vec{A}$ and $\vec{B}$ as in Figure 4. The resultant, $\vec{R}$, will be the diagonal of the parallelogram, as shown. As in the triangle method, the magnitude of $\vec{R}$ and the angle θ can be measured with a scale and a protractor, respectively. In these examples, forces $\vec{A}$ and $\vec{B}$ were chosen to be 3 lb and 4 lb, respectively. In all the graphical solutions, the resultant R should be 5 pounds, and the angle θ should be 37°.

MATHEMATICAL SOLUTION

Because the forces $\vec{A}$ and $\vec{B}$ act at right angles to each other and can be added as shown in Figure 2, the resultant can be computed from the right triangle formed by $\vec{A}$, $\vec{B}$ and $\vec{R}$, as indicated by Equations 1 and 2.

$$R^2 = A^2 + B^2 \tag{1}$$

where

R = resultant.
A = vertical force.
B = horizontal force.

$$R = \sqrt{A^2 + B^2} \tag{2}$$

Example A shows the use of Equation 2 in solving a problem.

EXAMPLE A: TWO FORCES AT RIGHT ANGLES - MATHEMATICAL SOLUTION

Given: A vertical force A of 3 lb and a horizontal force B of 4 lb, as shown in Figure 1.

Find:
(a) The resultant R.
(b) The angle θ.

Solution: (a) The resultant R can be found by Equation 2 as follows:

$$R = \sqrt{A^2 + B^2}$$

Because A = 3 lb and B = 4 lb, Equation 2 becomes:

$$R = \sqrt{(3\ \text{lb})^2 + (4\ \text{lb})^2} = \sqrt{9\ \text{lb}^2 + 16\ \text{lb}^2}$$

$$R = \sqrt{25\ \text{lb}^2}$$
$$R = 5\ \text{lb}$$

(b) The angle θ can be computed from either its sine, cosine or tangent, as follows:

$$\sin\theta = \frac{A}{R} = \frac{3\ \text{lb}}{5\ \text{lb}} = 0.6$$
$$\cos\theta = \frac{B}{R} = \frac{4\ \text{lb}}{5\ \text{lb}} = 0.8$$
$$\tan\theta = \frac{A}{B} = \frac{3\ \text{lb}}{4\ \text{lb}} = 0.75$$

The angle θ for each of these values is shown from angle tables to be as follows:

$$\theta = 37^\circ$$

Notice that these results are in agreement with those obtained graphically. In summary, a force of 5 lb acting at an angle of 37° with the horizontal will produce the same effect as the two forces of 3 lb vertically and 4 lb horizontally. The resultant is not the arithmetical sum of 3 lb and 4 lb. In other words, the two forces are not equivalent to a single force of 7 lb.

TWO FORCES NOT AT RIGHT ANGLES

$\vec{C}$ and $\vec{D}$ in figure 5 represents two forces at an angle α whose resultant is to be determined. The forces are not at right angles to each other.

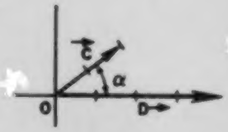


FIG. 5

GRAPHICAL SOLUTION

The two graphical methods used in the previous case will be applied here, as the techniques are very similar.

1. Triangle method. Choose a convenient scale and lay off force $\vec{C}$ with its tail at the head of force $\vec{D}$ as in Figure 6a. The force $\vec{D}$ can also be drawn with its tail at the head of $\vec{C}$ as in Figure 6b with the same results. The resultant $\vec{R}$ and the angle θ can then be measured with a scale and protractor respectively.

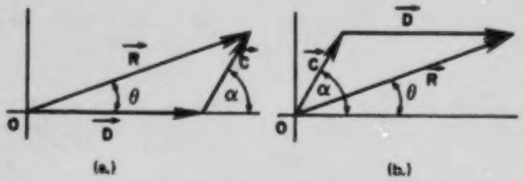


FIG. 6

2. Parallelogram method. Draw dashed construction lines parallel to $\vec{C}$ and $\vec{D}$, as in Figure 7. The resultant is the diagonal of the parallelogram as shown in this figure. The resultant $\vec{R}$ and the angle θ can be determined by using a

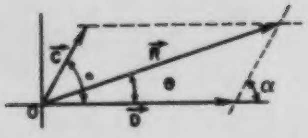


FIG. 7

scale and a protractor, respectively. In these examples, forces $\vec{C}$ and $\vec{D}$ are 2 lb and 4 lb, respectively, and are applied at 45° to each other. In all of the graphical solutions, the resultant R is found as 5.60 lb and the angle θ, 14.6°.

MATHEMATICAL SOLUTION

For two forces not at right angles, there are two mathematical approaches that can be used.

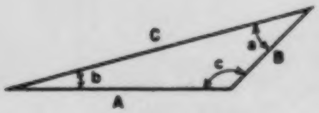


FIG. 8

1. Method law of sines and cosines. The law of sines and cosines for the oblique triangle in Figure 8 are indicated by Equations 3, 4 and 5.

$$C^2 = A^2 + B^2 - 2AB \cos C \quad \text{law of cosine} \tag{3}$$

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c} \quad \text{law of sines} \tag{4}$$

The law of cosines to the triangle that was formed in Figure 6a yields the results indicated in Equation 5.

$$R^2 = C^2 + D^2 - 2CD \cos (180° - \alpha) \tag{5}$$

These equations are used to solve a problem in Example B.

EXAMPLE B: TWO FORCES NOT AT RIGHT ANGLES - MATHEMATICAL SOLUTIONS

Given: The following values for use with Figure 6a: C = 2 lb, D = 4 lb and α = 45°.

Find: (a) The resultant R^2.
(b) The angle θ.

Solution: (a) The resultant R^2 can be found by Equation 5 as follows:

$$\begin{aligned} R^2 &= C^2 + D^2 - 2CD \cos (180° - \alpha) \\ &= (2)^2 + (4)^2 - 2(2)(4) \cos (180° - 45°) \\ &= 4 + 16 - 16 \cos 135° \end{aligned}$$

$$R^2 = 20 - 16\ (-0.707)$$
$$R^2 = 20 + 11.3 = 31.3$$
$$R = \sqrt{31.3} = 5.59 \text{ lb}$$

(b) The angle θ in Figure 6a can be computed from the law of sines as follows:

$$\frac{C}{\sin\theta} = \frac{R}{\sin\ (180° - \alpha)}$$
$$\sin\theta = \frac{C}{R}\sin\ (180° - \alpha)$$
$$\sin\ \ = \frac{2}{5.59}\sin\ (180° - 45°)$$
$$\sin\theta = 0.2530$$
$$\theta \simeq 14.6$$

2. Rectangular resolution method. The method of law of sines and cosines for finding the resultant is often awkward and, when applied to more than two forces, becomes very difficult to manage. The rectangular resolution method to be explained here is easier to handle, and can be used with any number of forces. Indeed, this method is probably the best for computational purposes to determine the resultant force in any situation; it can be outlined in 3 steps:

a. Resolve all forces into their rectangular components along the X and Y axis.
b. Find the algebraic sum of all X and Y components. The problem now has been reduced to one net force along X and one net force along Y.
c. Combine these two net forces to find the resultant, as for a right triangle explained in Figure 2 or 3.

As an example, the resultant of the two forces C and D used previously are computed in Example C.

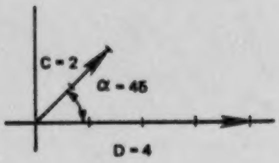
EXAMPLE C: TWO FORCES NOT AT RIGHT ANGLES - THE RECTANGULAR RESOLUTION METHOD

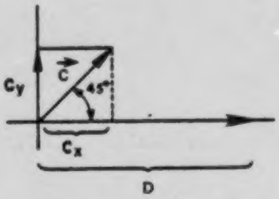
Given: The following values (same as in Example B) for use with first two sketches below: C = 2 lb, D = 4 lb and $\alpha = 45°$.

Find: (a) The resultant R.
(b) The angle θ.

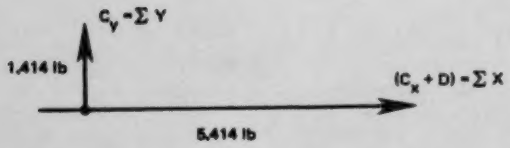
Solution: (a) The resultant R can be found with the rectangular resolution method as indicated below:

Since force D already lies on the X axis (first adjoining sketch), resolve force C into its X and Y components (second adjoining sketch). X components directed to the right are usually considered positive and those to the left negative. Similary, Y components directed upward are positive, and those downward are negative.

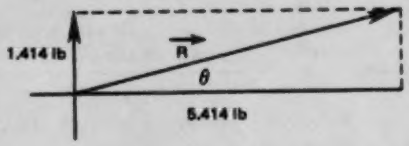




The X components of the 2-lb $\vec{C}$ force is 2 cos 45° = 2 (0.707) = 1.414 lb. This component added to the 4-lb $\vec{D}$ force already in the positive X direction yields a net force of 1.414 lb + 4 lb = 4.414 lb to the right. The Y component of the 2-lb $\vec{C}$ force is cos 45° = 2 (0.707) = 1.414 lb upward. Because the 4-lb force has no vertical component, the algebraic sum of all the Y components is simply 1.414 lb. These resultant X and Y components are shown in adjoining sketch.



The symbols ΣX and ΣY stand for the sum of the X and Y components, respectively. The resultant now can be calculated as before, as shown in adjoining sketch.



$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2}$$

$$= \sqrt{5.414^2 + 1.414^2} = \sqrt{29.31 + 2.000}$$

$$= \sqrt{31.31}$$

$$R = 5.60 \text{ lb}$$

This value agrees with the result previously calculated.

(b) The angle θ can be computed from any of the trigonometric functions, but only one will be used here.

$$\tan \theta = \frac{1.414 \text{ lb}}{5.414 \text{ lb}}$$
$$= 0.2611$$
$$\theta = 14.6°$$

θ also agrees with the result previously calculated.

BOTH FORCES IN THE SAME LINE

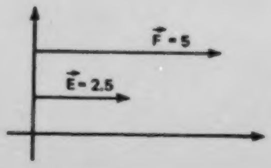
When both forces lie in the same straight line, the triangle shown in Figure 6a can be represented as flattening out into a straight line. The adding and subtracting of numbered line segments was used in Example C to obtain resultant X and Y components. All these forces are displaced slightly in sketches of Example D in order that they may be observed. When both forces are in the same line, the resultant R can be found as shown in Example D.

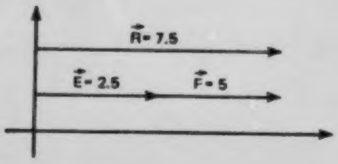
EXAMPLE D: BOTH FORCES IN THE SAME LINE

Given: Adjoining sketches and the following values: Force $\vec{E}$ = 2.5 lb, and Force $\vec{F}$ = 5 lb.

Find: Resultant $\vec{R}$.

Solution: $\vec{R}$ is therefore simply 2.5 lb + 5 lb = 7.5 lb, in the positive X direction. If the forces are in the same line, but are oppositely directed as in Figure 9, the results are obtained in a similar fashion. In this case, the resultant $\vec{R}$ is 2.5 lb - 5 lb = -2.5 lb. The minus sign simply indicates that the resultant is directed in the negative X direction.





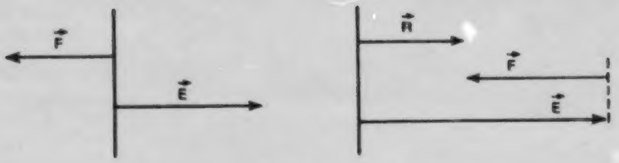


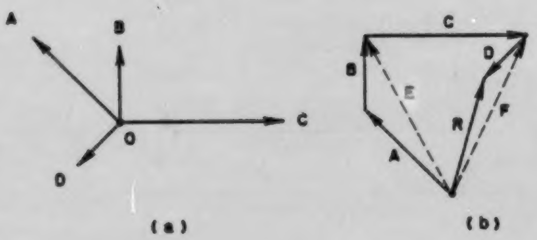
FIG. 9

MORE THAN TWO FORCES

When the resultant of two or more forces is being computed graphically or mathematically, methods are essentially the same as in Example D. This section shows how these methods can be extended to handle several forces acting together.

GRAPHICAL SOLUTION - POLYGON METHOD

As more than two forces are being resolved, the triangle method in reality becomes the better-known polygon method. This process, illustrated in Figure 10a, shows four forces $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$ acting concurrently at a point O. In Figure 10b, forces $\vec{A}$ and $\vec{B}$ are combined graphically, giving the resultant $\vec{E}$. The resultant force $\vec{E}$ is then combined graphically with force $\vec{C}$, giving the resultant $\vec{F}$. Finally, force $\vec{F}$ is combined graphically with force $\vec{D}$, giving the final resultant $\vec{R}$. This process could be simplified, because forces $\vec{E}$ and $\vec{F}$ need not have been drawn. The given forces are drawn in succession, with the tail of each at the head of the force preceding it. The resultant is then the force that completes this polygon, and is drawn from the tail of the first force to the head of the last. To illustrate that the order of adding the forces is unimportant, they are added in a different sequence in Figure 11. The same resultant would occur for any other sequence. For an interesting exercise, this claim should be verified. When the resultant has been obtained in this manner, its magnitude R and its direction θ can be determined with the aid of a scale and a protractor.



(a) (b)

FIG. 10

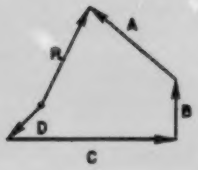


FIG. 11

MATHEMATICAL METHOD - RECTANGULAR RESOLUTION

Because the "law of sine and cosine method" requires computations involving oblique triangles and is awkward, only the "method of rectangular resolution" will be shown. The resultant of the four forces shown in Figure 10a will be computed in Example E. The magnitude and angles of these forces are indicated in Figure 12a.

Rectanglular resolution is demonstrated in Example E.

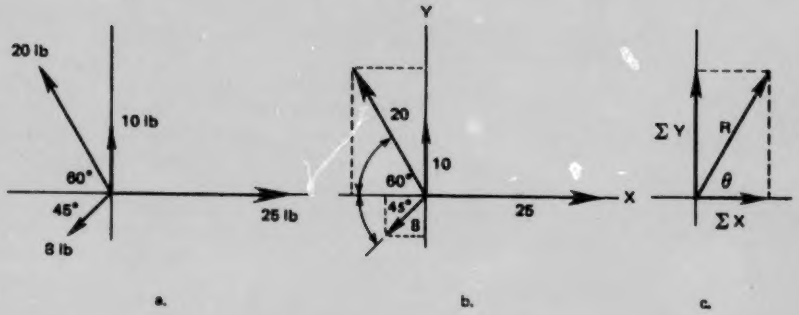


FIG. 12

EXAMPLE E: RECTANGULAR RESOLUTION OF MORE THAN TWO FORCES

Given: The below values for use with Figure 12b, which resolves each force into X and Y components.

Force Vector	X-component	Y-component
10 lb	0	+ 10 lb
25 lb	+ 25 lb	0
20 lb	-20 cos 60 = - 10 lb	+20 sin 60 = 17.32 lb
8 lb	-8 cos 45 = -5.66 lb	-8 sin 45 = -5.66 lb

Find:
(a) Algebraic sum of X and Y components.
(b) The resultant R.

Solution:
(a) The algebraic sum of the X component forces is 9.34 lb directed toward the right, and the algebraic sum of the Y component forces is 21.66 lb directed upward.

(b) Magnitude and direction of the resultant can be determined as before for a right triangle as shown in Figure 12c.

$$R = \sqrt{(\Sigma X)^2 + (\Sigma Y)^2} = \sqrt{(9.34)^2 = (21.66)^2} = \sqrt{556.39}$$

$$R = 23.6 \text{ lb}$$

$$\tan\theta = \frac{\Sigma Y}{\Sigma X}$$

$$= \frac{21.7 \text{ lb}}{9.34 \text{ lb}}$$

$$\tan\theta = 2.32.$$

$$\theta = 66.7°$$

These results are accurate. The results obtained graphically will agree well with these results if the drawings and measurements with scale and protractor are carefully done.

APPLICATION TO THE VELOCITY VECTOR

As previously stated, vector methods outlined in this module are not restricted to force vectors, but apply as well to any other vector quantities. Following is a typical example in which the vector is quantity of interest in velocity.

Suppose a pilot flies in a NE direction (45° north of east) at an indicated airspeed of 160 mi/h. During the flight, a wind is blowing at 50 mi/h due south. What is the resultant velocity of the airplane with respect to the ground?

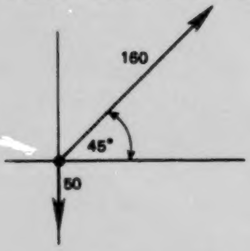
Vectorial solution to a velocity problem is shown in Example F.

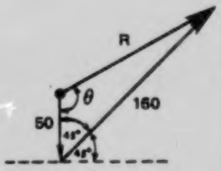
EXAMPLE F: VECTOR APPLIED TO VELOCITY

Given: A pilot flies in a northeastern direction (45° north of east), at an indicated airpseed of 160 mi/h. During the flight, a wind is blowing at 50 mi/h due south.

Find: The resultant velocity of the airplane with respect to the ground.

Solution: The first adjoining sketch shows the two velocities in the directions specified. The second is the vector triangle that must be solved for the resultant velocity R. Use the cosine law for the magnitude of R:





$$R^2 = 50^2 + 160^2 - 2(50)(160)\cos 45°$$

$$R^2 = 16786$$

$$R = 129.6 \text{ mi/h}$$

Use the sine law for the direction of R:

$$\frac{\sin\theta}{\sin 45} = \frac{160}{129.6}$$

$$\sin\theta = \frac{160}{129.6}(\sin 45°)$$

$$= 0.8730$$

$$\theta = 60.8°$$

The aircraft would be traveling at 129.6 mi/h in a direction 60.8° north of east. Vectors are used widely in navigation.

SUMMARY

Two or more forces (qualities of push or pull upon an object) can be exerted at right angles, at other than right angles or along the same line. The "resultant" is the resulting direction and magnitude of force, and can be determined mathematically or graphically. Many systems in technology and industry rely upon vectorial solutions of force problems and of numerous other problems.

DETERMINING RESULTANTS OF FORCES

MATERIALS

Scale.
Protractor.
Force table and accessories.

PROCEDURE

1. Level force table; attach weights; position pulleys to arrange force table (Figure 13) for Problem 1 as shown in Data Table.

2. Experimentally determine angle and weight required to balance given forces in order that small ring remains stationary when pin is removed. Because system is balanced, resultant of forces is equal in magnitude to this weight, but is oppositely directed. Record both magnitude and direction of resultant in appropriate boxes in Data Table for Problem 1.

3. Graphically determine magnitude and direction of resultant; record in appropriate boxes.

4. Mathematically determine magnitude and direction of resultant; record in appropriate boxes.

5. Repeat steps 1, 2, 3 and 4 for Problems II, III and IV as illustrated in Data Table.

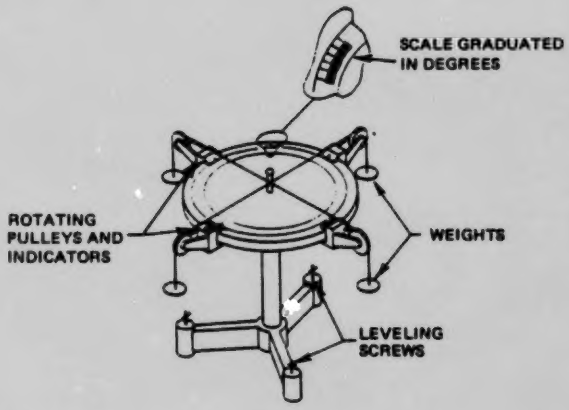
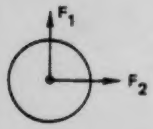


FIG. 13

6. Do the experimental, graphical and/or mathematical resultants for any one of the four problems differ? If so, why?

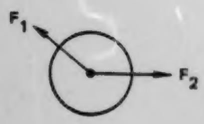
DATA TABLE



Problem I

	Gram Weights	Angle
F_1	150	90°
F_2	300	0°

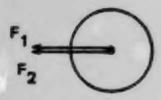
	Resultant
Experimental	
Graphical	
Mathematical	



Problem II

	Gram Weights	Angle
F_1	100	120°
F_2	350	0°

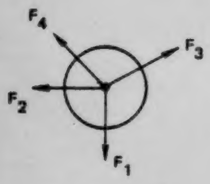
	Resultant
Experimental	
Graphical	
Mathematical	



Problem III

	Gram Weights	Angle
F_1	200	180°
F_2	150	180°

	Resultant
Experimental	
Graphical	
Mathematical	



Problem IV

	Gram Weights	Angle
F_1	200	270°
F_2	400	180°
F_3	50	45°
F_4	150	150°

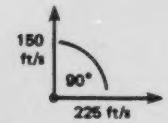
	Resultant
Experimental	
Graphical	
Mathematical	

SELF CHECK

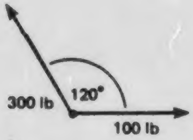
1. Define:

Contact forces.
Action-at-a-distance forces.
Concurrent forces.
Coplanar forces.
Resultant

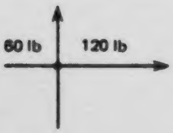
2. Determine the resultant of concurrent forces, graphically and experimentally for the following:


150
ft/s
90°
225 ft/s

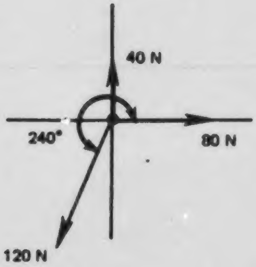
a.


300 lb
120°
100 lb

b.



c.

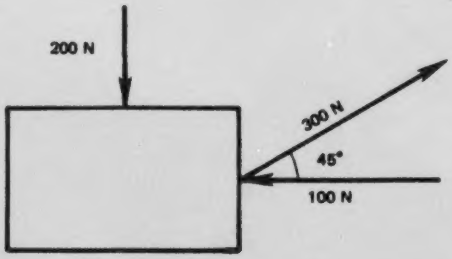

40 N
240°
80 N
120 N

d.

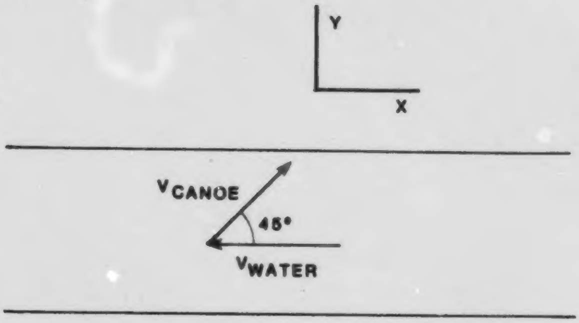
(e) 25 ft at 30°.
32 ft at 120°.
15 ft at 300°.

(f) 4.2 N at 37°.
6.8 N at 58°.
1.3 N at 198°.

3. What is the magnitude and direction of the force which must be applied to the block in the figure below in order to make the resultant force on the block zero?


200 N
300 N
45°
100 N

4. A man paddles a canoe diagonally across a river (see figure). He is paddling at a rate that would make the canoe travel 2 m/s in still water. The current in the river is 1 m/s. The canoe is traveling at an angle of 45° upstream. What is the velocity of the canoe with respect to the banks of the river?



1M3

ADDITION OF TORQUES

Torque (moment) is the force-like quality in the mechanical rotational system. The twisting action of torque results when all forces acting upon a body do not act through a single point; that is, force is applied off the center of mass, causing rotation. Normally, the word "torque" is applied to this rotational movement, and the term "moment" is employed to describe the product of the force and its moment arm (perpendicular distance from axis to point at which force is applied). Rotational equilibrium is the state at which the forces applied will not cause rotation about an axis.

In this module, the student will use the concept of torque to solve problems involving torque and equilibrium. In the laboratory, the student will determine the mass of an object from the forces applied to it, and will determine the position required to balance it, or bring it to equilibrium.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:

  Torque.
  Moment of a force.
  Moment arm.
  Center of gravity.

- Describe in words and with equations the two conditions necessary for equilibrium of a body.

- Given the forces acting on a body and the moment arm of each force, calculate the force and moment arm necessary to establish equilibium.
- Given the appropriate equipment, experimentally verify the solution to equilibrium problems and compare the calculated and experimental results.

SP

DISCUSSION

The first condition for equilibrium is given by Equation 1.

$$\Sigma\vec{F} = 0 \tag{1}$$

where
$\Sigma\vec{F}$ = the vector sum of forces acting upon a body.

When this condition is satisfied, the body is in "translational equilibium"; that is, the forces will not cause it to translate as a whole along a fixed direction. Figure 1 shows a mechanical system for which this condition is met. This system is not, however, in complete equilibrium. The two forces have equal magnitudes and opposite directions, but they do not act through the same point. As a result, the two forces cause a rotation of the body. In this figure, point P is an axis perpendicular to the paper about which the body is free to rotate. The torque (or moment) of a force is given by Equation 2. The word "torque" normally is used when movement results, and the word "moment" (or torque) of a force about an axis is the product of the force and its lever arm.

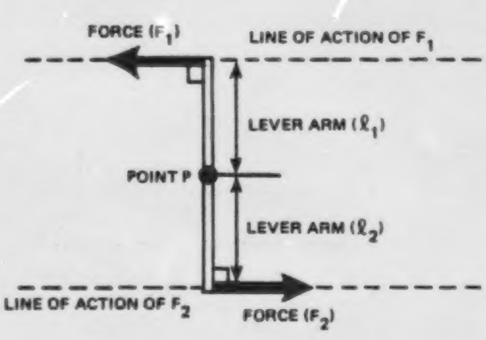


FIG. 1

$$\tau = F\ell \tag{2}$$

where
τ = torque.
F = force.
ℓ = lever arm.

The lever, or moment, arm is the perpendicular distance from the line of action of the force to the axis, or pivot point. Since a clockwise rotation is positive, the total torque for this system is indicated by Equation 3, in which both torques cause counterclockwise rotations.

$$\Sigma\tau = (-F_1\ell_1) + (-F_2\ell_2) \tag{3}$$

where

$\Sigma\tau$ = total torque.
F_1 = force 1.
F_2 = force 2.
ℓ_1 = lever arm 1.
ℓ_2 = lever arm 2.

The second condition of equilibrium is given in Equation 4, in which the sum of the moments on a body about any axis is zero.

$$\Sigma\tau = 0 \tag{4}$$

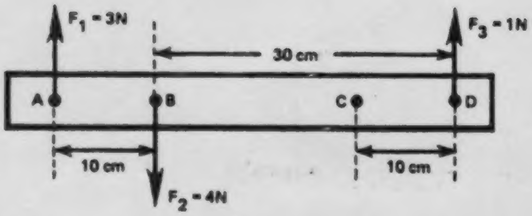


FIG. 2

When this condition is satisfied, the body is in "rotational equilibrium"; that is, the forces will not cause it to rotate about any fixed axis. When both conditions for equilibium (Equation 1 and Equation 4) are met, the forces will not cause either translation or rotation of the body.

Figure 2 shows a system which meets both conditions for equilibrium. Equilibrium in this situation can be verified by the use of Equations 1, 2 and 4 in Example A.

EXAMPLE A : EQUILIBRIUM SYSTEM CALCULATION

Given: The mechanical system in Figure 2.

Find: If the system is in eqiulibium.

Solution:
1. $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$ (Define up to be positive.)
   $= 3\ \text{N} + (-4\ \text{N}) + 1\ \text{N}$
   $\Sigma\vec{F} = 0.$ (First condition for equilibium is met.)
2. $\Sigma\tau = F_1\ell_1 + F_2\ell_2 + F_3\ell_3$ (Define clockwise to be positive.)
   (a) Choose point A as the pivot point - the axis.
   $\Sigma\tau = (3\ \text{N})(0\ \text{M}) + (+4\ \text{N})(0.1\ \text{M}) + (-1\ \text{N})(0.4\ \text{M})$
   $= +0.4\ \text{N}\cdot\text{m} - 0.4\ \text{N}\cdot\text{m}$
   $\Sigma\tau = 0$ (Second condition for equilibium is satisfied.)

Notice that F_1 causes no rotation about point A, F_2 gives a positive torque and F_3 gives a negative torque.

(b) Choose point D as the pivot point.
$\Sigma\tau = (3\ \text{N})(0.4\ \text{m}) - (4\ \text{N})(0.3\ \text{m}) + (1\ \text{N})(0\ \text{m})$
$(1.2\ \text{N}\cdot\text{m} - 1.2\ \text{N}\cdot\text{m}$
$\Sigma\tau = 0$

(c) Choose point C as the pivot point.
$\Sigma\tau = (3\ \text{N})(0.3\ \text{m}) - (4\ \text{N})(0.2\ \text{m}) - (1\ \text{N})(0.1\ \text{m})$
$= 0.9\ \text{N}\cdot\text{m} - 0.8\ \text{N}\cdot\text{m} - 0.1\ \text{N}\cdot\text{m}$
$\Sigma\tau = 0$

Notice that F_1 causes a clockwise rotation about point C and gives a positive torque. F_2 and F_3 both cause counterclockwise rotations and give negative torques.

Example A shows that if a body is in rotational equilibrium about some axis, it is in equilibrium about any axis; however, it is only necessary to calculate torques about one point. In systems in which a fixed axis of rotation is defined, that axis is usually chosen. (Any convenient point may be used in testing for equilibrium of torques.) In systems involving the action of gravitational forces, the center of gravity may be chosen.

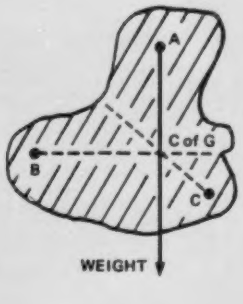
The center of gravity (C of C) of an object generally is defined as that point at which the entire weight of an object can be considered to be concentrated. An object will balance if supported or hung from its center of gravity. Example B shows how to find the C of G of nonuniform density body of irregular shape.

EXAMPLE B: CALCULATION OF IRREGULAR BODY C OF G

Given: The irregular object shown in the figure.

Find: C of G.

Solution: (a) Suspend the object near the periphery point A; drop a weighted line from A and mark its location. (b) Suspend the object near the periphery, say point B; drop a weighted line from B, and mark where it crosses line A. (c) Suspend the object near the periphery, say point C; drop a weighted line from C, and it should cross the intersection of lines A and B at the C of G.

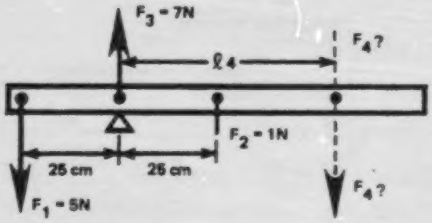


Example C is another problem using Equations 1, 2 and 4 in the solution. In this example, "down" and "counterclockwise" were chosen as positive. This was done to stress the point that positive and negative directions may be defined in any way, as long as they are consistent within any single problem.

EXAMPLE C: EQUILIBRIUM BEAM CALCULATION

Given: The system shown in figure.

Find: F_4 and ℓ_4 that will cause the system to be in equilibrium about Δ.



Solution: (a) $\Sigma\vec{F} = 0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$\vec{F}_4 = -\vec{F}_1 - \vec{F}_2 - \vec{F}_3$ (Define down to be positive.)

$= -(5\ N) - (1\ N) - (-7\ N)$

$= -5\ N - 1\ N + 7\ N$

$\vec{F}_4 = 1\ N$. (F_4 is 1 N downward.)

(b) $\Sigma\tau = 0 = F_1\ell_1 + F_2\ell_2 + F_3\ell_3 + F_4\ell_4$. [Define ccw (counterclockwise) as positive and the fulcrum as reference point.]

$$0 = (5\ N)\ (25\ cm) - (1\ N)\ (25\ cm) + (7\ N)\ (0) + (1\ N)\ (\ell_4)$$

$$= 125\ N{\cdot}cm - 25\ N{\cdot}cm + 0 - 1\ N \times \ell_4$$

$$0 = 100\ N{\cdot}cm - 1\ N \times \ell_4$$

$$-100\ N{\cdot}cm = 1\ N \times \ell_4$$

$$\frac{-100\ N{\cdot}cm}{-1\ N} = \ell_4$$

$$100\ cm = \ell_4$$

$$1\ m = \ell_4$$

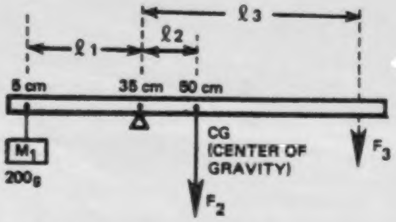
Since ℓ_4 equals one meter, F_4 is one meter to the right of the fulcrum.

Example D shows the solution to an additional equilibrium problem. Since mass units (grams) are given rather than force units (newtons), the acceleration due to gravity (g) is used in the solution even though it divides out.

EXAMPLE D: EQUILIBRIUM BEAM CALCULATION

Given: The system shown in figure.

Find: Where a 100 g weight should be placed to achieve equilibrium.



M_2 = MASS OF METER STICK = 50 g
CG = CENTER OF GRAVITY

Solution: $m_1 = 200$ g, $m_2 = 50$ g, $m_3 = 100$ g
$\ell_1 = 30$ cm, $\ell_2 = 15$ cm, $\ell_3 = ?$ (Choose fulcrum as reference point.)
$\Sigma\tau = 0 = F_1\ell_1 + F_2\ell_2 + F_3\ell_3 + F_4\ell_4$
$0 = -m_1g\ell_1 + m_2g\ell_2 + m_3\ell_3g + F_4\ell_4$ (F_4 is upward force at fulcrum since ℓ_4 is zero; F_4 need not be calculated.)
$= g\,(-m_1\ell_1 + m_2\ell_2 + m_3\ell_3)$ (Since g is in every term, it can be divided out of the equation.)
$= -\,(200\text{ g})(30\text{ cm}) + (50\text{ g})(15\text{ cm}) + (100\text{ g})(\ell_3)$
$= -6000\text{ g}\cdot\text{cm} + 750\text{ g}\cdot\text{cm} + 100\text{ g }\ell_3$
$0 = -5250\text{ g}\cdot\text{cm} + 100\text{ g }\ell_3$
$100\text{ g }\ell_3 = 5250\text{ g}\cdot\text{cm}$

$$\ell_3 = \frac{5250\text{ g}\cdot\text{cm}}{100\text{ g}}$$

$\ell_3 = 52.5$ cm

The 100 g weight should be placed 52.5 cm to the right of the reference point (fulcrum) or at the 87.5 cm mark.

In Example D, all forces have been applied in a "single" direction; that is, either up or down. In many "real" problems this is not the case. Figure 3 shows the forces F_1, F_2 and F_3, acting on a door, together with their X and Y components, and their lever arms.

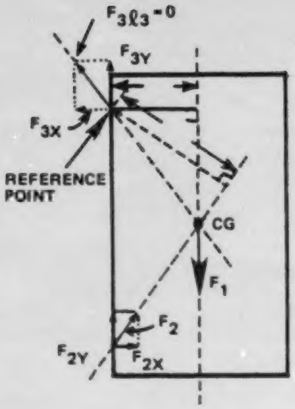


FIG. 3

The reference point chosen in Figure 3 is the upper hinge, and the equilibrium conditions are given by Equations 5, 6 and 7:

$$\Sigma F_X = 0 = F_{2X} - F_{3X} \tag{5}$$

where
X = equilibrium of X-component forces.

$$\Sigma F_Y = 0 = F_{2Y} + F_{3Y} - F_1 \tag{6}$$

where
Y = equilibrium of Y-component forces.

$$\Sigma \tau = 0 = F_1\ell_1 - F_2\ell_2 + F_3\ell_3 \tag{7}$$

Equilibium of torques is shown by Equation 7.

SUMMARY

Torque is the twisting, force-like quality in the mechanical rotational system. Torque is produced when all forces acting upon a body do not act through a single point, but act off the axis, causing rotation. The moment, or lever, arm is the point at which force is applied perpendicularly away from the axis. Rotational equilibrium is the state at which the forces applied will not cause rotation.

MASS OF METER STICK

MATERIALS

Meter stick (weight clamps optional).
String.
Set of metric weights.
Stand or mount on which system can be hung.

PROCEDURES

1. Suspend a 100-g weight from 10-cm point on meter stick with a small piece of string.
2. Tie a loop in a short length of string, and slip it over free end of meter stick. Exert an upward force on string. This string is shown as "F" in Figure 4. Move the loop of string to the position which balances the meter stick while hanging freely.
3. Record all necessary data in Data Table, and calculate mass of meter stick.

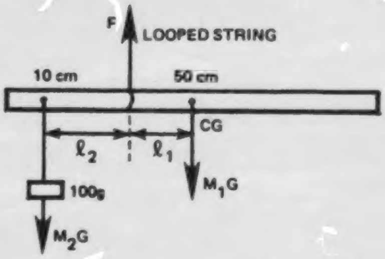


FIG. 4

MASS NECESSARY TO BALANCE METER STICK

MATERIALS

Meter stick (weight clamps optional).
String.
Set of metric weights.
Stand or mount on which system can be hung.

PROCEDURE

1. Hang weights from meter stick, as shown in Figure 5, and place string supporting meter stick at the 65-cm mark.

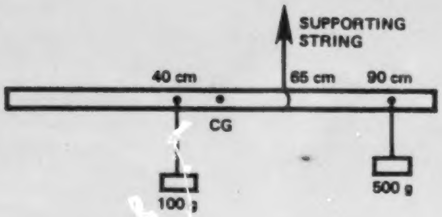

SUPPORTING STRING
40 cm
65 cm
90 cm
CG
100 g
500 g

FIG. 5

2. Calculate mass placed at the 15-cm mark necessary to balance meter stick, and record in Data Table.

3. Hang weights at 15-cm mark on stick until it is balanced. Record this mass as that necessary to balance experimentally this system.

4. What is the relationship between masses and moment levers?

POSITION OF GIVEN MASS TO BALANCE METER STICK

MATERIALS

Meter stick (weight clamps optional).
String.
Set of metric weights.
Stand or mount on which system can be hung.

PROCEDURE

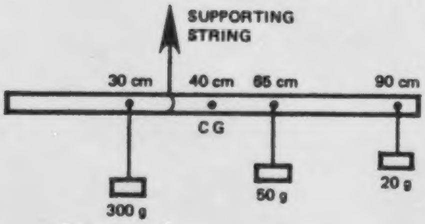

SUPPORTING STRING
30 cm
40 cm
65 cm
90 cm
CG
300 g
50 g
20 g

FIG. 6

1. Assemble experimental setup as shown in Figure 6.

2. Calculate location of 200-g weight that balances the stick. Record in Data Table.

3. Hang a 200-g weight from stick and move it until a balance is achieved experimentally. Record this position in Data Table.

4. In what state are torque forces?

DATA TABLE

Procedure 1

Mass of weight : $m_2 = 100$ g.
Position of looped string on meter stick that causes blance ________ cm.
Distance from looped string to center of gravity of stick: $\ell_1 =$ ________ cm.
Distance from looped string to weight: $\ell_2 =$ ________ cm.
Since $\Sigma\tau = 0$, $m_1 g \ell_1 = m_2 g \ell_2$,

Mass of meter stick: $m_1 = \frac{m_2 \ell_2}{\ell_1}$
= (______ g)(______ cm) ÷ (______ cm)
= ______ g.

Procedure 2

Mass calculated to balance meter stick: ______ g.
Mass that balances stick experimentally: ______ g.

Procedure 3

Location calculated to balance meter stick: ______ g.
Location that balances stick experimentally: ______ g.

SELF CHECK

1. Identify each of the following:
   a. The point at which the entire weight of an object can be considered to be concentrated.
   b. The perpendicular distance from the line of action of a force to the pivot point or axis of rotation.
   c. The product of a force and its lever arm when no motion occurs.
   d. The product of a force and its lever arm when rotational motion results.
2. Describe the two conditions necessary for equilibrium of a body.
3. A meter stick with a mass of 162 g is suspended by a string from the 40-cm mark. The following weights are hung from

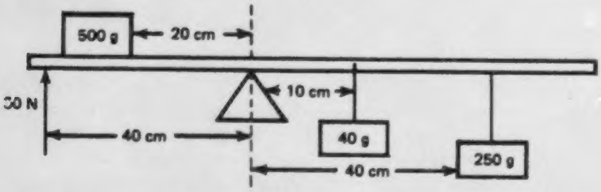
the stick at the specified locations. Find the missing quantity for equilibrium in each case.

a. $m_1 = 100$ g $\quad m_2 = 50$ g
$X_1 = 30$ cm $\quad X_2 =$ ______ cm

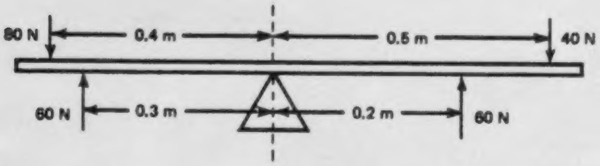
b. $m_1 = 500$ g $\quad m_2 =$ ______ g
$X_1 = 40$ cm $\quad X_2 = 10$ cm

c. $m_1 = 100$ g $\quad m_2 = 200$ g $\quad m_3 = 50$ g
$X_1 = 10$ cm $\quad X_2 = 50$ cm $\quad X_3 =$ ______ cm

4. What is the magnitude and direction of the net torque on the system shown below?

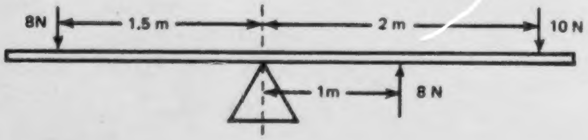


5. Is it possible to satisfy both conditions of equilibrium by applying one additional force to the system shown below? Justify your answer.



6. If two people try to push open a swinging door, from opposite sides, at the same time, which way will the door swing if one person pushes with a force of 20 N applied 30 cm from the hinge while the other person pushes with a force of 8 N applied 1 m from the hinge?

7. Is it possible to bring the system shown below to equilibrium by the addition of a single force? Justify your answer.



1M4

MECHANICAL EQUILIBRIUM IN STRUCTURES

In preceding modules, relationships of forces and motions were discussed. Force always tends to cause a change in motion of a body, and torques will cause a body to rotate about a selected point. Bridges, buildings, dams and houses are designed to withstand forces and torques of wind, earthquakes, internal weight and motion of water. Because forces and torques cause motion, it is the problem of the designer to distribute these forces and stresses in individual members that make up the structure so that they are all in equilibrium. This module considers application of forces and torques to everyday common structures to ensure that their design is within the safe limits of loading.

OBJECTIVES

To complete this module, the student should be able to:

- Define:
    - Static load.
    - Perfect structure.
    - Strut.
    - Tie.
    - Frame structure.
- Draw a simple loading diagram for a structural system, identifying load location and direction.
- Describe the two conditions of equilibrium and apply them to a perfect structure.

- Calculate forces in a perfectly framed structure using methods of vectors or parallelogram of forces covered in Modules 1M2 and 1M3.
- Perform the necessary laboratory procedure to find forces in the members of a given structure and verify by calculating the values to within ± 10% accuracy.

DISCUSSION

Bridges consist of structural members, as do roof trusses, box girders and steel framed buildings (Figure 1).

Each frame is to be considered as a series of structural members welded or pin-joined together in such a manner as to distribute the internal and external forces in the tension or compression loads (pull or push). Structures of this type resist static loads (dead loads) and dynamic loads (wind, cars, rain and snow). Tension members are called struts.

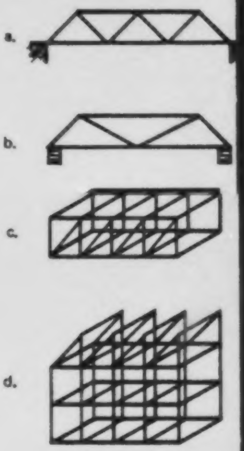


FIG. 1

In Figure 2, a simple frame structure is shown. Note the ties and struts (arrows indicate direction of resistance by the members). If the member AB (or AC) were cut, A would fall toward B (or C), denoting that members AB and AC are in compression. Such members are called struts. However, if member BC is cut, B would move away from C, and we call this a tension member or tie.

If a series of similar structures are connected and pin-joined together, a familiar structure is constructed, i.e., a bridge or roof truss as shown in Figure 1a and 1b respectively.

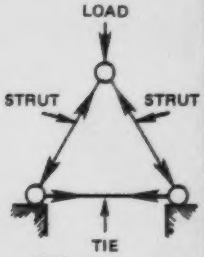


FIG. 2

Similarly, other familiar structures are shown in Figures 3 and 4. These will be analyzed using vector and mathematical solutions.

Each of the previous figures illustrates what is considered to be a perfect structure, that is one that has just sufficient members in it to keep it in equilibrium under the known system of loading. This could also mean that it is the most economic structure in terms of cost.

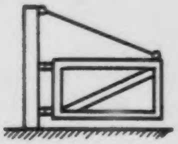


FIG. 3

LOADS IN A PERFECT STRUCTURE

All structural members must satisfy both conditions of mechanical equilibrium, i.e., $\Sigma F = 0$, $\Sigma \tau = 0$. By using either of the two methods covered in previous modules, graphical or mathematical (rectangular resolution method), the loads in a structural member can be found (see Examples A, B, C and D which follow). Consequently, size of the member can be chosen so that it can withstand the forces exerted upon it without deforming or breaking under the given load. Example A shows a simple structural member under the influence of three unknown forces T_1, T_2, and T_3. (Solution of this problem requires knowledge and formulas contained in Modules 1M2 and 1M3.)

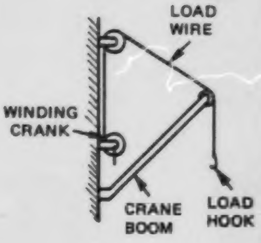
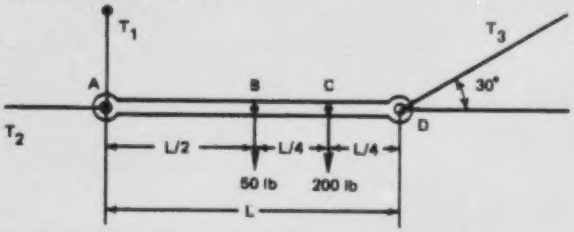


FIG. 4

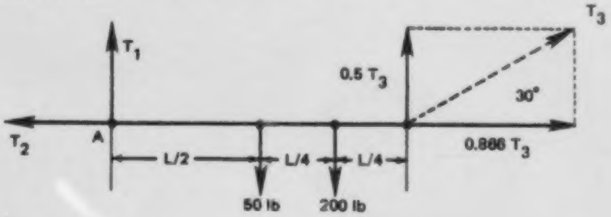
EXAMPLE A: SIMPLE STRUCTURAL MEMBER

Given: A uniform structural member (AD) with three rigid connecting members, T_1, T_2, and T_3 as shown in figure below. Each member weighs 50 lb. The member supports a load of 200 pounds at C.



Find: Tension in each member T_1, T_2, T_3.

Solution: To construct a force diagram, show the forces acting on the member AD and replace the force in T_3 by its vertical and horizontal components. This is shown below:



$\Sigma F_x = 0$:

$0.866\ T_3 - T_2 = 0.$ (1)

$\Sigma F_y = 0$:

$T_1 + 0.5\ T_3 - 50 - 200 = 0$ (2)

Choose point A as the rotational pivot as it will eliminate T_1 and T_2 from the equation of $(\Sigma\tau)_A$ and leave only T_3 to be solved.

From Solution figure of this example, $(\Sigma\tau)_A = 0$ gives

$$- 50\ \frac{L}{2} - 200\ \frac{3L}{4} + 0.5\ T_3\ L = 0 \quad (3)$$

Assume clockwise rotation is (-) and anticlockwise rotation is (+).

Force 0.86 T_3 is also eliminated because its line of action passes through the rotation at pivot point A and so has no lever arm.

Solve for T_3 by using Equation 3 after dividing both sides of the equation by L.

$$-25 - 150 + 0.5\ T_3 = 0$$

$$T_3 = 350\ \text{lb}.$$

Substituting T_3 in Equation 1

$$0.866 \times 350 = T_2$$

$$T_2 = 303.1 \text{ lb.}$$

Substituting T_3 in Equation 2

$$T_1 = -0.5 \times 350 + 50 + 200$$

$$= -175 + 250$$

$$T_1 = 75.$$

As all calculated forces are positive, their directions were correctly chosen in figure under Solution of this example.

Check: Sum torques about another point D:

$$(\Sigma\tau)_D = 0$$

$$+ T_1L - 50(\tfrac{L}{2}) - 200(\tfrac{L}{4}) = 0$$

$$\therefore T_1 - \frac{50}{2} \; \frac{-200}{4} = 0$$

$$75 - 25 - 50 = 0$$

$$75 - 75 = 0 \text{ (check).}$$

Example B shows the mathematical analysis of the loads in several members of a roof truss using a technique known as "method of sections." In this technique each point of the truss is isolated by cutting a plane through several of the members that meet at a selected point in such a fashion that no more than two unknown loads need to be found. From the basic equations of equilibrium, the horizontal and vertical forces may be resolved.

EXAMPLE B: ROOF TRUSS ANALYSIS

Given: A truss system as shown in figure at right.

Find: R_A, R_E, F_{AB}, F_{BH}.

Solution:

Total load on truss = 1500 + 1500

= 3000 lb (down).

This is supported by an upward reaction provided by the two supporting pillars. Because the bridge is symmetrical about the pillars,

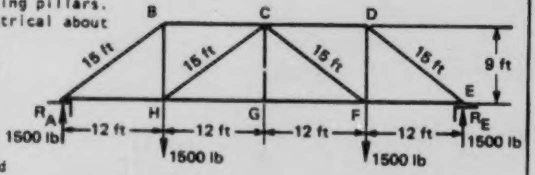
$R_A + R_E = 3000$ lb, and

$R_A = R_E$

therefore

$R_A = 1500$ lb upward and

$R_E = 1500$ lb upward, as shown in the Figure.


B
C
D
15 ft
15 ft
15 ft
15 ft
9 ft
E
R_A
H
G
F
R_E
1500 lb
12 ft
12 ft
12 ft
12 ft
1500 lb
1500 lb
1500 lb

Now, joint A may be isolated by making the first cut through $\overline{AB}$ and $\overline{AH}$, and shown as a free body diagram as in figure at right.

F_{AB} can be resolved into horizontal and vertical components as shown. From the similar triangle showing dimensions of the truss, we have

$\sin \theta = 9/15 = 3/5$

$\cos \theta = 12/15 = 4/5$

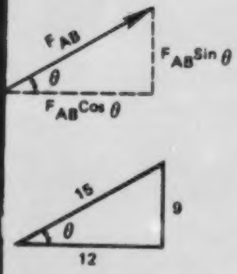
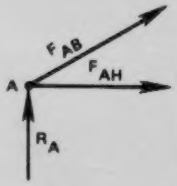

F_{AB}
$F_{AB}\sin\theta$
θ
$F_{AB}\cos\theta$
15
9
θ
12

FIG. 5

The force F_{AB} in member AB can be resolved into:

Vertical component = $F_{AB} \sin \theta = 3/5\ F_{AB}$.

Horizontal component = $F_{AB} \cos \theta = 4/5\ F_{AB}$.



The vertical and horizontal conditions of equilibrium are then:

$\Sigma F_H = 4/5\ F_{AB} + F_{AH} = 0.$

$\Sigma F_V = R_A + 3/5\ F_{AB} = 0.$

The second can be solved for F_{AB}:

$3/5\ F_{AB} = -R_A = -1500.$

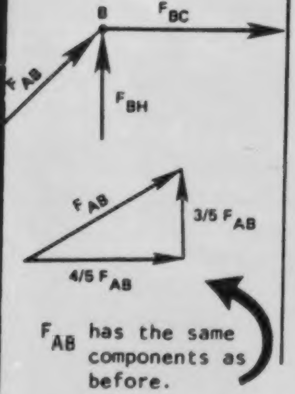
$F_{AB} = 5/3\ (-1500) = -2500$ lb.

The negative sign means that the direction of F_{AB} is opposite that shown.

The other equilibrium equation now yields a value for F_{AH}:

$F_{AH} = -4/5\ F_{AB} = -4/5\ (-2500) = 2000$ lb.

Next, joint B can be isolated and treated in a similar fashion. F_{AB} is now known to be 2500 lb. The force diagram is shown in margin.


B
F_{BC}
F_{AB}
F_{BH}
F_{AB}
3/5 F_{AB}
4/5 F_{AB}

F_{AB} has the same components as before.

FIG. 6

Equilibrium conditions are:

$\Sigma F_H = 4/5\ F_{AB} + F_{BC} = 0.$

$\Sigma F_V = 3/5\ F_{AB} + F_{BH} = 0.$

Using $F_{AB} = 2500$ lb, these equations become:

$2000 + F_{BC} = 0$

$1500 + F_{BH} = 0$

which give: $F_{BC} = -2000$ lb.

$F_{BH} = -1500$ lb.

Again, the negative signs simply mean the directions assumed for these forces on junction B were incorrect and should be reversed. Their magnitudes, however, are correct. The desired answers are F_{AB} = 2500 lb and F_{BH} = 1500 lb. These strut members must be built to withstand the tensile forces calculated.

In this way, continue to consider points C, H, G, etc., until the tension in each truss member is calculated.

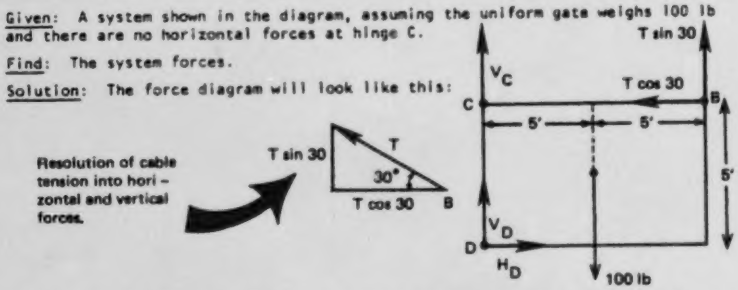
Example C shows the force diagram and a mathematical solution for a hinged fence gate that utilizes the second law of equilibrium, i.e., $\Sigma\tau = 0$.

EXAMPLE C: HINGED FENCE GATE

Given: A system shown in the diagram, assuming the uniform gate weighs 100 lb and there are no horizontal forces at hinge C.

Find: The system forces.

Solution: The force diagram will look like this:



Equilibrium of forces and torques acting on the gate require three conditions:

$$\Sigma F_H = H_D - T \cos 30° = 0. \tag{4}$$

$$\Sigma F_V = V_C + V_D + T \sin 30° - 100 = 0. \tag{5}$$

$$\Sigma \tau_D = (T \cos 30°)5 + (T \sin 30)10 - (5) = 0. \tag{6}$$

Equation 6 can be solved for T:

$$T\ (5 \cos 30° + 10 \sin 30°) = 500.$$

$$T\ (4.33 + 5) = 500.$$

$$9.33\ T = 500 \text{ or } T = 53.6 \text{ lb.}$$

Substituting this value into Equation 4 gives H_D:

$$H_D = T \cos 30° = 53.6\ (0.866).$$

$$H_D = 46.4 \text{ lb.}$$

Finally, Equation 5 can be solved for $(V_C + V_D)$, the sum of the vertical loads at the hinges:

$$V_C + V_D = 100 - T \sin 30° = 100 - (53.6)(0.5).$$

$$V_C + V_D = 73.2 \text{ lb.}$$

From this information for the loads, the hanging gate and post could be designed.

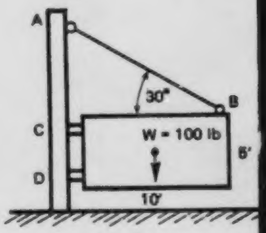
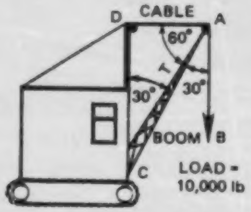


FIG. 7

Example D covers the mathematical analysis of another familiar structure, a crane boom. In addition, this problem may be solved using the graphical methods covered in Module 1M2, with particular reference to vector forces and triangle solutions.

EXAMPLE D: CRANE BOOM



Given: Crane system shown in the diagram.

Find: Tension in the crane cables.

Solution: The analysis of a crane boom would be made as follows: Assume the crane is lifting a weight of 10,000 lb. Calculate the tension in the cable F_H when the crane boom is at 30° to the vertical, as well as the tension T in the boom.

A force diagram would look like figure in margin.

For equilibrium, $F_H = T \sin 30 = 1/2\ T$

and $T \cos 30 = 10,000$.

Then: $T = \frac{10,000}{0.866} = 11,547$ lb

and $F_H = 1/2\ T = 1/2\ (11,547)$

$F_H = 5,774$ lb.

A crane boom is generally designed to withstand stresses of a maximum weight with a restricted angle of the boom to the vertical. If the boom were to exceed the angle, it could collapse due to higher bending stresses than it was designed for. As the crane boom angle increases, the load it is designed to carry must decrease. A safety alarm, fitted to the boom, will sound in the crane operator's cab when the limiting angle is approached.

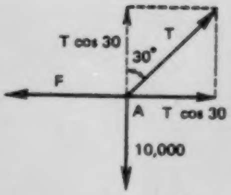


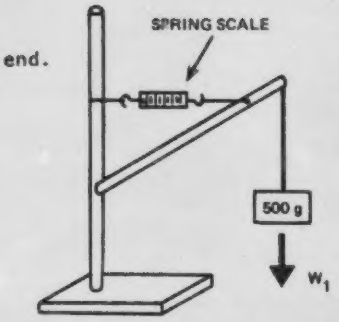
FIG. 8

When the load in a structural member has been ascertained, the member must be selected that can resist this load without deformation or failure. Module 1M5, "Stress and Strain," will explain the selection of suitable members for this purpose.

LABORATORY

MATERIALS

Spring scale with a range of 0-2000 g.
Support stand.
Wooden dowel 60 cm long with hole drilled 10 cm from one end.
1.5 m length of fine strong wire or string.
Protractor.
Two 500 g weights.
Triple beam balance.



CONTAINING SPRING SCALE

FIG. 9

PROCEDURE

1. Weigh dowel and record mass in grams in Data Table.
2. Set up the apparatus as shown in Figure 9 using a weight of 500 g.
3. Adjust length of string containing spring scale to bring it to true horizontal.
4. Use the diagram of forces on the dowel shown in Figure 10 to develop the equations to solve for T and F.
5. Measure and record the actual value of T.
6. Repeat with a weight W_1 of 1000 g and a larger angle θ.

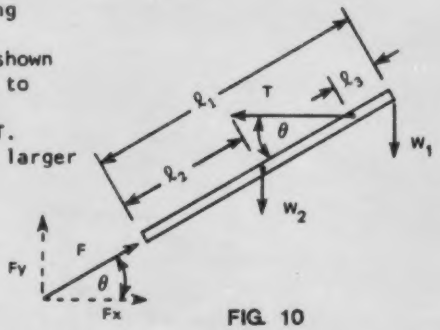


FIG. 10

DATA TABLE

Case 1

W_1 = ________ g ℓ_1 = ________ cm ℓ_3 = ________ cm

W_2 = ________ g ℓ_2 = ________ cm θ = ________

Calculated values: F_x = ________ g

F_y = ________ g

F = ________ g

T = ________ g

Measured value: T = ________ g

Case II

W_1 = ________ g ℓ_1 = ________ cm ℓ_3 = ________ cm

W_2 = ________ g ℓ_2 = ________ cm θ = ________

Calculated values: F_x = ________ g

F_y = ________ g

F = ________ g

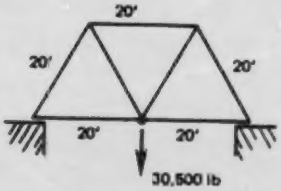
T = ________ g

Measured value: T = ________ g

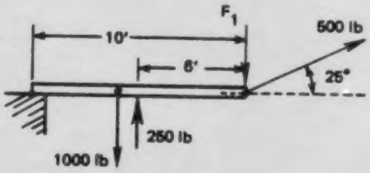
SELF CHECK

1. Define:
    - Static load.
    - Perfect structure.
    - Strut.
    - Tie.
    - Frame structure.

2. Draw simple loading diagrams for the following structural system, identify load and direction:

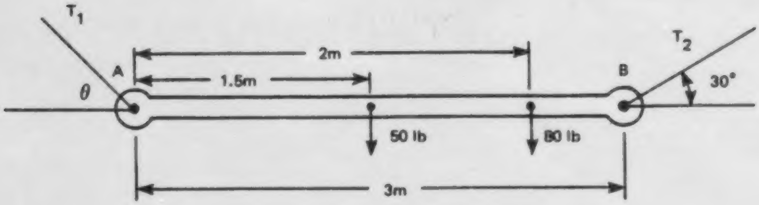


3. List the two conditions of equilibrium and apply them to the following structure.

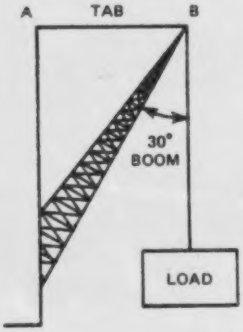


4. Calculate the forces in the framed structure given in Example 2 above, using methods of vectors.

5. A uniform structural member (AB) and two rigid-connecting members (T_1, T_2) are shown in the figure below. AB weighs 75 lb and supports a load of 80 lb as shown. Determine the tensions in T_1 and T_2 and the angle θ.


T1
2m
A
1.5m
B
T2
30°
θ
50 lb
80 lb
3m

6. Given the crane shown in the figure below. What load is the crane supporting by the tension in the cable AB if the compression on the boom is 12,000 lb?


A
TAB
B
30°
BOOM
LOAD

1M5

STRESS AND STRAIN

Module 1M4, "Structural Forces and Torques," develops the principles necessary for computing tensile and compressive forces in members of a structure. Practical use of this information requires knowledge of how large a member is needed to withstand these forces and how much the structure will distort under load. Elasticity is the study of the way actual materials are changed in shape by forces applied to them.

The elastic properties of materials are described in terms of two concepts known as "stress" and "strain." The concepts of stress and strain of elastic materials will be investigated in this module.

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Stress.
  - Strain.
  - Hooke's law.
  - Elastic limit.
  - Yield point.
  - Breaking point.
  - Modulus of elasticity.
  - Shear modulus.

  - Bulk modulus of elasticity.
  - Safety factor.
  - Ultimate breaking strength.
- Given appropriate forces and surface area of an object, calculate stresses produced.
- Given appropriate changes in dimensions of an object, calculate strain produced on that object.
- Given stress and strain on an object, derive the correct modulus of elasticity.
- Given the necessary equipment, perform the following laboratory procedures:
  - Experimentally verify Hooke's law.
  - Determine ultimate breaking strength of several different materials.
  - Measure deflection of a wood beam.

DISCUSSION

STRESS

If a body does not move away when a force or pressure is applied to it, it undergoes stress. No material is perfectly rigid, so the force will cause the material to deform. It will bend, twist, compress or change in some manner.

Stress depends on two factors: the force applied and the area over which the applied force is distributed. This relation is given by Equation 1.

$$\text{Stress} = S = \frac{F}{A} = \frac{\text{force applied}}{\text{area of application}}. \qquad (1)$$

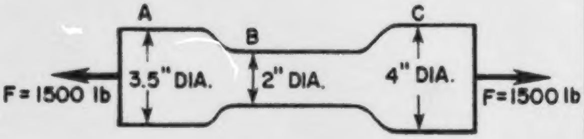
Example A illustrates use of this equation.

EXAMPLE A: STRESS PRODUCED IN A ROD OF VARYING DIMENSION

Given: A bar of size shown in diagram subjected to the applied loads.

Find: Stress produced in each section of the bar under tension.

Solution: The tensile force is the same for all three diameters = 1500 lb.



$$S_A = \frac{F}{A} = \frac{1500}{\frac{\pi}{4} \times (3.5)^2} = 156 \text{ lb/in}^2.$$

$$S_B = \frac{F}{A} = \frac{1500}{\frac{\pi}{4} \times (2)^2} = 477 \text{ lb/in}^2.$$

$$S_C = \frac{F}{A} = \frac{1500}{\frac{\pi}{4} \times (4)^2} = 119 \text{ lb/in}^2.$$

The area of each diameter at points A, B and C is different.

STRAIN

Strain measures change in dimension or dimensions due to some kind of stress. It is a pure number with no dimensions. Strain is defined by Equation 2.

$$\text{Strain} = e_\ell = \frac{\Delta \ell}{\ell_o} = \frac{\text{change in length}}{\text{original length}}$$

$$= \frac{(\text{final length} - \text{original length})}{\text{original length}}$$

or

$$e_V = \frac{\Delta V}{V_o} = \frac{\text{change in volume}}{\text{original volume}}$$

$$= \frac{(\text{final volume} - \text{original volume})}{\text{original volume}}$$

$$e_\ell = \frac{\Delta \ell}{\ell_o}$$

or

$$e_V = \frac{\Delta V}{V_o}. \tag{2}$$

Note: Strain can be (+) or (-) depending on the situation. Examples B and C illustrate the use of this equation.

EXAMPLE B: STRAIN IN A PIECE OF WIRE

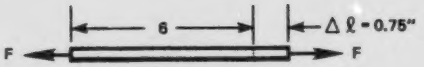
Given: A 6-inch wire increases to 6.75 inches when subjected to a certain force.

Find: Strain in the wind.

Solution: $e_\ell = \frac{\Delta \ell}{\ell_o}$

$= \frac{(6.75 - 6.00)}{6}$

$= 0.125.$



EXAMPLE C: STRAIN IN A CUBE OF METAL

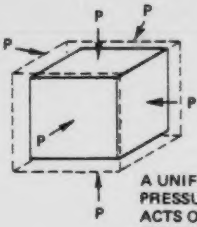
Given: Cube 10 inches on a side decreases to a cube 9.85 inches on a side when subjected to an external pressure.

Find: Strain in the cube.

Solution:

$$e_V = \frac{\Delta V}{V_o} = \frac{(9.85)^3 - (10)^3}{10^3} = \frac{956 - 1000}{1000}$$

$$= \frac{-44}{1000} = -0.044.$$



A UNIFORM PRESSURE ACTS OVER THE ENTIRE SURFACE

Therefore:

$$e_V = -0.044.$$

The negative sign indicates that the strain has produced a decrease in volume.

HOOKE'S LAW

Hooke's law states that, within limits the stress divided by the strain is constant. This constant is called the modulus E and is defined by Equation 3.

$$\text{Modulus} = E = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\Delta\ell/\ell} = \frac{F\ell}{A\Delta\ell}. \qquad (3)$$

One set of dimensions might be lb/in^2, but other units of force divided by area may be used.

The modulus for various materials is tabulted under "E" in Table 1.

ELASTICITY AND THE ELASTIC LIMIT

The modulus E is constant only over a certain range of loading for materials in tension. Beyond the point of critical loading, the material is no longer elastic. Increased tension will cause more rapid elongation and eventual breaking of the material (Figure 1).

TABLE 1. TYPICAL MECHANICAL PROPERTIES OF METALS

Metal	Ultimate Strength ($x\ 10^3$ lb/in^2) Tension S_t	Compression, S_c	Shear S_s	Tension Yield S_y	Modulus of Elasticity, 10^6 lb/in^2 E	Modulus of Rigidity, 10^5 lb/in^2 G
Wrought iron	47	47	38	28	28	10.0
AISI 1020 steel (structural)	65	65	50	45	30	11.5
AISI 1045 steel	95	95	70	60	30	11.5
AISI 1095 steel	142	142	105	83	30	11.5
AISI 2340 steel	135	135	100	116	30	11.5
302 stainless steel	140	140	1[illegible]	100	28	11.0
Cast iron class 20	20	80	32	---	11[illegible]	4.5
Cast iron class 40	40	125	55	---	16	5.5
Cast iron class 60	60	170	65	---	19	8.0
Red brass	57	---	37	40	15	5.7
Yellow brass	61	---	40	50	15	5.8
17 S aluminum alloy	62	---	38	40	10.4	3.8
24 S aluminum alloy	68	---	41	46	10.4	3.8
Monel	80	---	56	40	26	9.5
AZ 31X magnesium alloy	40	---	19	30	6.5	2.4

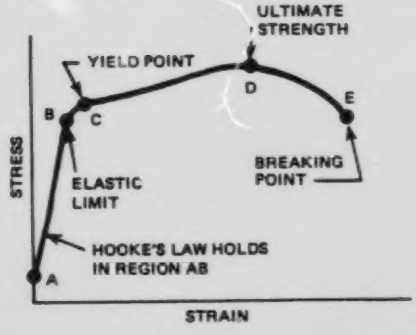


FIG. 1

The part of the curve between A and B is the proportional section of the curve. If at any point between A and B the stress is released, the material will return to its original length. Hooke's law is valid over this region.

When more heavily loaded (beyond B), the material will not return to its original length. The point B is called the "elastic limit."

Further increase in loading beyond B reaches point C which is the yield point. From there on, a small increase in loading stress gives a rapid elongation and strain.

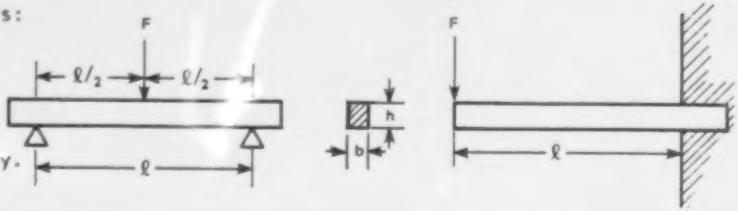
Next reached is point D, ultimate strength and finally, as load is further increased, the material breaks at point E. In the region from D to E, the cross-sectional area of the material is decreasing rapidly. This change must be considered in any calculations of stress for a material in region DE. However, this region is to be avoided in the design of structures.

DEFLECTION - MOVEMENT UNDER STRESS

Often the limiting factor in a design is not how much it is stressed or whether it will break and fail, but rather the deflection due to loading. This situation is exemplified by the frame of glass windows which must be rigid to prevent excess flexing and consequent breaking of the window panes.

In general, deflection depends on the size of the applied force, the modulus of elasticity E, and the length and cross section of the beam. Consider the two cases shown in Figure 2. Deflection, y, of a beam depends upon a number of factors:

F = force applied in pounds.
ℓ = length in inches.
E = modulus of elasticity.
I = "moment of cross section" = $bh^3/12$.



ENDS SUPPORTED

BEAM CROSS SECTION

OVERHANG-CANTILEVER

FIG. 2

Formulas giving the deflection are:

$y = \frac{F\ell^3}{48EI}$: Both ends supported, force at center, deflection at center. (4)

$y = \frac{F\ell^3}{3EI}$: Overhanging cantilever, force at end, deflection at unsupported end. (5)

These equations will be used later in a laboratory exercise.

SHEAR MODULUS

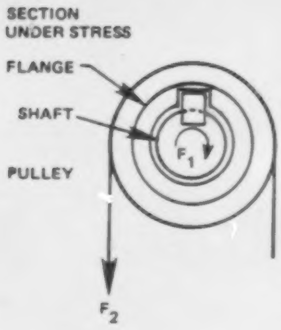


FIG. 3

In the rotating shaft and pulley arrangement (Figure 3), there are different stresses and strains on the shaft, pulley and key. One of the major cross sections of maximum stress and failure is in the key and is shown by the dotted line A-B. This "shear" stress is caused by the force of the shaft, F_1 being resisted by the pulley force F_2. These forces tend to stress the material, causing the top half of the key to shear off the bottom half. Values of allowable shear stresses in various materials are shown in Table 1.

The formula for shearing stress is the same as that given by Equation 1, namely, the shearing force divided by the area under stress. Often the units are lb/in^2. Example D illustrates calculation of shearing stress.

EXAMPLE D: SHEAR STRESS IN AN ALUMINUM BLOCK

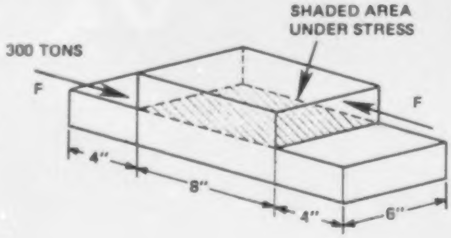
Given: A force of 300 tons applied as shown to the aluminum block. The top portion is fixed in place.

Find: Shearing stress. Will the allowable stress be exceeded? Can the block be used as shown? Assume 24S aluminum alloy (Table 1).

Solution: Actual stress $= \frac{F}{A}$

$= \frac{300 \times 2000}{8 \text{ in} \times 6 \text{ in}} = 12{,}500 \text{ lb/in}^2$.

(8 in x 6 in is the area under stress.)



Allowable stress from Table 1 is 41,000 lb/in^2. The block is usable in this application, because the actual stress, 12,500 lb/in^2 is well below the allowable.

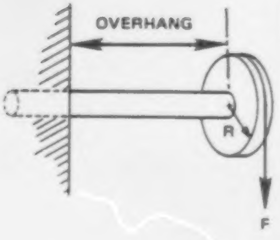


FIG. 4

SHAFTS IN ROTARY TORSION (TWIST)

The shaft shown in Figure 4 is fixed on the left and has a pulley with a force applied at radius, R. The shaft will have a torque stress induced in it, which can be calculated. The shaft will twist on the right end in the direction shown by the arrow. The angle of twist can be predicted by using a mathematical formula. If the turning force is too much and actual stress exceeds the allowable value, the shaft could break. If the shaft overhang is long, there will be a bending stress tending to break it in addition to the torque stress.

BULK MODULUS

Normal changes in air pressure do not perceptibly affect volume of a solid object. However, if a solid is acted on by a large pressure (for example, the pressure 1 mile deep in the ocean), it can be slightly compressed. The amount of compression is predicted by using the "bulk modulus of elasticity," B.

$$B = \frac{\text{stress}}{\text{strain}} = \frac{\Delta P}{\Delta V/V} = \frac{\text{extra pressure applied}}{\text{change in volume/original volume}}.$$

Typical values of B are listed in Table 2 and a calculation is shown in Example E.

TABLE 2. BULK MODULUS FOR SOLIDS AND LIQUIDS

Material	Bulk Modulus B	
	newtons/$m^2 \times 10^{10}$	lb/$in^2 \times 10^6$
Solids:		
Aluminum	7.7	11
Brass	6.1	8.5
Copper	14	20
Glass	3.7	5.3
Iron, cast	9.6	14
Lead	0.77	1.1
Steel	16	23
Liquids:		
Ethyl alcohol	0.110	0.16
Kerosene	0.13	0.19
Lubricating oil	0.17	0.25
Mercury	2.8	4.0
Water	0.21	0.31

EXAMPLE E: COMPRESSION DUE TO WATER PRESSURE

Given: An object two miles under the surface of the ocean, where added pressure is 4,600 lb/in^2.

Find: At this depth how much a cubic foot of brass would decrease in volume.
Note: 1 cubic foot equals 1,728 cubic inches.

Solution: Since,

$$B \text{ (from Table 2)} = \frac{\Delta P}{\Delta V/V}$$

$$\Delta V = \frac{\Delta P V}{B} = \frac{4600 \times 1728}{8.5 \times 10^6} = 0.94 \text{ in}^3.$$

The decrease of about 1 cubic inch would make the volume 1727 in^3. The % decrease in volume is 0.94/1728 or only 0.05%.

The compressibility, C, which is the reciprocal of the bulk modulus, B, is often referred to in the literature.

$$\text{Compressibility} = C = \frac{1}{B} = \frac{1}{\text{bulk modulus of elasticity}}.$$

LABORATORY

HOOKE'S LAW

MATERIALS

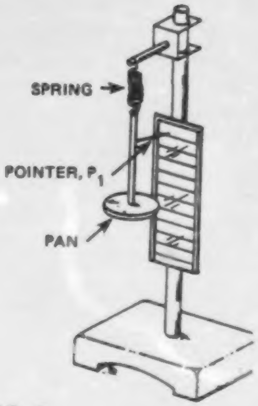


FIG. 5

Hooke's law apparatus (Figure 5).
Set of metric weights.
Lengths of wire:
  Brass #14.
  Copper #14.
  Aluminum #10.
  Aluminum #18.
Micrometer or wire gage.
Graph paper.

PROCEDURE

1. Refer to Figure 5.
2. Set spring and pointer. Record scale reading opposite pointer with no load except pan; record in Data Table 1 as P_1 under trial 1.
3. Add a weight so pointer moves about 2 cm. Record weight and pointer reading (P_2).
4. Calculate stretch (S) which is $P_2 - P_1$.
5. Repeat steps 2 and 3 for increasing loads.
6. Plot a graph of stretch on horizontal axis versus weight on vertical axis as shown in Figure 6.
7. What is nature of curve? Does it agree with section AB of Figure 1?

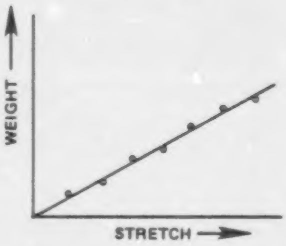


FIG. 6

DATA TABLE 1

Trial	Weight On Pan	Pointer Reading With No-Load P_1	Pointer Reading With Load P_2	Stretch of Spring $S = P_2 - P_1$
1	0 g			
2	g			
3	g			
4	g			
5	g			
6	g			

ULTIMATE BREAKING STRENGTH

MATERIALS

Set of metric weights.
Lengths of wire:
    Brass #14.
    Copper #14.
    Aluminum #10.
    Aluminum #18.
Micrometer or wire gage.
Ultimate strength tester (Figure 7).

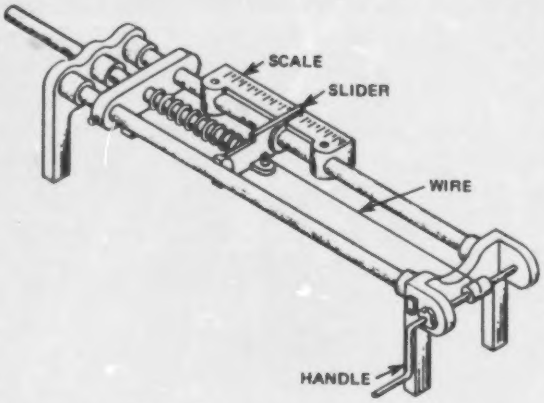


FIG. 7

PROCEDURE

1. Refer to Figure 7.
2. Measure diameter of a section of brass wire with micrometer; record in Data Table 2.
3. Place length of brass wire in machine. Turn handle until wire breaks.
4. In Data Table 2, record force indicated by slider at breaking point (Figure 8).
5. Repeat for copper and aluminum wires.
6. Calculate breaking strength by using Data Table 2.
7. Look up table value and compare.

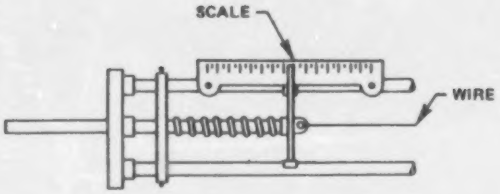


FIG. 8

DATA TABLE 2						
Wire Material	Gage	Diameter (in)	Area = 0.785 D^2	Breaking Force, F (lb)	Strength = F/A	*Table Value

*Metric to English conversions may be required.

WOOD BEAM DEFLECTION

MATERIALS

Set of metric weights.
Flexure apparatus (Figure 9) including two knife-edge clamps, weight pan, micrometer screw with a 1/2-mm pitch and clamp to hold micrometer ridigly, test rods, bell and lead wires, weights: 1" x 2" pine beam approximately 1 yard long.

PROCEDURE

1. Refer to Figure 9.
2. Set up an end-supported wooden pine beam with ℓ = 20 inches. Position beam so that its thickness in vertical direction is two inches. Load beam with weights; measure deflection. Double and triple weight; again record deflection (measurements 1A, 1B, 1C). See Data Table 3.
3. Repeat 1 with ℓ = 30 inches (measurements 2A, 2B, 2C).

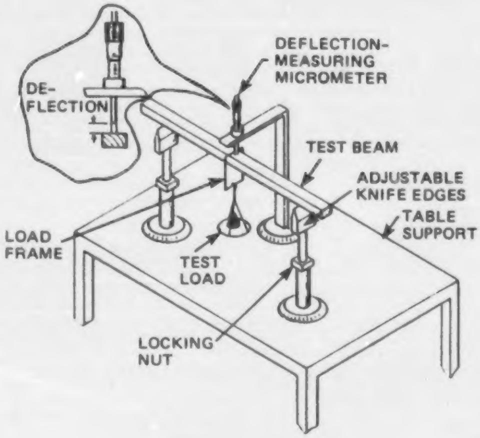

DEFLECTION-MEASURING MICROMETER
DE-FLECTION
TEST BEAM
ADJUSTABLE KNIFE EDGES
TABLE SUPPORT
LOAD FRAME
TEST LOAD
LOCKING NUT

FIG. 9

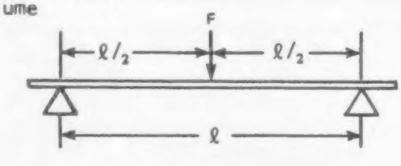
4. Repeat 3 with 1 inch beam dimension as vertical dimension (3A, 3B, 3C).
5. Set up a cantilever 25 in long. Load beam with weights; measure deflection. Double and triple weight; record deflection (4A, 4B, 4C).

DATA TABLE 3

Trial	Applied F Pounds	Measured y Inches	ℓ Inches	$I = \frac{bh^3}{12}$	Calculated y From Formula (Equations 4,5)
1A 1B 1C			20		
2A 2B 2C			30		
3A 3B 3C			30		
4A 4B 4C			25		

SELF CHECK

1. Define:
    - Stress.
    - Strain.
    - Hooke's law.
    - Elastic limit.
    - Yield point.
    - Breaking point.
    - Modulus of elasticity.
    - Shear modulus.
    - Bulk modulus of elasticity.
    - Safety factor.
    - Ultimate breaking strength.
2. Find the stress placed on an object with an area of 8 in^2 subjected to 3000 pounds of force.
3. A plastic rod 6 inches long subjected to 2000 pounds of force changed the length to 5.73 inches, determine the strain that has been put on the rod.
4. Determine the modulus of elasticity of an object with 1500 lb/in^2 stress and 0.125 strain factor.
5. From the laboratory experiment on Hooke's law, answer the following questions.
    - What is the shape of the line that best fits the points plotted on the graph?
    - The graph shows that the stretch of the spring (strain) is __________ (inversely, directly) proportional to the applied force (stress).
    - If, after removing a weight from the pan, the spring scale does not return to the no-load reading, the __________ limit of the spring has been exceeded.
6. Answer the following questions based on observations of the beam deflection experiment.
    - Does the deflection double and triple with double and triple weights?
    - By what approximate factor does deflection change when the beam is used with the 1-inch beam thickness h compared with the 2-inch thickness?
    - How do measured and calculated value of deflection compare?
7. What material is an object made of if; when it is subjected to a uniform, external pressure of 6400 lb/in^2, its fractional change in volume (AV/V) is 4.57×10^{-4}? (See Table 2.)
8. Determine applied force to the beam shown if volume of the beam is 9 in^3, its dimensions are $b = h = 1$ in, $\ell = 9$ in (where b, h, ℓ are as shown in Figure 2), and deflection at the center of the beam is 3.6×10^{-4} in. The beam is made of Monel.



1M6

TENSION IN WIRES AND CABLES

Forces are applied to objects to cause them to move or to resist impending motion. Cables, wires, strings and ropes are often used to apply force to an object. The force applied to objects using these flexible force agents is called tension. Automobiles, airplanes, boats and bicycles use cables to control the motion of the vehicles. Elevators, cranes, power lines and radio tower support lines have tension forces within them. This module discusses tension and its general characteristics and applications. Tension in weighted flexible cables, such as power lines, also is discussed. In the laboratory, the student will determine tension in a weighted line.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define a tension force.
- Describe the effects of a tension force applied to a wire, cable or rope.
- Given a set of cables attached to a load in such a manner that a concurrent force system is established, solve for the tension forces in all cables, using vector conditions of static equilibrium.
- Calculate the weight required to produce a given sag in a weighted cable.
- Given the appropriate equipment, determine tension at the center of a line and at the supports.

DISCUSSION

Forces are applied to many objects through cables, wires, ropes, strings and threads. (See Figure 1.) Man applies cables and strings, and uses forces to control, contain, or move various objects. They allow the transfer of forces over long distances. In industry, cables and ropes are so important in moving heavy equipment that an identifiable occupation has been named. Many large engineering companies employ "riggers." These men are in charge of controlling and preparing heavy loads to be moved by crane (Figure 2). To attach cables to several million dollars worth of heavy equipment that must be moved safely requires the expert knowledge possessed by riggers. They must know how to attach cables to loads and what safe working loads are.

FIG. 1

TENSION FORCES IN CABLES

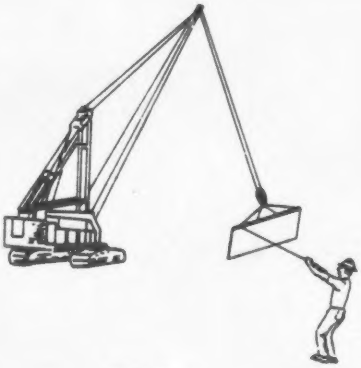


FIG. 2

A tension force tends to pull and stretch a body on which it acts. Actually, tension forces come in pairs and act together simultaneously upon the rope or cable. Figure 3 shows a tension force (pair) acting upon a rope segment. The tension forces are equal and oppositely directed. They are acting on the rope as well as within the rope. A frequent misinterpretation of tension forces arises when considering two forces pulling on a rope in opposite directions. For example, if there is a 100-lb force in opposite directions, a human tendency is to expect the tension in the rope to be 200 lb, although experiment would verify that the tension is 100 lb. Another important characteristic of tension in cables is that the direction of the tension force vector is always parallel to the direction of the central axis of the rope. This direction is the line of action, illustrated in Figure 3.

The line of action may be bent or the direction of tension forces in ropes and cables may be changed by use of a pulley. The pulley does not affect the magnitude of the tension force but affects only the direction of tension force (Figure 4).

TENSION FORCES IN STATIC LOADING

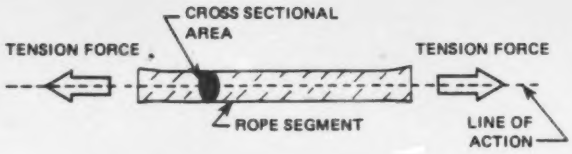


FIG. 3

Often, ropes and cables are attached to loads in such a manner that they form a concurrent coplanar force system. The forces involved in a concurrent coplanar force system are in the same geometrical plane and, when the rope or cable is stationary, the lines of action of all the forces intersect at a common point. The forces can be assumed to be acting at this point. An example of a concurrent force system is shown in Figure 5. The concurrency point is a key feature in identifying concurrent force systems. Note that in Figure 5 three forces are acting on a ring, and the point of concurrency is in the space at the center of the ring. Because the forces act through the same point, the load is prevented from rotating.

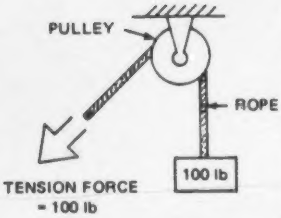


FIG. 4

When the load, as shown in Figure 5, remains stationary, it is in static equilibrium. When a system of forces is in equilibrium, the algebraic sum of the components of the forces along any mutually perpendicular coordinate axes is zero. For most static loading the vertical-horizonal coordinate axis system shown in Figure 5 is used. In this system, unknown elements (such as tension forces) of the system may be determined entirely by Equations 1 and 2 for force equilibrium.

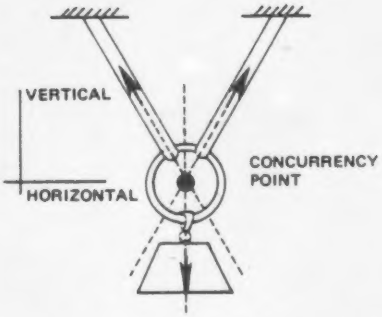


FIG. 5

The sum of all vertical components = 0

$$\Sigma F_V = 0 \qquad (1)$$

(Σ is a mathematical symbol meaning "sum of.")

and

The sum of all horizontal components = 0

$$\Sigma F_H = 0 \qquad (2)$$

Equations 1 and 2 are general equations that must be applied to the specific onditions of each individual problem to give equations which apply to that problem. To apply Equations 1 and 2, the horizontal and vertical components of each vector must be found. A sign convention is applied to each component. Upward-directed components are designated positive (+) forces; downward-directed components are designated negative (-) forces. Similarly, horizontal force components directed to the right are considered

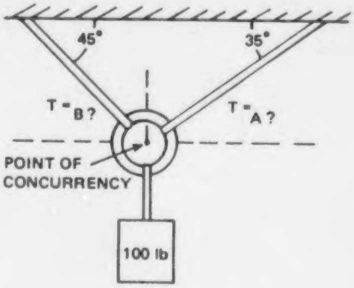
(+) forces, and those to the left (-) forces.

Equations 1 and 2 are used to solve a problem in Example A.

EXAMPLE A: EQUILIBRIUM OF CONCURRENT FORCE SYSTEM

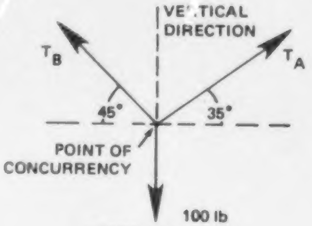
Given: A 100 lb weight supported by two ropes, A and B, at the angles shown.

Find: The tension in Rope A and Rope B. Note: Neglect weight of the rope.



45°
35°
T = B ?
T = A ?
POINT OF CONCURRENCY
100 lb

Solution: 1) As an aid, draw a vector diagram illustrating all forces acting on the point of concurrency. Label the magnitudes of the forces on the diagrams, giving letter names for unknown values. Label all known angles.



VERTICAL DIRECTION
T_B
T_A
45°
35°
POINT OF CONCURRENCY
100 lb

VECTOR DIAGRAM

2) Resolve all vectors into horizontal and vertical components. Write component equations for each direction, even though these equations may contain unknown values:

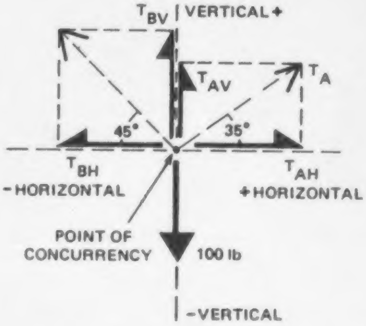
HORIZONTALLY:

$$T_{AH} = T_A \cos 35° = T_A\ (0.819)$$
$$T_{BH} = T_B \cos 45° = T_B\ (0.707)$$

VERTICALLY:

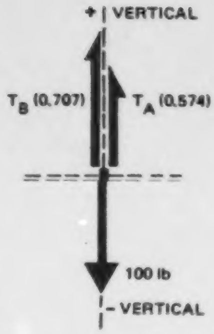
$$T_{BV} = T_B \sin 45° = T_B\ (0.707)$$
$$T_{AV} = T_A \sin 35° = T_A\ (0.574)$$

Known force = 100 lb



T_{BV}
VERTICAL +
T_{AV}
T_A
45°
35°
T_{BH}
- HORIZONTAL
T_{AH}
+ HORIZONTAL
POINT OF CONCURRENCY
100 lb
- VERTICAL

3) By using Equation 1,



The sum of all vertical components = 0,

focus attention on vertical direction only. Write an equation representing the sum of all vertical forces, observing force sign convention:

$$\Sigma F_V = T_{BV} + T_{AV} - WT = 0$$

$$T_B\ (0.707) + T_A\ (0.574) - 100\ \text{lb} = 0$$

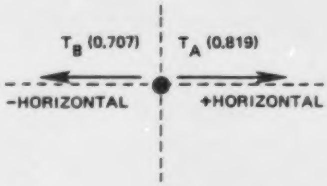
This equation is a specifc one, true for this problem. Note that it contains two unknowns, T_B and T_A, the desired tension forces. The horizontal direction furnishes an additional needed equation.

4) In the horizontal direction, the forces are as follows:

$$\Sigma F_V = -T_{BH} + T_{AH} = 0$$

$$-\ T_B\ (0.707) + T_A\ (0.819) = 0$$

According to this equation, horizontal components balance each other.



5) Solve the horizontal equation for T_B.

$$T_B = \frac{(0.819)\ T_A}{(0.707)} = (1.16)\ T_A$$

6) Substitute T_B above into the specific vertical equation.

$$+\ [(1.16)\ T_A]\ (0.707) + T_A\ (0.574) - 100\ \text{lb} = 0$$

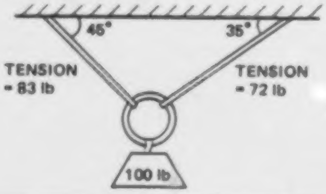
$$T_A = 71.7\ \text{lb}$$

7) Substitute this value in the equation of Step 5,

$$T_B = \frac{(0.819)}{(0.707)} (71.7\ \text{lb})$$

$$T_B = 83\ \text{lb}$$

8) In summary, the tension in rope A is 72 lb, and the tension in rope B is 83 lb.



The steps for finding tension forces in cables of a concurrent force system can be summarized as follows:

- Identify the point of concurrency in the problem.
- Draw a sketch of the problem, labeling unknown and known forces.
- Draw a vector diagram.
- Resolve all forces into horizontal and vertical components.
- Apply the general conditions of equilibrium to develop specific equations for the problem.
- Solve for the values of the unknowns.

FLEXIBLE CABLES , HORIZONTALLY–UNIFORM LOADS

Telephone wires, power wires, and trolley wires whose ends are lying at or near the same level are examples of uniform, horizontal distribution of load weight. In such distribution, shown in Figure 6, load on the cable is described as weight per unit length. Sag produced in the cable is due to the cable's own weight distributed along the length between the supports. The amount of sag (distance the cable drops down from the support level at its lowest point) is affected also by the initial tension placed on the wire. For metal power lines, sag also is affected by temperature changes, which cause the lines to sag more in hot weather than in cold weather. In hot weather, lines expand and increase in length, therefore, sagging more. In cold weather, lines contract and decrease in length, therefore, sagging less.

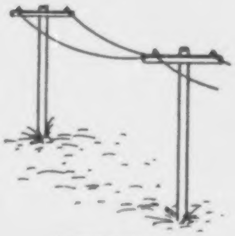

POWER LINES

FIG. 6

Tension force in a cable is always parallel to the cable axis, but unlike earlier tension forces, it varies in magnitude along the cable length. Figure 7 illustrates the variables involved in describing the tension forces in flexible cables. The cable is suspended between two supports a distance ℓ apart. The tension varies in magnitude from H at the center C to a maximum of T_{max} at the ends of cable supports A and B. The sag S is measured from the support level to point C. The horizontal tension at the bottom of the cable sag is given by Equation 3.

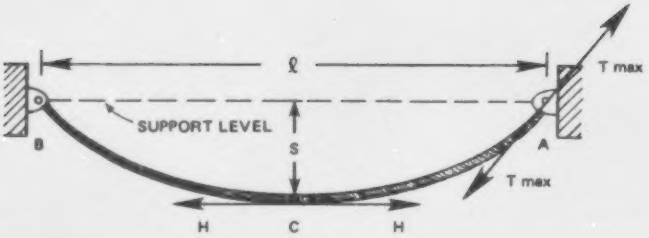


FIG. 7

$$H = \frac{W\ \ell^2}{8\ S} \qquad (3)$$

where

H = horizontal tension.
W = weight distribution per unit length (lb/ft, n/m, etc.).
ℓ = distance between supports.
S = sag.

The maximum tension (T_{max}) in the cable at the supports is given by Equation 4.

$$T_{max} = \frac{W\ \ell}{2}\sqrt{\frac{\ell^2}{16\ S^2} + 1} \qquad (4)$$

Example B illustrates the use of Equations 3 and 4 in determing the tension in a flexible cable.

EXAMPLE B: TENSION IN A POWER LINE

Given: A power line weighing 2 lb per ft is stretched between two supports at equal elevations 100 ft apart. The sag at the center is 10 ft.

Find: Tension H at the center and maximum tension at one support.

Solution: 1) Using Equation 3:

$$H = \frac{W\ \ell^2}{8\ S}$$

$$H = \frac{(2\ \text{lb/ft})(100\ \text{ft})^2}{8\ (10\ \text{ft})}$$

$H = 250$ lb = horizontal tension at center

2) Using Equation 4:

$$T_{max} = \frac{W \ell}{2}\sqrt{\frac{\ell^2}{16\ S^2} + 1}$$

$$T_{max} = \frac{(2\ \text{lb/ft})(100\ \text{ft})}{2}\sqrt{\frac{(100\ \text{ft})^2}{16\ (10\ \text{ft})^2} + 1}$$

T_{max} = 269 lb = maximum tension at supports

In actuality, all cables, wires, ropes and strings possess a uniform distributed load; however, when the weight of a cable or rope is very small compared to the lumped load or weight attached to them, the weight of the cable or rope is usually neglected in a force analysis.

SUMMARY

Tension is the force which exists in cables, wires and other lines under load. The calculation of tension, sag and weight applied to lines has practical applications in power lines, phone lines, in general all lines bearing a load.

TENSION IN A WEIGHTED LINE

MATERIALS

Two vertical support stands.
Two thermometer supports.
20 meters of heavy manila twine.
A sheave or pulley.
Weight hanger.
Set of slotted weights.
Meter stick.
Metric steel tape (30 m).
Triple beam balance.

PROCEDURE

1. Arrange experimental apparatus as shown in Figure 8.

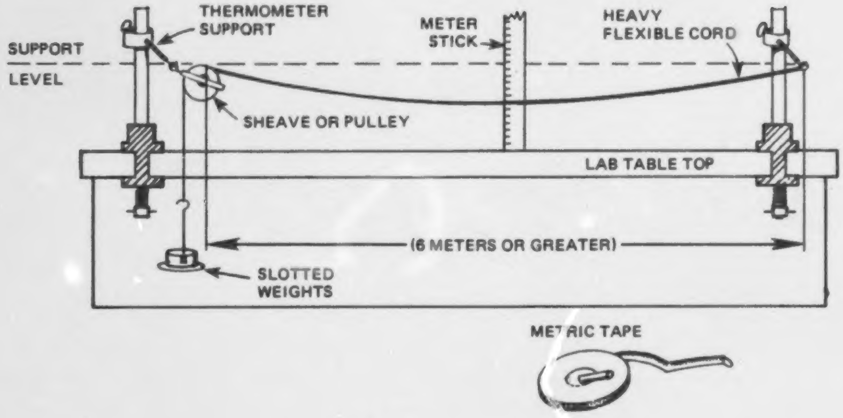

THERMOMETER SUPPORT
SUPPORT LEVEL
METER STICK
HEAVY FLEXIBLE CORD
SHEAVE OR PULLEY
LAB TABLE TOP
(6 METERS OR GREATER)
SLOTTED WEIGHTS
METRIC TAPE

FIG. 8

2. Place a 100 g mass on 50 g weight hanger. Pull weight hanger down a small distance and release. This procedure allows the system to come to equilibrium.

3. Measure a 10 m segment of heavy twine or cord. Roll into a small, secure roll, and place it on the beam balance. Record mass (kg) of 10 m segment in Data Table.

4. Measure distance from thermometer stand on one end to top of pulley at other end. Record this distance, ℓ, in the Data Table.

5. Measure amount of sag using meter stick. This measurement should occur at half the distance between supports. Record amount of sag in Data Table.

6. Divide the mass in kilograms obtained in step 3 by the 10-meter length to obtain the mass per unit length. Multiply by acceleration of gravity ($g = 9.8$) to obtain weight per unit length in newtons/meter.

7. Using Equation 4, calculate maximum tension in cord at top of the pulley. Record this value in Data Table.

8. Record total weight of loaded mass plus mass of weight hanger. (Determine weight in Newtons by multiplying number of kilograms by g, the acceleration of gravity.)

9. Calculate % difference between results of step 7 and results of step 8.

10. Repeat steps 4 through 9 for several larger separation distances of supports and loaded masses.

11. How is tension in the line related to sag, distance between supports, and weight?

DATA TABLE

Distance Between Supports, ℓ (m)	Sag in Cable at Center, C (m)	T_{max} Calculated (N)	Load on Hanger Plus Hanger (N)	% Difference

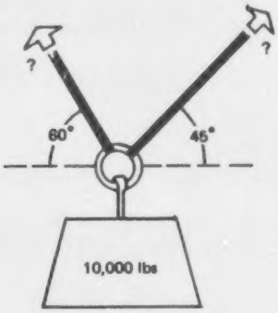
Mass of 10 m length = __________ kilograms
Weight per unit length, W = mg = __________ N/m

SELF CHECK

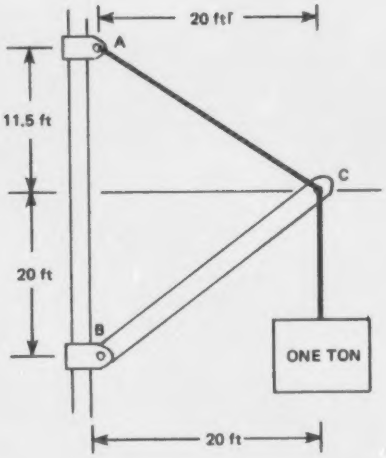
1. Define a tension force.

2. List two effects on a wire, cable or rope when a tension force is applied.

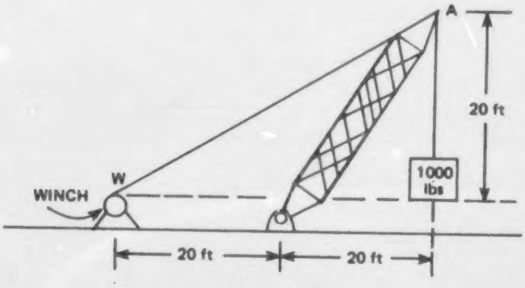
3. Two cables are used to lift a load of 10,000 lb. One cable pulls to the right, making a 45° angle with the ground. The other cable pulls to the left, making a 60° angle with the ground. Find the tension forces in the cables.



A simple derrick is shown which supports a load of one ton. Calculate the tension force in cable AC. Neglect weight of cable and strut BC.

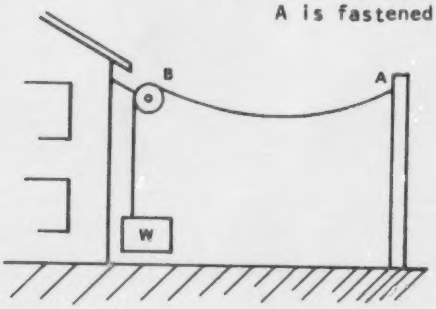


5. The simple arrangement shown must support a 1000 lb vertical load by means of a rope attached at A. What is the tension in rope AW? Neglect weight of the rope.



6. Compute the maximum tension in a flexible wire weighing 0.4 lb per ft stretched between two supports 200 ft apart and at equal levels, if the sag is 10 ft.

7. In order to control the sag in a radio aerial wire, one end A is fastened to the top of a tower, as shown in the accompanying figure, while the other end passes over a small sheave B and supports a weight W. If the wire weighs 0.05 lb/ft, and if A and B are 80 ft apart and at the same level, compute the weight W required to keep the sag at 2 ft. Neglect the diameter of the sheave B and the weight of the wire between B and W.



1F2

DENSITY, BUOYANCY, AND SPECIFIC GRAVITY

This module introduces the methods of determining the density and specific gravity of fluids by means of floating objects placed within those fluids. Such suspended objects are acted upon by the downward force of gravity (weight) and by the upward buoyant force of the fluid. Specific gravity is applied in determining the amount of antifreeze in radiators, the amount of acid in batteries, etc. In the laboratory, the student will use a hydrometer to determine the specific gravity of several fluids.

OBJECTIVES

Upon completion of this module, the student should be able to:

- State Archimedes' principle.
- State and apply formulas for density and specific gravity.
- Explain what causes the byoyant force on a floating object.
- Explain how one can determine specific gravity and density of an object that sinks in water.
- State, explain and apply formula for specific gravity and density of liquids and floating objects.
- Describe the function and operation of a hydrometer.
- Given the appropriate equipment, determine the specific gravity of several fluids.

DISCUSSION

DENSITY

Figure 1 shows a wooden cube 2 cm on each side with a mass of 4 grams. Its volume is 8 cm^3 (found by multiplication of length x width x height: 2 cm x 2 cm x 2 cm = 8 cm^3). Density is determined by one of the formulas shown in Equation 1. One possible set of units for each is given.

$$\rho_m = \frac{m}{V}$$

or

$$\rho_W = \frac{W}{V} \qquad (1)$$

where

m = mass in grams.
ρ_m = mass density.
v = volume in cm^3.
w = weight.
ρ_w = weight density.

This wooden block with a density of 0.5 g/cm^3 is one-half as dense as water. If the block is placed into water, it will float with one-half the block above water and one-half the block under water. It will be buoyed up by the water.

BUOYANCY

Weight of fluid displaced equals buoyant force on any object floating or submerged in a fluid. If the block is floating in water (Figure 1), the bottom will be at a depth of 1 cm. The buoyant force at that depth is found simply by multiplying volume of the water displaced (4 cm^3) by the weight density of water. (1 cm^3 of water has a mass of 1 gram, and 1 gram weighs 980 dynes.) Note: 10^5 dynes = 1 Newton.

Buoyant force = $(4\ cm^3)(1\ g/cm^3)(980\ \frac{dynes}{g})$
Buoyant force = 3920 dynes.

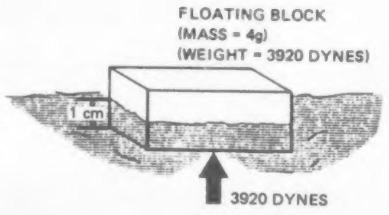


FIG. 1

Objects that do not float are also buoyed up by a force equal to the weight of the water displaced. A 10-pound fish can be pulled to the surface with an 8-pound test line; but a dip net must be used to lift the fish from the water, because the weight of the fish above water (10 lb) would snap the 8-pound test line.

An object in a fluid experiences two forces: One force is the downward force due to gravity, which is simply the weight of the object; the other force is the upward buoyant force of the fluid. For the object to float, there must be an upward force exactly equal to the downward force. Figure 2 illustrates a common instance in which the buoyant force causes an object to float.

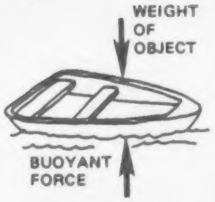


FIG. 2

These examples illustrate Archimedes' principle, which states the following:

> A body partially or entirely submerged in a fluid will experience a buoyant force equal to the weight of the fluid displaced.

The king had given Archimedes the task of determining whether or not his crown was made of pure gold. While taking a bath, Archimedes realized that impure gold would lose a greater proportion of its entire weight than would pure gold when submerged in water. It is said that he was so excited with this discovery that he jumped up from the tube and ran through the streets yelling, "Eureka! I've got it, I've got it!"

USING BUOYANCY IN DETERMINING DENSITY

Now that the same block referred to earlier is placed in methanol antifreeze, a liquid having a density of 0.80 g/cm^3. To support the 4-gram block, the buoyant force must equal its weight, 3920 dynes, as shown before. But the block will sink lower in the methanol antifreeze than in the water, because a larger volume of the less-dense methanol must be displaced to equal 3920 dynes. The formula for density and Archimedes' principle is applied to find the volume of displaced fluid as follows:

$$\text{Weight of displaced fluid} = \text{weight of floating block}$$

$$\text{Vol of displaced methanol} \times \text{weight density} = \text{weight of floating block}$$

$$V \times (0.80 \times 980)\ \frac{\text{dynes}}{\text{cm}^3} = (4 \times 980)\ \text{dynes}$$

$$V \times 784\ \frac{\text{dynes}}{\text{cm}^3} = 3920\ \text{dynes}$$

$$V = \frac{3920\ \text{dynes}}{784\ \text{dynes/cm}^3} = 5.0\ \text{cm}^3$$

As 3920 dynes of methanol antifreeze must be displaced by the 4-g block for the block to float, volume of the fluid displaced must be 5 cm^3. This means that 5 cm^3 of the 8-cm^3 block (5/8 or 62.5%) lies below the surface.

In practice, one may need to determine the mass or weight density of a fluid knowing the fraction of the object lying above or below the fluid surface. Suppose the block is floating in a fluid of unknown density. If it is determined that 5/8 or 62.5% of the block is below the surface, weight density of the fluid can be calculated from Equation 1 as follows:

$$\rho_W = \frac{W}{V}$$

where

W = weight of fluid displaced.
V = volume of fluid displaced.

Since the mass of the block is 4 g, and since the block is in equilibrium (therefore supported by a buoyant force of 4 × 980 dynes), weight of the fluid displaced (according to Archimedes' principle) is 3920 dynes. Then the weight density is as follows:

$$\rho_W = \frac{W}{V}$$

$$= \frac{3920 \text{ dynes}}{8 \text{ cm}^3 \times 0.625} \quad \text{(As 62.5\% is below the surface.)}$$

$$\rho_W = 784 \frac{\text{dynes}}{\text{cm}^3}$$

Mass density is determined as follows:

$$\rho_m = \frac{m}{V}$$

$$= \frac{784 \text{ dynes/cm}^3}{980 \text{ dynes/g}}$$

$$\rho_m = 0.80 \frac{\text{g}}{\text{cm}^3}$$

Example A shows another kind of problem that uses the Equation 1 density formula in its solution.

The density of other mixtures could be determined by use of similar calculations.

EXAMPLE A: DENSITY OF A MIXTURE

Given:	A car with a 16-quart capacity radiator. It is filled with a 50%/50% mixture of ethylene glycol (antifreeze) and water; that is, 8 quarts antifreeze and 8 quarts water. Mass density of water is 1 g/cm^3. Mass density of ethylene glycol is 1.11 g/cm^3. One quart = 946.5 cm^3.
Find:	Mass density of the mixture.

Solution: Use Equation 1 for mass density:

$$\rho_m = \frac{m}{V}$$

$$\rho_{mixture} = \frac{\text{mass of mixture}}{\text{volume of mixture}}$$

$$\text{mass of mixture} = \text{mass of water} + \text{mass of ethylene glycol}$$

$$m_w = \text{mass of water} = (\text{density of water})(\text{volume of water})$$

$$m_w = (1\ \text{g/cm}^3)(8\ \text{qt})(946.5\ \frac{\text{cm}^3}{\text{qt}}) = 7572\ \text{g}$$

$$m_e = \text{mass of ethylene glycol} = 1.11\ \text{g/cm}^3\ (8\ \text{quarts})(946.5) = 8405\ \text{g}$$

$$\rho_{mixture} = \frac{7572 + 8405\ \text{g}}{(16\ \text{qt})(946.5\ \text{cm}^3/\text{qt})}$$

$$\rho_{mixture} = 1.055\ \text{g/cm}^3$$

Thus, density of the 50-50 mixture is 1.055 g/cm^3.

SPECIFIC GRAVITY

Specific gravity of a substance is a number comparing density of the substance to density of water. This relationship is shown in Equation 2.

$$\text{S.G.} = \frac{\rho_s}{\rho_w} \qquad (2)$$

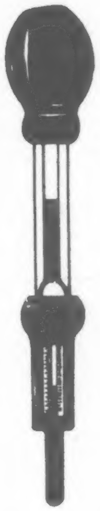


FIG. 3

where

S.G. = specific gravity of a substance.
ρ_s = density of substance.
ρ_w = density of water, 1.0 g/cm^3.

In the metric system of units, as density of water is 1.0 g/cm^3, the specific gravity of a substance will be equal numerically to the density of the substance. Specific gravity has no units, because it represents a ratio of two numbers with like units. As specific gravity has no units, it must have the same value, regardless of the system of units in which one is working.

Sometimes a more convenient approach is to simplify Equation 2 and to compare the weights or masses of equal volumes of two substances rather than their densities. The ratio still will be the same, as here indicated:

$$\text{S.G.} = \frac{\rho_s}{\rho_w} = \frac{W_s}{W_w}$$

$$S.G. = \frac{m_s}{m_w}$$

where

W_s = weight of substance.
W_w = weight of equal volume of water.
m_s = mass of substance.
m_w = mass of equal volume of water.

A hydrometer such as the one shown in Figure 3 often is used to measure specific gravity of liquids.

The hydrometer is a glass bulb weighted and calibrated in such a manner that the level at which it floats in a fluid will indicate directly the specific gravity of the liquid. Density of the liquid is determined from the scale in the units — g/cm³.

Most antifreeze testers in commercial use have hydrometer scales divided into two parts. The upper scale is used for water-methanol mixtures (specific gravities from 0.79 to 1.00), and the lower scale is used for water and ethylene-glycol mixtures (specific gravities from 1.00 to 1.16).

Specific gravity of antifreeze is important, because it gives an indication of the concentration of the mixture of antifreeze and water. On the back of an antifreeze container, the freezing temperatures for different mixtures of antifreeze and water are listed. Table 1 indicates the freeze-protection temperatures for antifreeze protection.

TABLE 1: APPROXIMATE TABLE FOR ANTIFREEZE PROTECTION

Type	Parts Of Water	Parts Anti-freeze	Ratio	Specific Gravity	Freezing °F	Temperature °C
Water	16	0	--	1.0	32°	0°
Methanol	6	10	3 to 5	0.0875	-67	-55
	8	8	1 to 1	0.90	-38	-39
	10	6	5 to 3	0.925	-15	-26
	12	4	3 to 1	0.95	+5	-15
Ethylene-glycol	6	10	3 to 5	1.062	-65	-54
	8	8	1 to 1	1.055*	-34	-37
	10	6	5 to 3	1.041	-8	-22
	12	4	3 to 1	1.027	+10	-12

*As calculated in Example A.

Battery testers are the same as antifreeze testers, with a little additional weight to the hydrometer bulb. They measure specific gravity of the battery acid and water mixture (in most cases sulfuric acid). The scale indicates specific gravities from 1.10 to 1.30. If the reading is between 1.26 and 1.30 (colored green), the battery is charged. If the reading indicates less than about 1.23 (red) the battery is discharged.

A laboratory method for determining specific gravity of a solid object often is used; it is based upon Equation 3.

$$\text{S.G.} = \frac{m_A}{m_w} \tag{3}$$

or

$$\text{S.G.} = \frac{W_A}{W_w}$$

where

m_A = mass of object.
m_w = mass of an equal volume of water.
W_A = weight of object.
W_w = weight of an equal volume of water.

The specific gravity of objects that sink in water is given by Equation 4.

$$\text{S.G.} = \frac{W_A}{W_A - (W_A)_w} \tag{4}$$

where

$(W_A)_w$ = weight of submerged object in water.

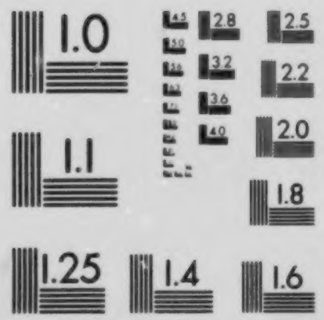
Note that the denominator $W_A - (W_A)_w$ is equal to the weight of the volume of water displaced by the submerged object. The latter claim follows because the loss of weight of the object in water is equal to the weight of water displaced, by Archimedes' principle. As the object was totally submerged, this is the weight of the body's volume of water. Hence Equation 4 follows at once from the general definition for specific gravity.

A similar method can be used to determine specific gravity of a given fluid by use of Equation 5.

$$\text{S.G.} = \frac{W_A - W_L}{W_A - (W_A)_w} \tag{5}$$

where

W_L = weight of an object submerged in an unknown liquid.


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

Because $W_A - W_L$ is the loss of weight when the object is submerged in a given liquid, it will also equal the buoyant force in that liquid. From Archimedes' principle, $W_A - W_L$ must be the weight of the liquid displaced by the object; therefore, $W_A - W_L$ divided by $W_A - (W_A)_w$ is the ratio of weight of the liquid displaced to weight of the same volume of water displaced. This ratio, of course, is equal to specific gravity of the given liquid.

SUMMARY

When an object is suspended in a fluid, the object is acted upon by two forces: (1) the force of gravity (weight) downward, and (2) the force of buoyance upward. The specific gravity of a fluid can be determined, therefore, by the degree of suspension of an object in the fluid. The hydrometer contains an object in a calibrated tube; the level at which the object floats indicates the specific gravity of the fluid tested.

DETERMINING SPECIFIC GRAVITIES OF FLUIDS WITH A HYDROMETER

MATERIALS

Laboratory balance.
Meter stick.
Block of wood such as a short piece of 2 x 4 (not a cube).
Three containers (preferably transparent) large enough for the wooden block to float with its large surface in a horizontal position.
Ethylene-glycol and water mixture (about 1/2 gallon).
Methanol (methyl alcohol) and water mixture (about 1/2 gallon).
Laboratory hydrometer (capable of reading specific gravity from 0.8 to 1.2).
Automobile radiator hydrometer (antifreeze tester).

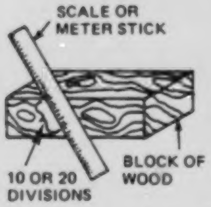


FIG. 4

PROCEDURE

1. Weigh block of wood on a laboratory balance. Record in Data Table.

2. Measure length, width, and height of block of wood; calculate its volume. Record.

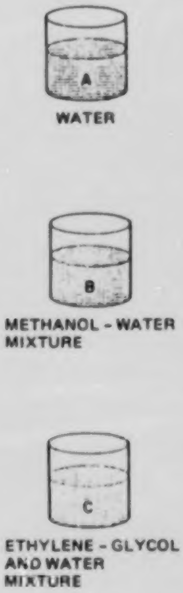
3. With a scale slanted, as shown in Figure 4, side of block can be divided into 10 or 20 divisions. Mark these divisions with a pencil.

4. Arrange three containers of liquids as shown in Figure 5.

5. Place block of wood in container A. Count number of divisions below surface; divide by total number of divisions; multiply by 100. This calculation gives percent of block below surface. (Note: Block should not be left floating long enough to become water-logged.)

6. Calculate volume of block below surface. (Multiply total block volume by decimal equivalent of percent below surface.) Record.

7. Record buoyant force. (It must be equal to weight of block, as block is floating and is therefore in equilibrium.)



WATER

METHANOL - WATER MIXTURE

ETHYLENE - GLYCOL AND WATER MIXTURE

FIG. 5

8. Record weight of fluid displaced. (Apply Archimedes' principle.)

9. Record volume of fluid displaced (same as volume of block below surface).

10. Calculate fluid density.

11. Repeat for fluids in containers B and C.

12. Use hydrometer to determine specific gravity of each fluid. Convert this reading to density for each fluid.

13. Find percent error in each case, assuming density as determined with hydrometer to be correct.

14. Using value of density obtained with hydrometer, look up approximate freezing temperature for fluids A, B and C in Table 1.

15. Use antifreeze tester to measure freezing temperature of fluids A, B and C.

16. Why were the hydrometer balk suspended higher in some fluids than in others?

DATA TABLE

Weight of wooden block ______.

Length ______. Width ______. Height ______. Calculated Volume ______.

	A	B	C
Percent of block below surface			
Volume of block below surface			
Buoyant force			
Weight of fluid displaced			
Volume of fluid displaced			
Density of fluid			
Density measured by hydrometer			
Percent error			
Approximate freezing temperature (from table)			
Approximate freezing temperature (from antifreeze tester)			

SELF CHECK

1. In your own words, describe Archimedes' principle.

2. Is this a correct formula for density? If not, what must be changed?

$$\rho_m = \frac{V}{m}$$

3. Would it be possible to lift a 12-pound weight from the bottom of a lake to the surface with a line of 10-pound test strength? Why or why not?

4. If the weight of an object and the weight of an equal volume of water were known, could the specific gravity be determined? If not, what other information would be needed? If so, describe the solution in an equation format.

5. The specific gravity of aluminum is 2.7. The weight of water is 62.4 lb/ft^3. Calculate the weight in pounds of a two cubic foot solid piece of aluminum.

6. A popular antifreeze tester is the kind that has several little balls, each having a slightly different density. A sample is taken from a radiator in which ethylene-glycol is used, and all of the balls are found to be floating.

   Would the fluid in the radiator be in danger of freezing?

   Explain your answer. You probably will wish to use these terms: buoyant force, weight of displaced fluid, and density (or specific gravity).

7. What is the volume of an object if it displaces 2 kg of water when it is placed in water and its mass density is 850 kg/m^3.

8. Find the mass density of an unknown liquid if when it is mixed with water in a 3 to 1 ratio (3 parts unknown liquid to 1 part water) the mixture has a mass density of 1.16 g/cm^3.

1F4

MANOMETERS

The manometer measures the force of fluid pressure, and is the basic pressure-measuring instrument because of its simplicity and inherent accuracy. The actual measurement consists simply of measuring the heights of liquids in columns. The manometer is unique in being both a pressure-measuring device and a standard for calibrating other instruments. It is inherently accurate, since it contains no gears, no pressure-tight bearings, no bellows, no diaphragms, and no parts subject to wear, age or fatigue.

In this module, the student will be introduced to the basic concepts of pressure measurement with manometers. General characteristics of various U-tube manometers are described; and measurement of positive pressure, vacuum and differential pressure is illustrated. Industrial variations - the well-type manometer and the inclined manometer - also are presented. In the laboratory, the student will use a U-tube manometer to verify the calibration of a pressure gage.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the terms:

    Differential pressure.
    Concave meniscus.
    Convex meniscus.
    Vacuum.

- Given the height of a manometric fluid and its density, calculate the differential pressure in the manometer.

- With the equations provided and several values for height of column level, manometer fluids, differential pressure and density values, solve typical manometer problems.

- Diagram, with suitable labeling, the following manometers:

  U-tube.
  Well-type.
  Inclined tube.
  State the advantages/disadvantages of each type.

- Convert pressure in "centimeters of mercury" or "inches of mercury" to equivalent pressure units in pounds/in^2 and vice versa.

- Given the appropriate equipment, verify the calibration for a pressure gage with a U-tube manometer.

DISCUSSION

The word "manometer" is derived from "mano" (rare or thin) and "meter" (measure) and literally means the measurement of something rare, such as air or gas. Manometers, indeed, are used in the measurement of air or gas pressure, as well as in the measurement of pressure of liquids in motion.

BASIC CONCEPTS OF PRESSURE MEASURE MEASUREMENTS

Pressure is a measure of force applied to an area, and is defined according to Equation 1.

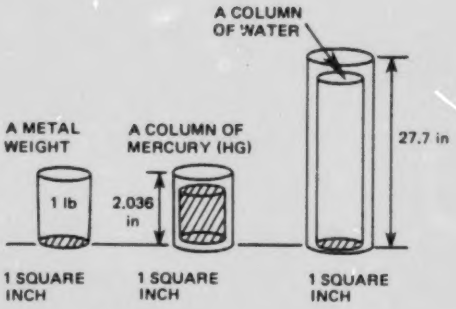
$$p = \frac{F}{A} \qquad (1)$$

where

p = pressure.
F = force.
A = area.

Some common units of pressure are pounds per square inch, pounds per square foot, newtons per square meter and dynes per square centimeter. Figure 1 illustrates how a pressure of one pound per square inch can be produced using different types of weights.

Pressure in a liquid increases with depth and is the same in all directions. A diver practicing at a deep swimming pool experiences increase in pressure in his ear drums as he dives to the bottom of the pool. In a column of fluid, such as shown in Figure 2, the pressure depends not only upon the depth, but also upon the weight density of the fluid. The difference in the pressures p_1 and p_2 shown in Figure 2 depends only upon the distance between the two points (levels) and upon the weight density of the fluid. This pressure difference is stated in Equation 2.



PRESSURE = 1 POUND PER SQUARE INCH

FIG. 1

$$p_2 - p_1 = \rho h \tag{2}$$

where

p_1 = pressure at level 1.
p_2 = pressure at level 2.
ρ = weight density of liquid.
h = height of liquid.

Equation 2 is the basic equation that governs all manometers.

Table 1 illustrates some common manometer fluids and their densities.

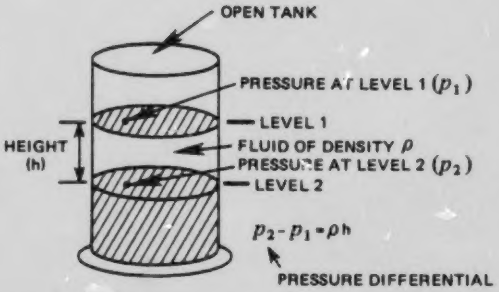

OPEN TANK
PRESSURE AT LEVEL 1 (p_1)
LEVEL 1
HEIGHT (h)
FLUID OF DENSITY ρ
PRESSURE AT LEVEL 2 (p_2)
LEVEL 2
$p_2 - p_1 = \rho h$
PRESSURE DIFFERENTIAL

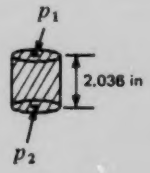
FIG. 2

TABLE 1 WEIGHT DENSITY OF COMMON MANOMETER FLUIDS

Substance	Density
Water	62.4 lb/ft^3 0.0361 lb/in^3 1 g/cm^3 (mass density)
Mercury	0.491 lb/in^3 13.60 g/cm^3 (mass) density)

Example A illustrates the use of Equation 2 and Table 1 in solving a problem.

EXAMPLE A: CALCULATION OF PRESSURE DIFFERENCE


p_1
2.036 in
p_2

Given: Two points (p_1 and p_2) in a column of mercury are separated by 2.036 in.

Find: Difference in pressure between p_1 and p_2.

Solution: $p_2 - p_1 = \rho h$

$$= (0.491\ \frac{\text{lb}}{\text{in}^{32}})(2.036\ \text{in})$$

(See Table 1.)

$p_2 - p_1 = 1 \text{ lb/in}^2$ (Recall Figure 1.)

The metric, SI units of pressure are the Pascal and the bar. A Pascal (Pa) is a newton per square meter, a relatively small unit of pressure. Therefore, units kPa (1000 N/m^2) frequently are used. A simplifying practice in manometry is to express pressures in terms of heights of specific manometric fluid, such as "60 cm of mercury" or "20 inches of water." When expressed in this form, the addition of pressure is simplified: the heights of liquid columns simply are added. A set of unit relationships for different pressures is given in Table 2.

TABLE 2: UNIT CONVERSIONS FOR PRESSURE

1 Pa = 1 N/m^2, 1 bar = 10^5 Pa.
1 inch of water = 0.0361 lb/in^2.
1 inch of Hg = 0.491 lb/in^2.
1 mm of Hg = 0.133 kPa ≃ 1/8 kPa.
1 inch of water = 0.249 kPa ≃ 1/4 kPa.
1 inch of Hg = 3.3858 kPa ≃ 3.4 kPa.
1 psi = 6.895 kPa ≃ 7 kPa.
1 atmosphere = 14.7 lb/in^2.

AIR PRESSURE, VACUUM, ATMOSPHERIC PRESSURE

The force per unit area which is exerted upon any body immersed in air at one atmosphere is approximately 14.7 lb/in^2. One standard atmosphere is defined as follows:

1 standard atmosphere (air pressure at sea level) = 760 mm (29.92 in) of mercury (Hg) at 0°C (32°F) having a density of 13.6 g/cm^3
= 14.7 lb/in^2
= 1.013×10^5 N/m^2
= 1.013×10^6 dynes/cm^2

Barometric pressure is atmospheric pressure at a particular location.

Partial vacuum is a space in which there are very few atoms or molecules. A perfect vacuum is a theoretical space in which no atoms are present. The term, "vacuum" is generally taken to mean a space containing air or gas at well below atmospheric pressure, although, literally speaking, this space should be referred to as a partial vacuum. "Vacuum" is sometimes spoken of as "negative pressure" relative to a local barometric pressure.

Gage pressure is the difference in a pressure being read from a gage and the equivalent atmospheric pressure. Absolute pressure is the actual pressure of air or vacuum being measured. Manometers may measure gage pressure or absolute pressure, depending upon their construction. Figure 3 illustrates the relationship between the scales for pressure measurement.

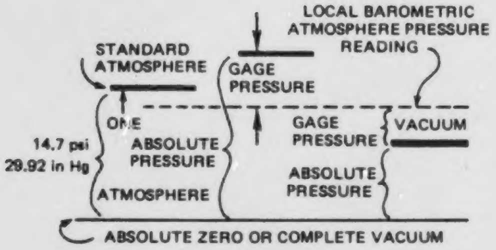

LOCAL BAROMETRIC ATMOSPHERE PRESSURE READING
STANDARD ATMOSPHERE
GAGE PRESSURE
ONE ATMOSPHERE
ABSOLUTE PRESSURE
14.7 psi
29.92 in Hg
GAGE PRESSURE
VACUUM
ABSOLUTE PRESSURE
ABSOLUTE ZERO OR COMPLETE VACUUM

FIG . 3

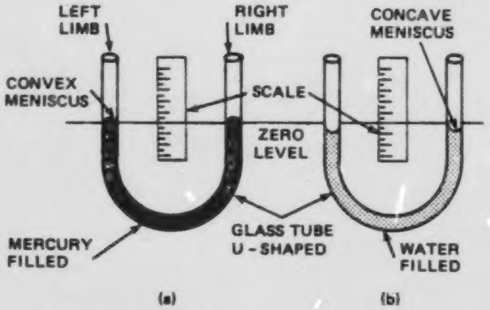

LEFT LIMB
RIGHT LIMB
CONCAVE MENISCUS
CONVEX MENISCUS
SCALE
ZERO LEVEL
MERCURY FILLED
GLASS TUBE U-SHAPED
WATER FILLED
(a)
(b)

FIG . 4

THE MANOMETER

The simplest reliable device for measuring moderate static pressure differences is the U-tube manometer. A simple U-tube manometer consists of a tube made of glass or other transparent material bent into the shape of a U, with both ends left open. A few spoonfuls of water poured into the tube is all that is required to make a manometer.

The U-tube generally is filled about halfway with distilled water, oil, mercury or any other liquid which flows easily, although mercury and water are most common. Figure 4 shows two liquid-filled manometers.

For low pressures, the choice of liquids can be oil or water. The densities of such liquids are very low compared with mercury; therefore, the liquids are more sensitive to slight change in pressure.

Each liquid has its advantages and disadvantages as a manometric fluid. Water tends to evaporate; and if not distilled or demineralized, it tends to leave a lime deposit on the tube wall, which makes future readings difficult. Water also wets the wall of the glass tube causing the meniscus to be lower at the center than at the side, as shown in Figure 4. For higher pressure ranges, mercury is used. Clean, highly-refined mercury will not wet the walls of the tube, but will form a menisucs higher in the center than at the edges. Regardless of the shape of the meniscus, the manometry liquid levels are determined at the point at which the liquid meets the tube wall, because this point can be determined accurately by visual inspection. Consistency is all that is required, however, since pressure measurements always involve a difference in two level readings.

When no pressure is applied to either limb of a manometer, the liquid remains at zero reading on the scale. The manometer is a self-balancing instrument because it basically balances gravitational forces and applied pressures on the two limbs of the U-tube. When some outside pressure is applied to the left limb, the manometer fluid is pushed downward in the left limb and upward in the right limb until equilibrium or static conditions are achieved. This condition is illustrated in Figure 5.

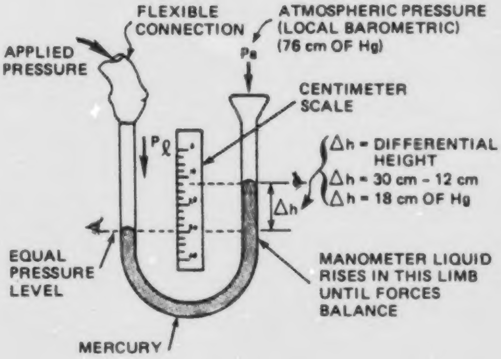

FLEXIBLE CONNECTION
ATMOSPHERIC PRESSURE (LOCAL BAROMETRIC) (76 cm OF Hg)
APPLIED PRESSURE
Pa
CENTIMETER SCALE
Pℓ
Δh = DIFFERENTIAL HEIGHT
Δh = 30 cm - 12 cm
Δh = 18 cm OF Hg
Δh
EQUAL PRESSURE LEVEL
MANOMETER LIQUID RISES IN THIS LIMB UNTIL FORCES BALANCE
MERCURY

FIG. 5

When the equilibrium condition shown in Figure 5 is established, pressure in the left limb is balanced by (and equal to) the total pressure in the right limb. Stated in a balanced equation form:

$$p_{\text{left}} = \Delta h + p_a \qquad (3)$$

where

p_{left} = pressure in left limb of manometer.
Δh = differential pressure height of fluid in right column.
p_a = atmospheric pressure.

Use of Equation 3 in solving a problem is shown in Example B. The simple relationships expressed by Equations 2 and 3 facilitate the use of the manometer to calibrate other types of pressure gages. If a manometer and some other pressure gage are subjected to the same applied pressure and the same atmospheric pressure, the differential height of the column of manometric fluid can be read and compared to the gage reading of the other pressure-reading device.

EXAMPLE B: CALCULATION OF PRESSURE FROM MANOMETER READING

Given: Information shown in Figure 5.

Find: (a) Applied pressure in centimeters of mercury.
(b) Applied pressure in lb/in^2.

Solution: (a) Use Equation 3:

$$p_{left} = \Delta h + p_a$$
$$= 18 \text{ cmHg} + 76 \text{ cmHg}$$
$$p_{left} = 94 \text{ cmHg}$$

(b) From Table 1, density of mercury = 0.491 lb/in^3

$$p_{left} = p_2 - p_1 = \rho\Delta h \qquad \text{(Equation 2)}$$
$$= (0.491 \text{ lb/in}^3)(94 \text{ cm} \times \frac{1 \text{ in}}{2.54 \text{ cm}})$$
$$p_{left} = 18.2 \text{ lb/in}^2 \text{ (absolute pressure)}$$

If the direction of an "applied" pressure is reversed, a pressure less than atmospheric is produced in the left-hand limb; i.e., a partial vacuum. This measurement is shown in Figure 6.

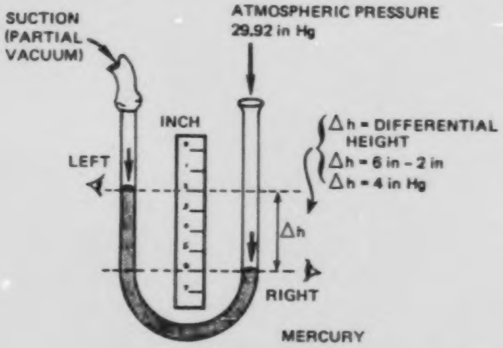

SUCTION (PARTIAL VACUUM)
ATMOSPHERIC PRESSURE 29.92 in Hg
INCH
LEFT
Δh = DIFFERENTIAL HEIGHT
Δh = 6 in - 2 in
Δh = 4 in Hg
Δh
RIGHT
MERCURY

FIG. 6

Pressure on the left must equal pressure on the right when static conditions are achieved. The balanced condition is illustrated by Equation 4.

$$p_{left} + \Delta h = p_{right} = p_a,$$

or

$$p_{left} = -\Delta h + p_{right} \qquad (4)$$

where

p_{right} = pressure in right limb of manometer.

Equation 4 is utilized to solve a problem in Example C.

EXAMPLE C: CALCULATION OF PRESSURE FROM MANOMETER READING

Given: Information shown in Figure 6.

Find:
(a) Calculate the "vacuum" gage pressure in lb/in^2.
(b) Calculate the absolute pressure, p_{left}, in inches of Hg and in lb/in^2.

Solution: (a)

$$\text{Gage pressure} = p_{left} - p_{right}$$
$$= \rho_{Hg}h$$
$$= (0.491\ \text{lb/in}^3) \times (4\ \text{in})$$
$$\text{Gage pressure} = 1.964\ \text{lb/in}^3$$

(But note p_{left} is at a pressure below p_{right}.)

(b) Use Equation 4:

$$p_{left} = -\Delta h + p_{right}$$
$$p_{left} = -4 \text{ inches of Hg} + 29.92 \text{ inches of Hg}$$
$$= 25.92 \text{ inches of Hg}$$
$$= (25.92\ \text{in}) \times (0.491\ \text{lb/in}^3)$$
$$p_{left} = 12.7\ \text{lb/in}^2$$

(p_{right} is 14.7 lb/in^2.)

The U-tube manometer also is used to measure differential pressures in which the difference in the pressures of the fluids is of prime interest. Differential pressure measurement is illustrated in Figure 7.

The differential pressure measurement can be expressed in terms of ($p_1 - p_2$) units or in terms of the differential height of manometer fluid column, Δh.

In the measurement of fluid differential pressure with a manometer, two immiscible fluids, each of different density, are present in the manometer tubes (Figure 8). As before, the total pressure on the left-hand side must balance the total pressure on the right-hand side in a static situation. The differential pressure which results is given by Equation 5:

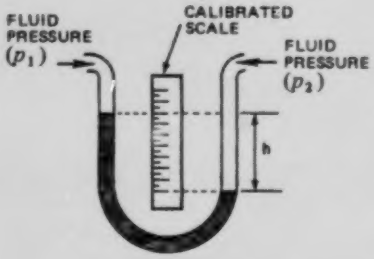


FIG. 7

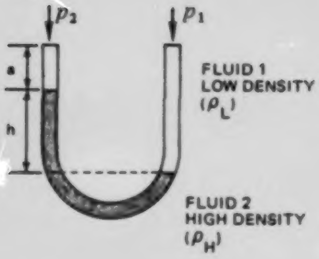


FIG. 8

$$p_1 - p_2 = \Delta h\ (\rho_H - \rho_L) \qquad (5)$$

where

ρ_H = weight density of 1st liquid (higher).
ρ_L = weight density of 2nd liquid (lower).

Equation 5 is used to solve a problem in Example D. The difference in densities of the two liquids is known as the wet-leg correction.

EXAMPLE D: CALCULATION OF PRESSURE DIFFERENTIAL

Given: A U-tube manometer filled with mercury is used to measure the differential pressure across a venturi tube in a water line. While a steady water flow is occurring, a height, Δh, is measured to be 10 inches.

Find: The differential pressure ($p_1 - p_2$) in lb/in².

Solution: From Table 1:

Mercury density (ρ_H) = 0.491 lb/in³

and

Water density (ρ_L) = 0.0361 lb/in³

Therefore:

$$\begin{aligned} p_1 - p_2 &= \Delta h\ (\rho_H - \rho_L) \\ &= (10\ \text{in})(0.491\ \text{lb/in}^3 = 0.0361\ \text{lb/in}^3) \\ &= (10\ \text{in})(0.455\ \text{lb/in}^3) \end{aligned}$$

$$p_1 - p_2 = 4.55\ \text{lb/in}^2$$

Notice that if the wet-leg correction had not been made, the pressure obtained would have been 4.91 lb/in² (uncorrected).

INDUSTRIAL VARIATIONS OF THE MANOMETER

WELL-TYPE MANOMETERS

U-tube manometers require that two readings be taken and then added algebraically before the desired final pressure can be obtained. To simplify this procedure, a well, or reservoir, is substituted for one of the columns of the U-tube. The liquid level in the reservoir has a much larger surface area, as shown in Figure 9. Pressure applied to the reservoir forces the level down slightly while the fluid enters the measuring column. If the diameter of the reservoir is much larger than the diameter of the measuring tube (in commercial models, a ratio of about 250 to 1), then the drop in the reservoir level is considered insignificant; and only the level in the measuring tube need be read to determine Δh directly from the calibrated scale. The pressure differential is given by Equation 6.

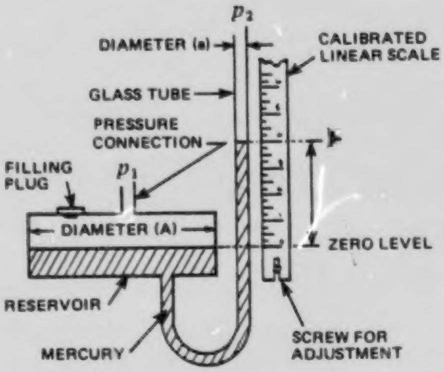


FIG. 9

$$p_1 - p_2 = \rho_{Hg}\, h \qquad (6)$$

(if a << A).

In actual practice, the well manometer should be zero adjusted before readings are taken. It also should be leveled by an attached level gage.

THE INCLINED-TUBE MANOMETER

Extremely small differential pressures frequently are measured with a special U-tube manometer. One tube is inclined at an angle (θ) to the horizontal and to the vertical tube (that consists of a relatively large well), as shown in Figure 10. The main purpose of the inclined tube is to achieve a longer scale than possible with the ordinary vertical U-tube manometer for the same pressure differential.

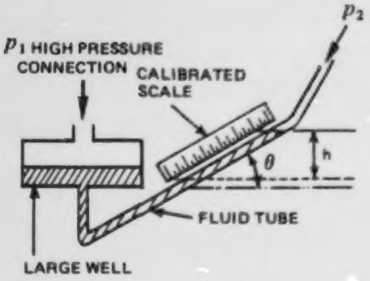


FIG. 10

For example, if the tube is inclined to the horizontal at some angle in such a manner that 20 units measured along the tube equals 1 unit vertical rise, then a rise, h, in the level of the liquid in the tube will produce a movement of the liquid along the tube 20 times as great. Therefore, the movement for a small change in level is more readily detected than in a vertical manometer. The increased sensitivity of an inclined manometer can be illustrated by a six-inch vertical range, expanded by an inclined manometer to cover a scale 48 inches in length. An instrument of this type easily could be graduated into hundredths of an inch. With water as the indicating manometer fluid, a tenth of an inch on the scale would correspond to a pressure differential of approximately 0.0005 psi.

The pressure differential measured by an inclined manometer is given by Equation 7.

$$p_1 - p_2 = \rho dF \qquad (7)$$

where

d = inclined scale reading.

F = scale factor (vertical height/unit scale length), $\sin\theta$.

The inclined manometer is used mostly for low-pressure differential measurements of boiler draft, furnace pressure, static pressures in air-conditioning systems, and for determining differential pressures across air filters and fans.

In this module, the method of reading the manometer has been limited to visual inspection and direct reading of a scale. Modern industry, however, has developed a variety of more sophisticated and accurate methods for following the rise and fall of a manometer fluid in a column.

SUMMARY

The manometer measures the force of fluid pressure. Because of its simplicity and accuracy, the manometer is the basic pressure-measuring device. It can measure positive pressure, vacuum and differential pressure, and can be used to calibrate other instruments. The inclined-tube manometer is an extremely accurate and useful design for industrial uses.

LABORATORY

MANOMETRIC VERIFICATION OF PRESSURE-GAGE CALIBRATION

MATERIALS

Manometer (vacuum and pressure) - A U-shaped glass tube with arms one meter in length, both ends open, mounted on meter-stick scale clamped to vertical support. (See Figure 11.)

About 175 g (6 oz) of mercury.

Thistle tube for filling manometer with mercury.

Vacuum pump, hand-operated, high-impact polystyrene plastic, with attached pressure gage calibrated in centimeters and inches. Pressures up to 7 inches positive and 25 inches (vacuum) are capable. Pumping rate is 15 cm^3 per stroke (Sargent-Welch Scientific Company). (See Figure 11.)

Two-foot length of transparent vinyl plastic tubing 1/4" inside diameter (connection from pump to manometer).

PROCEDURE

1. Arrange equipment as shown in Figure 11.
2. Attach connection tube to positive pressure outlet of hand pump, and gently squeeze handle until needle on pressure gage registers one centimeter of Hg. Hold this pressure while reading column level scale readings. Record each scale reading in the Data Table.
3. Increase the pressure in approximately one centimeter steps, as indicated on the pressure gage, holding pressure while reading column heights. Record each column level scale reading for each pressure in Data Table until the maximum calibrated pressure on the gage is reached.
4. Carefully and slowly release pressure on manometer. Change connection tube to vacuum outlet of hand pump.

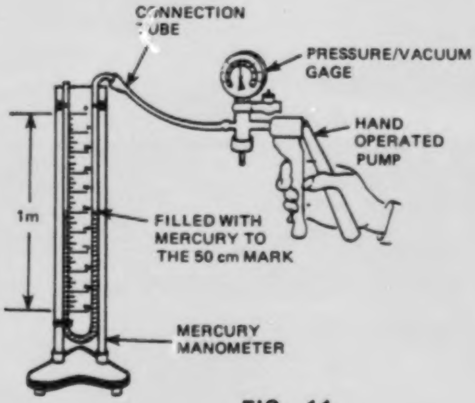


FIG. 11

5. Gently squeeze handle of pump to produce a one-centimeter "vacuum" on pressure gage. Hold this "vacuum," and record manometer levels in Data Table. Repeat this procedure until maximum "vacuum" for which the pressure gage is calibrated is reached.

6. Carefully release "vacuum."

7. From observations recorded in Data Table, calculate manometer height for each pressure reading by subtracting level of right limb from level of left limb. Record results in Data Table. Do these results differ from those indicated on the pressure-gage dial?

8. With Equation 2, calculate the pressures in lb/in^2 for each reading. Record results in Data Table.

9. Suggest a method by which the pressure gage could be recalibrated in lb/in^2 without breaking into the body of the gage.

DATA TABLE

Pressure Gage Reading cm of Hg	Manometer Left Level cm of Hg	Manometer Right Level cm of Hg	Manometer Pressure (1) cm of Hg	Pressure (2) lb/in^2
Vacuum				

Notes: (1) Manometer pressure = (left level) - (right level).
(2) Pressure = (0.491 lb/in^3) x manometer height.

SELF CHECK

1. Define the terms:

   Differential pressure.
   Concave meniscus.
   Convex meniscus.
   Vacuum/partial vacuum.

2. The differential height of a mercury column in a barometer is 76 cm. Calculate the differential height in meters, if oil (density 0.90 g/cm^3) is used instead of mercury (density 13.6 g/cm^3).

3. If a mercury manometer is used to measure a differential pressure of 2.0 psi, calculate the differential height between the two levels of mercury columns in inches.

4. In question 3 above, what would be the difference in column levels if water were used instead of mercury?

5. A pressure was applied to one arm of an open tube manometer (mercury filled) and the level difference was found to be 12.0 inches. Compute the applied pressure in pounds per square inch.

6. Sketch a diagram of a U-tube manometer, and label its parts.

7. Sketch a diagram of a well-manometer labeling its principal parts. State its advantages over the U-tube manometer.

8. Sketch a diagram of an inclined-tube manometer. State its advantages over the U-tube manometer.

9. For problems 3, 4, and 5, convert the results to metric and/or English systems units.

1E3

ELECTRIC FORCES AND FIELDS

This module introduces electric forces and electric fields encountered in the electromagnetic energy system. Electric forces exist between electrically-charged particles in much the same way that gravitational forces exist between different masses. Electric forces and electric fields are important factors in the operation of electrical current in conductors, electrical discharge, breakdown voltage, capacitors, particle accelerators, resistance-capacitance d.c./a.c. circuits and in the field of electronics in general.

This module presents the calculation and graphical representation of electric forces between particles and the electric fields around capacitors. The relationships between charge, voltage, and capacitance also is presented. In the laboratory, the student will produce and describe line patterns of electric fields.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Name the two different types of electric charge and give an example of a particle with each type of charge.
- Determine the electric force between charged particles of known charge and separation.
- Draw electric field lines around point charges and charged conducting plates, and describe how field lines indicate direction and charging strength of the electric force on charged particles located in the field.
- Draw a parallel-plate capacitor including electric field lines; and describe the relationships between plate charge,

potential difference and capacitance and potential difference, plate separation and electric field strength.

- Describe the use of parallel plate capacitors and the role of the electric field in television tubes and electrostatic precipitators.

- Given the appropriate equipment, map the electric field between simple conductors carrying opposite charge.

DISCUSSION

Electric force is a simple push or pull which has its origin in electrical charge. There are two fundamental kinds of electrical charge, positive and negative. The proton is an example of positive charge; the electron is an example of negative charge. Several commonly-known particles with their appropriate electric charge (coulombs) are tabulated in Table 1.

TABLE 1. SOME ELEMENTARY CHARGED PARTICLES

Particle	Charge (coulombs)	Mass (kilograms)
Electron	-1.602×10^{-19} C	9.108×10^{-31} kg
Proton	$+1.602 \times 10^{-19}$ C	1.672×10^{-27} kg
Alpha	$+3.204 \times 10^{-19}$ C	6.648×10^{-27} kg
Neutron	No Charge	1.675×10^{-27} kg

The electron and the proton have the same charge, although the proton is about 1836 times heavier. The alpha particle has twice the charge of the proton and is about 4 times heavier. The neutron has no charge; it is an electrically neutral particle. Table 1 also shows that all the charged particles listed are very light (almost massless) and carry very little charge. For example, about 6.24×10^{18} electrons (6,240,000,000,000,000,000 electrons) are required to equal one coulomb of electrical charge; therefore, the charge on an electron or proton, the smallest charge known, is a very small part of one coulomb of charge.

FORCES BETWEEN ELECTRIC CHARGES

Different masses, such as the sun and the earth, only can attract, never repel, each other. Thus, gravitational forces - the forces between different masses - are attractive. Electrical charges, on the other hand, both attract and repel one another. Figure 1 shows that "like-charges" repel and "unlike-charges" attract. A positive charge repels another positive charge, but attracts a negative charge. Similarly, a negative charge repels another negative charge, but attracts a positive charge. The magnitude of the force between charged particles is given by COULOMB'S FORCE LAW, as

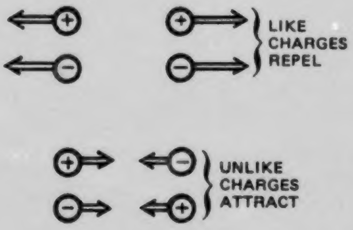


FIG. 1

indicated by Equation 1.

$$F = \frac{kQ_1Q_2}{d^2} \qquad (1)$$

where

F = the attractive or repulsive force in newtons.

k = a constant equal to $9 \times 10^9 \frac{\text{newton meter}^2}{\text{coulomb}^2}$.

Q_1, Q_2 = the charges of the particles in coulombs.

d = the separation in meters.

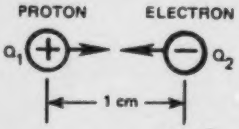
The use of Equation 1 in solving problems is illustrated in Examples A and B:

EXAMPLE A : ATTRACTIVE FORCE BETWEEN UNLIKE CHARGES

Given: An electron is separated from a proton by a distance of one cm.

Find: The force with which the electron pulls on the proton and with which the proton pulls on the electron.

Solution: The electron pulls on the proton with the same force that the proton pulls on the electron. This force is given by Equation 1. Thus,



$$F = \frac{kQ_1Q_2}{d^2}, \qquad k = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$$

$Q_1 = Q_{proton} = 1.602 \times 10^{-19} C$

(See Table 1.)

$Q_2 = Q_{electron} = -1.602 \times 10^{-19} C$

(See Table 1.)

$d = 1 \text{ cm} - 10^{-2} \text{ m}$

$$\therefore F = 9 \times 10 \frac{N \cancel{m^2}}{\cancel{C^2}} \times \frac{(1.602 \times 10^{-19} \cancel{C}) \times (-1.602 \times 10^{-19} \cancel{C})}{(10^{-2})^2 \cancel{m^2}}$$

$$\therefore F = -2.3 \times 10^{-24} \text{ N}$$

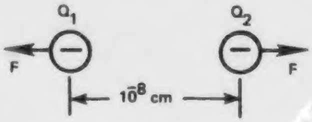
Since about 4.5 N are required to lift a one-pound weight, the electrical force here is very small. Electrical forces between charged particles are often very small.

EXAMPLE B: REPULSIVE FORCES BETWEEN LIKE CHARGES

Given: Two electrons in adjoining copper atoms are separted by 10^{-8} cm.

Find: The force with which the electrons repel each other.

Solution:



(From Equation 1.)

$\therefore F = \frac{kQ_1Q_2}{d^2}$

$k = 9 \times 10^9 \frac{N \cdot m^2}{C^2}$

$Q_1 = Q_2 = 1.602 \times 10^{-19}$ C (See Table 1.)

$d = 10^{-8}$ cm $= 10^{-10}$ m

$$\therefore F = \frac{(9 \times 10^9 \frac{N \cdot m^2}{\cancel{C^2}})\ (1.602 \times 10^{-19})^2\ \cancel{C^2}}{(10^{-10})^2 \cancel{m^2}} =$$

$F = 2.3 \times 10^{-8}$ N

This force is ten million billion times larger than the force in Example A, but is still small compared to one newton.

Notice that although the forces are often very small, as shown in both Examples A and B, they often act on very small particles (see masses of electron and proton in Table 1, for example) and therefore are quite effective in pushing or pulling these particles around.

ELECTRIC FIELDS

An electric charge in space presumably will change the space around it; the charge sets up an ELECTRIC FIELD. The electric field intensity E at a position P is defined as equal to the electric force per unit positive charge at position P, as shown by Equation 2.

$$\vec{E} = \frac{\vec{F}}{Q} \qquad (2)$$

where

$\vec{E}$ = the electric field intensity in N/C at position P.
$\vec{F}$ = the force in newtons on a charge Q at position P.
Q = the charge at position P.

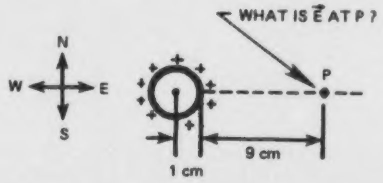
The electric field intensity $\vec{E}$ and force $\vec{F}$ are written with an arrow to show that each is a vector. Since a vector has both a magnitude and a direction, the electric field intensity $\vec{E}$ will have both a magnitude and direction, just as does the force $\vec{F}$. The magnitude of $\vec{E}$ (in newtons/coulomb) always will be given by the ratio F/Q in Equation 2. Its direction always will be given by the direction of the electric force $\vec{F}$ on a small positive test charge Q placed at the position in question. The use of Equation 2 is illustrated in Example C.

EXAMPLE C : ELECTRIC FIELD NEAR A CHARGED SPHERE

Given: A hollow metal sphere is charged to a high voltage, thereby producing an electric field in the space around it. The sphere has a charge of +10 coulombs and a radius of one centimeter.

Find: The electric field intensity $\vec{E}$ at a position 9 cm due east of the sphere.

Solution:



Note: Any charged conducting sphere can be replaced by a single-point charge of the same magnitude at the center of the sphere, so long as force and field calculations external to the sphere are to be determined; thus, point P is effectively 10 cm away from a 10 coulomb point charge. Now arbitrarily choose 0.01 coulombs as the small test charge at P. The choice of 0.01 coulombs for the test charge is not important, since its value does not affect the final value of E calculated below. (See note below.)

Then from Equation 1:

(a) $F = \dfrac{k\, Q_{sph} Q_{test}}{d^2}$

$$k = 9 \times 10^9 \ \frac{N \cdot m^2}{C^2}$$

$$Q_{sph} = 10 \text{ coul}$$

$$Q_{test} = 0.01 \text{ coul}$$

$$d^1 = 1 \text{ cm} + 9 \text{ cm} = 10 \text{ cm}$$

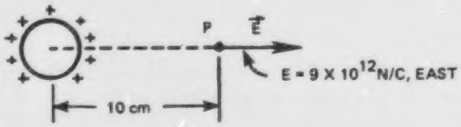
$$d^2 = 0.01 \text{ m}^2$$

$$F = \frac{(9 \times 10^9)\ (10)\ (\cancel{\tfrac{1}{100}})}{\cancel{\tfrac{1}{100}}} = 9 \times 10^{10} \text{ N}$$

and from Equation 2:

(b) $E = \frac{F}{Q_{test}} = \frac{9 \times 10^{10}}{1/100\ C}\ N = 9 \times 10^{12}\ N/C$

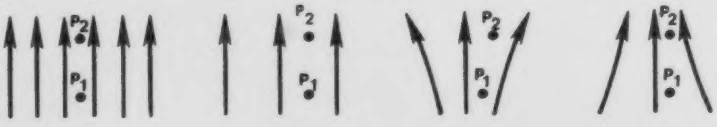
The complete answer for the required electric field intensity $\vec{E}$ at point P would then be 9×10^{12} N/C in an eastward direction. The direction of the field at a point is always taken as the direction of the force on a small positive test charge placed at that point. Thus:



Notice that the choice of 0.01 coulombs for the magnitude of the charge does not affect the final answer. The effect of Q_{test} on the answer is part (a) is cancelled out in part (b); therefore, a value of Q_{test} = 1 C can be taken more easily.

As illustrated in Example C, the direction of the electric field at any point is the direction of the force on a positive charge at that point. The advantage of having the concept of an E field is that one can measure the strength of the electric force without knowing anything about the charges that cause the force. Since forces which arise from many charges can be mathematically quite complicated, to work with the electric field is usually simpler than with the detailed charge configurations and the resulting forces between them.

In portraying an electric field around a charge distribution, whether the distribution is a single point charge or an assembly of charges on a conductor, imaginary lines are used to represent the electric field, both in direction and strength. These lines always begin on positive charges and end on negative charges. The direction of a line to any position in space around the charges is the direction of the force on a positive charge at that position. The strength of the field is represented by the density of lines; that is, the closer the lines, the stronger the field, as shown by the four separate drawings in Figure 2. Different representations of an electric field near two positions, P_1 and P_2, are shown in the figure.



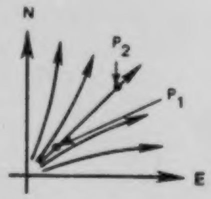
(a) (b) (c) (d)

FIG. 2

Figure 2a and 2b represent uniform or constant-strength fields, since the field lines remain the same distance apart. Figure 2a represents a field twice as strong as Figure 2b, since the lines are spaced twice as close together. In each case, Figure 2a and 2b, the field is constant, and the force on a positive charge at points P_1 and P_2 would be identical and directed upwards. In Figure 2c the field weakens in going from P_1 to P_2, since the lines spread. In Figure 2d the field intensifies in going from P_1 to P_2 since the lines crowd closer together. In Figure 2c the direction of the field at P_2 is mostly north but somewhat east. In Figure 2d the field is mostly north but slightly west. These principles are used to solve a problem in Example D.

EXAMPLE D : ELECTRIC FIELD PROBLEM

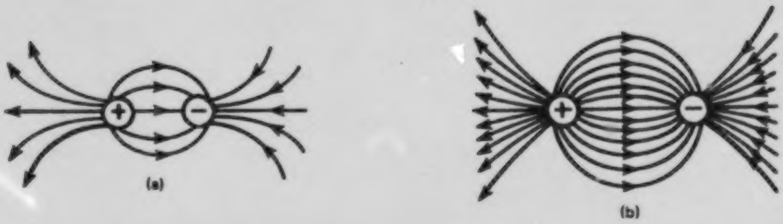
Given: Electric field configuration as shown. A point charge of $+2 \times 10^{-7}$ coulombs at position P_2 experiences a force of 6 newtons.



Find:
(a) The electric field at P_2.
(b) Whether force on the same charge is greater or smaller at position P_1.

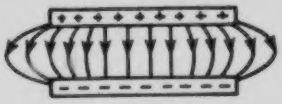
Solution:
(a) $E = \frac{F}{Q} = \frac{6\ N}{2 \times 10^{-7} C} = 3 \times 10^{7}$ N/C, direction northeast.
(b) The force on the same charge is greater at P_1, since field E is greater (lines are closer together.

Electric field lines around various charge configurations are shown in Figure 3. Electric field lines at any position always point in the direction of force on a small positive charge placed at that point. Figure 3a shows the field lines around an equal positive and negative charge-pair. In Figure 3b field lines are shown around another equal positive and negative charge pair as in a; but here the charges are larger, leading to a stronger electric field (thus lines are drawn closer together). Figure 3c shows the cross section of two oppositely-charged parallel plates. There are fringing, or bulging, effects of the field at the plate edges, and the field lines are evenly spaced at the center and throughout the interior region between the plates. These features indicate



(a) (b)

a constant strength field, as in Figure 2a and 2b.

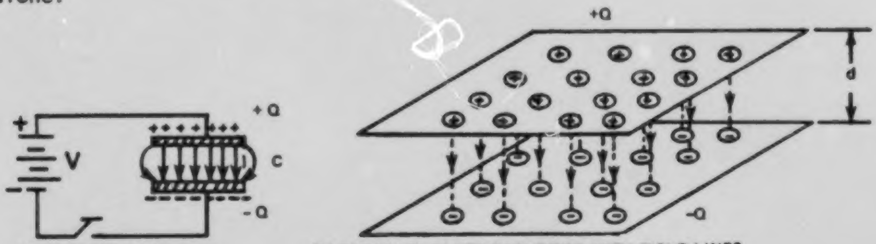


(c)

FIG. 3

PARALLEL PLATE CAPACITORS

A capacitor is defined as two conducting surfaces separated by a dielectric (insulating) medium. The simplest version of a capacitor, and one which illustrates the important relationships, is the so-called parallel plate air-capacitor. Figure 4a shows the charging circuit, and Figure 4b shows a three-dimensional view of the capacitor alone.



(a) CHARGING CURRENT

(b) PARALLEL PLATE CAPACITOR WITH FIELD LINES

FIG. 4

In Figure 4a, a battery with voltage V is used to charge a capacitor to a potential difference of V volts with a positive charge of +Q coulombs on one plate and a negative charge of -Q coulombs on the other plate. If the capacitor has a capacitance, the relationship between Q, C and V is given by Equation 3.

$$Q = CV \tag{3}$$

where

C = capacitance in farads.

V = volts.

In Figure 4c, the constant E field (fringing near the edges is neglected here) between the plates is shown by the dotted line with the direction pointing from +Q (top plate) to -Q (bottom plate). If the plates were charged with a battery of voltage V and the distance, in meters, between the plates is d, then the strength of the E field is given by Equation 4.

$$E = \frac{V}{d} \qquad (4)$$

where

E = electric field strength, expressed equally well in volts/meter or newtons/coulomb.

Example E illustrates the application of Equations 3 and 4 in a typical capacitor problem.

EXAMPLE E : CAPACITOR PROBLEM

Given: A parallel-plate capacitor whose plates are separated by 1 cm is charged to a voltage of 50 volts.

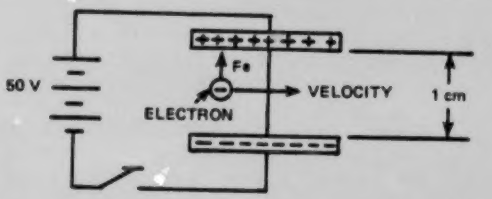
Find:

(a) The electric field strength between the plates.

(b) The force F_e exerted on an electron by the electric field if the electron is passing through the air space between the capacitor?

(c) If the plates each carry a charge of 50×10^{-6} C, what is the capacitance of the capacitor?

Solution:



(a) From Equation 4, $E = \frac{V}{d}$:

$E = \frac{50V}{10^{-2}m} = 5000$ V/m $= 5000$ N/C

(b) From Equation 2, $E = \frac{F}{Q}$ $\quad Q = 1.6 \times 10^{-19}$ C (See Table 1.)

$F = 5000$ N/C.

$F_e = EQ = (5000 \text{ N/C})(1.6 \times 10^{-19} \text{ C})$

$F_e = 8 \times 10^{-16}$ N, directed upward toward + plate.

(c) From Equation 3, $Q = CV$

$C = \frac{Q}{V} = \frac{50 \times 10^{-6} \text{ C}}{50V} = 10^{-6} \frac{C}{V} = 10^{-6}$ farad

C = 1 microfarad

APPLICATIONS OF ELECTRIC FIELDS AND CAPACITORS

Example E indicates that a charged particle, such as an electron, experiences a force because of the existence of an electric field in the region between the parallel plates of the capacitor. Such a force can be used to deflect moving electrons and thereby change their paths. This basic idea is used in controlling the path of electrons from the electron gun to the screen in a television tube. Figure 5 illustrates deflection of an electron beam with charged capacitor plates in a television tube/cathode ray tube.

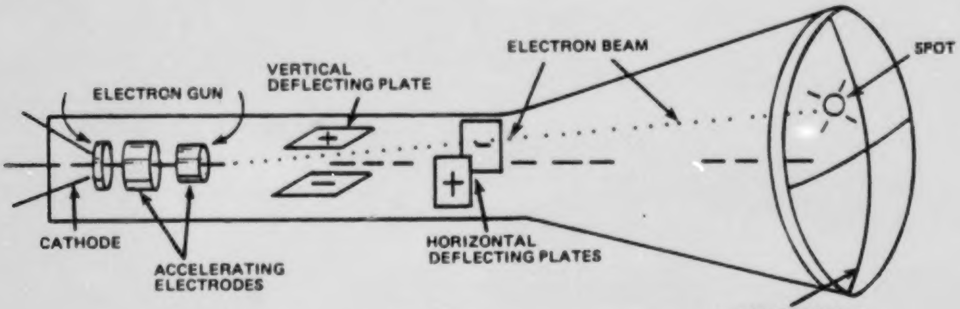


FIG. 5

The electron beam leaves the accelerating electrodes along the central axis of the tube. If the vertical deflecting plates are charged in such a manner that the top plate is positive and the bottom plate is negative, at the instant the electron passes through, it will be deflected upwards. Then, if the horizontal deflecting plates are charged with front plate positive, rear plate negative, as the electron passes between these plates, it will be deflected further toward the positive plate. The combined deflections will result in the electron beam striking the screen in the upper left quadrant, as shown. By changing voltage on the horizontal and vertical deflecting plates in a pre-arranged manner, the electron beam can be directed to various positions on the screen. In each instance, as in Example E, the existence of an electric field between the capacitor plates gives rise to an electric force on the charged particles (electron beam), which in turn deflects the motion of the particles.

In another basic application, known as electrostatic precipitation, the strong electric field between oppositely charged capacitor plates is used to precipitate out smoke, dust, fume, fog, haze, smog, aerosols, etc., from polluted air. In these days of environmental control, the purification of air in industrial environments is of great importance.

In the electrical precipitation process, three fundamental steps are required:

(a) electrical charging of foreign particles, that is, polluation.
(b) collection or deposition of charged particles in an electric field.
(c) removal of precipitated material from collecting electrodes to an external receptacle.

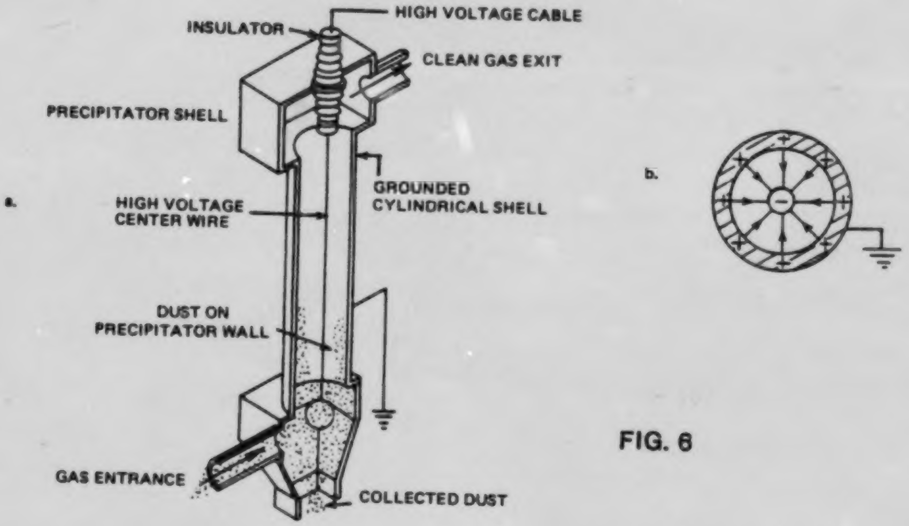

HIGH VOLTAGE CABLE
INSULATOR
CLEAN GAS EXIT
PRECIPITATOR SHELL
GROUNDED CYLINDRICAL SHELL
a.
HIGH VOLTAGE CENTER WIRE
DUST ON PRECIPITATOR WALL
GAS ENTRANCE
COLLECTED DUST
b.
−
+

FIG. 6

Figure 6a illustrates the general configuration in a single-state precipitator. In this arrangement the capacitor differs in geometry from the parallel-plate capacitors described above. The capacitor here consists of a high-voltage wire located along the central axis of a long pipe (cylinder). The center wire forms one "plate" of the capacitor; the cylinder wall forms the other "plate." Figure 6b shows the cylindrical capacitor in cross section, with the appropriate electric field lines.

The high voltage on the central wire (usually from 10,000 to 100,000 volts) creates a corona discharge region near the wire. The particle pollutants passing through the corona region pick up a negative charge of 300 electron charges for one-micron size particles and as high as 30,000 electron charges for 10-micron size particles. The strong electric field between the central wire and the cylindrical wall exerts a considerable force on the charged particles, driving them from the region of the corona discharge towards and onto the cylindrical walls. As shown in Figure 6, the pollutants are collected on the walls. The foreign particles are then shaken loose by some clever mechanical means and collected at the bottom of the precipitator. Clean air (or gas) is drawn out through the top.

A somewhat more sophisticated, two-stage arrangement of an electrostatic precipitator is shown in Figure 7. The left stage is the charging region where the pollutants flowing by the corona wires B and grounded cylinders G pick up electric charge. The right stage contains a series of parallel-plate capacitors labeled alternately C and D. The grounded collector plates are labeled C, the charged plates D. As the charged pollutants pass through the electric field region between plates C and D, the particles are deflected, collected on the plate surfaces C and removed. Purified air then emerges further downstream.

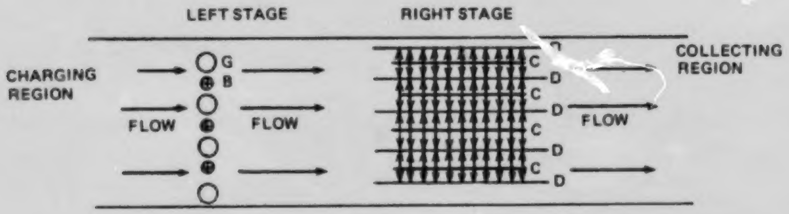


FIG. 7

SUMMARY

Electric forces between electrically charged particles form the basis of operation for various electronic components and devices. Such electric force produces electric fields around a component such as a capacitor. Some electric fields are used for deflecting electron beams in cathode-ray tubes; others, for electrostatic precipitation and other purposes.

LINE PATTERNS OF ELECTRIC FIELDS

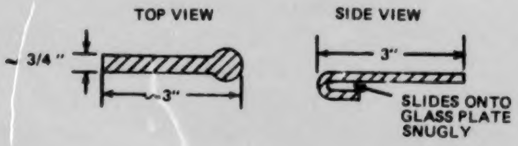
MATERIALS

8 x 8-inch glass plate.
4 rubber suction cups (or 4 rubber stoppers).
1 Induction coil or Wimshurst-type electrostatic machine.
Assorted electrodes made of copper or aluminum -
    (see experiment for details).
Chopped, clean straight hair (1/4 to 1/2 inch lengths).

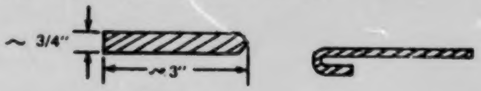
PROCEDURE

This experiment is designed to show the electric field line patterns around various charged electrodes. The electrodes to be used should be constructed from thin aluminum or copper metal stock; however, if these are unavailable, aluminum foil can be used. The shapes required, with essential dimensions, are shown below in Figure 8:

(a) Circular-end electrodes (2 each)



(b) Pointed-end electrodes (2 each)



(c) Parallel cross-strip electrodes (2 each) to simulate parallel-plate capacitors

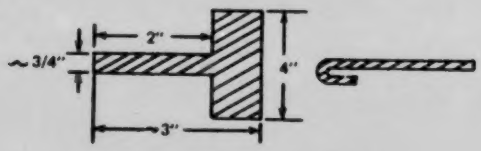


FIG. 8

1. Arrange apparatus as shown in Figure 9 with a clean glass plate lying on four clean rubber stoppers or four clean rubber suction cups. (Cleanliness is required to prevent leakage of charge.)

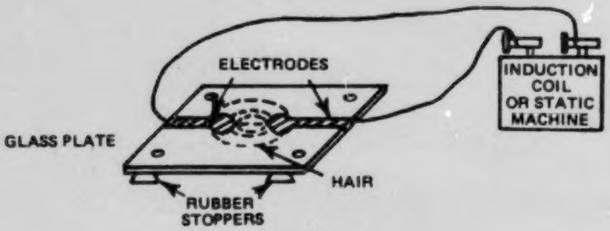


FIG. 9

2. Center the circular end electrodes on the glass plate as shown, ensuring that end clips fit snugly onto glass plate.

3. Attach wires from terminals of induction coil or static machine to electrodes.

4. Plug in induction coil or crank static machine to ensure a high voltage on electrodes. (Caution: Avoid touching electrodes and lead wires in operation.)

5. While voltage is on and electrodes are charged, sprinkle chopped hair lightly in central region between electrodes. Chopped hair should align itself along clearly-visible field lines. Tap glass plate lightly and repeatedly with rubber end of a pencil to facilitate orientation of chopped hairs.

6. Draw a top view of glass plate, electrodes and field lines (roughly to scale) on a clean sheet of paper. The drawing should be roughly to scale and should indicate clearly the electric field line pattern.

7. Remove induction coil or static machine wires, remove electrodes and collect chopped hair. Repeat steps 2 through 6 with the pointed-end electrodes and parallel cross-strip electrodes.

8. Explain any differences between the field patterns of step 6 and step 7.

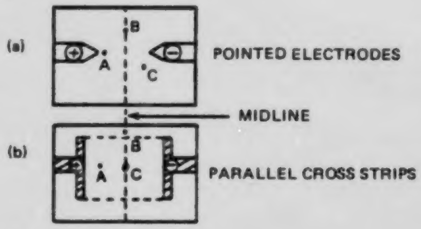
SELF CHECK

1. State one similarity and one difference in the charge of an electron and that of a proton.

2. Determine the force in newtons experienced by two protons separated by a distance of one angstrom (1A° = 10^{-10} m).

3. A parallel-plate capacitor of 10µ farad capacitance is charged to a voltage of 100 volts. The plates are separated by one millimeter of air. What is the electric field in the region between the plates in volts/meters? in newtons/coulomb? How much charge in coulombs was deposited on each plate in the charging process? If an alpha particle (see Table 1) is moving along a midplane between the capacitor plates, what is the force on the alpha particle? Is the force the same at the ends of the capacitor as it is near the center?

4. Refer to the electric fields mapped in the laboratory section of this module. For the electrode geometries shown in the adjoining figure and for the point A, B, and C, as marked, indicate with arrows the relative strength of the field, the direction of the electric field and the direction of the force on an electron placed at those points. Assume electrodes charged as shown.



5. Define:

   Coulomb force.
   Electric field line.
   Electric field.
   Parallel plate capacitor.

6. In words, express the basic equation for the following:

   Force between two charged particles.
   Relationship between electric field $\vec{E}$, charge Q, force $\vec{F}$.
   Relationship between E, V and d for a parallel plate capacitor.
   Relationship between Q, C and V for a parallel plate capacitor.

7. Describe how parallel plate capacitors and electric fields play central roles in television tubes and electrostatic precipitators.

8. What is the electric field intensity due to a charged sphere 30 cm away from the center of the sphere if the charge on the sphere is 2.5×10^{-4} coulombs?

9. How far apart are the plates of a parallel plate capacitor if the charge on the plates are plus and minus 1.5×10^{-6} C respectively; the capacitor is valued at 2 μf and E = 2500 N/C?

10. A test charge is 40 cm away from a sphere having a net charge of (+)5×10^{-6} C. If the test charge experiences a force of 10 N radially away from the center of the sphere, what is the charge of the test charge?

1E4

MAGNETIC FORCES AND FIELDS

Magnetic forces and fields are vital to the operation of generators, motors, induction coils, inductors, particle accelerators and to the general field of electronics. This module discusses the origin of magnetic forces, the representation of magnetic forces in terms of magnetic fields, and the mapping of magnetic fields around bar magnets, horseshoe magnets, iron-core solenoids and long, straight wires. It also discusses how forces develop on current-carrying conductors in the presence of a magnetic field and how this principle is used in the construction of generators and motors. In the laboratory, the student will map magnetic fields, observe the results of forces upon current-carrying conductors in magnetic fields, and will construct and operate a simple electromagnet.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Explain how moving charges produce magnetic fields in a permanent magnet and electromagnet.
- Explain the nature of attractive and repulsive forces between north and south poles of permanent magnets.
- Draw the magnetic-field lines around bar magnets, horseshoe magnets, a long, straight wire carrying a current and a solenoid carrying a current.
- Predict the location of the "equivalent" north and south poles of a solenoid from the direction of flow of electron current through the windings.
- Given the direction of the magnetic field, the relative field

strength and direction of motion of a charged particle in that field, predict the direction and relative strength of magnetic force exerted on the particle by the magnetic field.

- Given the direction of a magnetic field and the direction of motion of a conducting wire through that field, predict direction of electron current flow in the wire.
- Given the direction of a magnetic field and direction of current flow along a wire in that field, predict direction of the magnetic force on the wire.
- Explain with use of diagrams how the combination of a magnetic field and a moving, current-carrying conductor leads to the construction of a motor and a generator.
- Given the appropriate equipment, (1) show the magnetic-field patterns around different geometries of permanent magnets; (2) demonstrate the interaction between current-carrying conductors and magnetic fields involving long, parallel wires carrying currents; and (3) construct a simple lifting electromagnet, and determine its lifting force.

THE ORIGIN OF MAGNETISM

First experiences with magnetism and magnetic forces usually come from contact with bar magnets and compasses. Bar magnets and compasses have north and south poles that strongly attract one another, while two north poles or two south poles strongly repel one another. Also, some magnets, such as lodestone, are "permanent" in behaviour; that is, they exhibit magnetic properties seemingly forever. On the other hand, some magnets are "temporary" in behaviour, exhibiting magnetic properties only while a current is flowing near a "magnetic substance" such as soft iron. Electromagnets - wires wrapped around soft iron cores - are good examples of temporary magnets.

The origin of natural, permanent magnetism - as found in the substance called lodestone - or of temporary magnetism, as produced in electromagnets, can be traced in either case to the existence of microscopic atomic currents deep within matter itself. A tiny current, such as that formed by an electron whirling around inside an atom, gives rise to a magnetic force field in a region surrounding the electron's orbit.

In Figure 1a, an electron is whirling around in an orbit with axis tilted as shown. In Figure 1b, the magnetic field created by the movement of the electron in its orbit is shown as if it were produced by a tiny magnet aligned along the same axis with appropriate N and S poles.

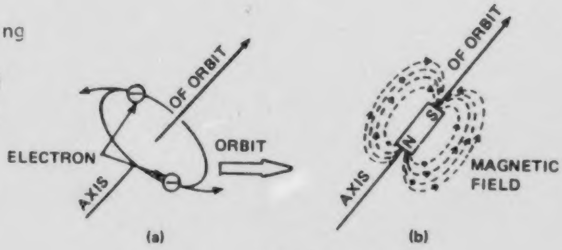


FIG. 1

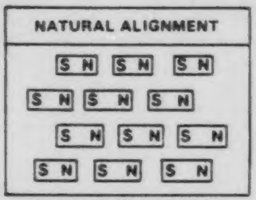
In effect, as illustrated in Figure 1, the tiny electron current can be thought of as a tiny bar magnet with an appropriate N and S pole giving rise to a magnetic field around the bar magnet. In natural or permanent magnets such as lodestone, therefore, all must be permanently aligned in such a manner that all the tiny equivalent magnets will add up to one large effecttive magnet. For permanent magnets, then, the key idea is clearly in the natural alignment of clusters of atoms - called magnetic domains - which in turn assures that the overall addition of billions and billions of the tiny equivalent magnets will result in one large effective magnet. (See Figure 2a.) On the other hand, initially unmagnetized materials somehow must lack the precise alignment present in the magnetic domains of permanent magnets; that is, here the alignment must be random in nature. (See Figure 2b.) The addition of billions and billions of randomly-ordered, unaligned, tiny equivalent magnets results in an overall effect of no magnetism, as with ordinary soft iron. But in unmagnetized substances - such as soft iron - which can be magnetized, these billions and billions of initially-disordered tiny magnets can be rearranged and aligned (as in permanent magnets) by an external magnetic force threading through the substance. In this manner, an unmagnetized substance often can be magnetized as shown in Figure 2c.

MAGNETIC FORCE AND FIELD

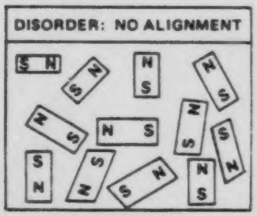
As indicated in the introductory discussion of the origin of magnetism, all magnetic substances can be represented in terms of two poles, a north (N) and south (S) pole. Any attempt continually to subdivide a bar magnet - as shown in Figure 3 - with the intention of separating and isolating one pole from the other, is doomed to failure. Subdivision of a magnet simply results in a smaller magnet of less strength, but always with two poles, N and S, just as in the original magnet. To this date, although scientists have been trying for years, no one has succeeded in isolating a free magnetic pole, either a south or north pole. Thus one is forced to work with both N and S poles when studying magnetic forces. (When electric forces were studied in Module 1E3, no trouble separating electric charge of positive and negative character from one another was encountered.)

If two simple bar magnets are arranged with their north poles (or south poles) next to one another, the magnets will strongly repel each other. This observation is summarized by the statement, "like magnetic poles repel." On the other hand, if the same two bar magnets are arranged with their N and S poles next to each other, they will attract each other strongly, which demonstrates that "unlike magnetic poles attract."

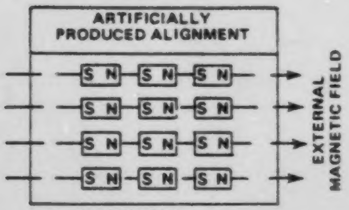
Figure 4 shows a single bar magnet with clearly labeled N and S poles. Drawn in the space surrounding the magnet are a series of lines to represent the direction of force on a tiny N pole of a very long magnet. (The force on the distant south pole is unimportant here.) The series of lines, just as in the discussion of electric charges and electric force in Module 1E3, constitutes a field, in this case, a magnetic field. Figure 4 shows the magnetic-field lines around a simple bar magnet. Solid lines represent the field outside the magnet; dotted lines represent the field inside the magnet. The lines always point in the direction of the force on a small north pole located at the given point in space. At the left end (N) of the bar magnet in Figure 4, field lines are directed away from the N pole (since two N poles repel); and at the right end (S), the field lines are directed toward the S pole (since a N and S pole attract). And here, as in Module 1E3 on the electric field, the strength of the magnetic field is indicated by the density of lines drawn. Closely-spaced



PERMANENT MAGNET (STRONG MAGNETIC FIELD)



UNMAGNETIZED SUBSTANCE (NO MAGNETIC FIELD)



MAGNETIZED SUBSTANCE (STRONG MAGNETIC FIELD)

FIG. 2

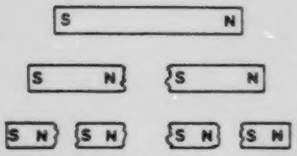


FIG. 3

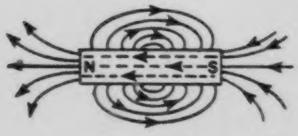
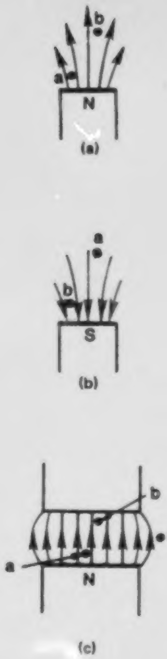


FIG. 4

lines indicate a strong field; widely-spaced lines indicate a weak field. Uniformly-spaced lines indicate a magnetic field of constant strength, and diverging or converging field lines indicate a decreasing or increasing magnetic field strength, respectively.

In Figure 5a, the magnetic field weakens in going from point a to b since the field lines spread. In Figure 5b, the field intensifies in going from point a to point b, since the lines are more densely packed. In Figure 5c, the field remains constant in going from a to b, since the lines there remain equally spaced. In going from a to c in Figure 5c, the field would, of course, weaken.



(a)
(b)
(c)

FIG. 5

The pattern of magnetic-field lines for various geometries is shown in Figure 6. The first two drawings, 6a and 6b, illustrate the magnetic field lines near adjoining N-S poles and N-N poles. The third drawing maps the field lines near a horseshoe magnet; that is, a simple bar magnet bent in a U-shape. The fourth drawing shows the field lines around a long, straight wire carrying electron current, and the fifth sketch shows the field lines around an air core and iron core solenoid (coil).

The patterns of the field lines in Figures 6a and 6b are vastly different, and the patterns in 6a and 6c are quite similar. Note that the magnetic field created by a long wire carrying a current I is a series of concentric rings encircling the wire, as shown in Figure 6d. The direction of the magnetic field lines can be determined easily as follows. If the thumb of the left hand is placed along the wire in the direction of electron current flow, the fingers of the left hand wrap themselves around the wire in the direction of the circular magnetic field lines. In Figure 6d, therefore, the thumb would point downwards in the direction of current flow, and the fingers would encircle the wire in a counterclockwise direction (looking down from the top), as shown.

For the ion-core and air-core solenoids (Figure 6e), the shape of magnetic field lines is similar to that shown in Figure 4 for a simple bar magnet. For electron current flowing around the core of the solenoid as shown, each solenoid "behaves" like a simple bar magnet, with a N pole at the left end and a S pole at the right end. Which end of the solenoid behaves like a N pole can be determined from the following simple rule. If the fingers of the left hand are wrapped around the solenoid in the direction of electron current flow, the extended thumb will point to the end of the solenoid which behaves like a N pole. If this rule is applied to Figure 6e, the left ends of the solenoids are the indicated N poles.

A comparison of magnetic-field sketches for the air-core and iron-core solenoids shows that the field is intensified in the center of the iron-core solenoid by the addition of the iron core. A stronger magnetic field can be obtained with a solenoid of given current and windings simply by inserting an iron core through the solenoid.

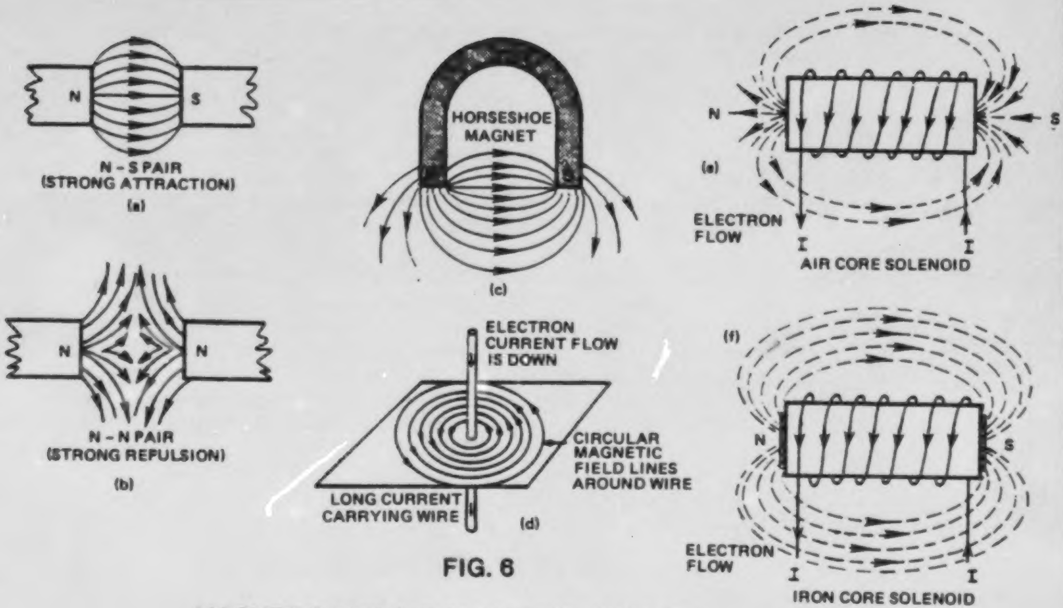
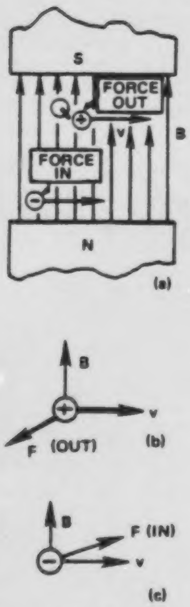

N
S
N - S PAIR
(STRONG ATTRACTION)
(a)
N
N
N - N PAIR
(STRONG REPULSION)
(b)
HORSESHOE
MAGNET
(c)
ELECTRON
CURRENT FLOW
IS DOWN
CIRCULAR
MAGNETIC
FIELD LINES
AROUND WIRE
LONG CURRENT
CARRYING WIRE
(d)
N
S
(e)
ELECTRON
FLOW
I
I
AIR CORE SOLENOID
(f)
N
S
ELECTRON
FLOW
I
I
IRON CORE SOLENOID

FIG. 6

MAGNETIC FORCES ON MOVING CHARGES


S
FORCE
OUT
Q
+
v
B
FORCE
IN
−
N
(a)
B
+
v
F
(OUT)
(b)
B
F (IN)
−
v
(c)

Simple forces exerted on electric charges moving in a magnetic field are essential for the operation of the generator and the motor.

In the upper half of Figure 7a, a positively-charged particle of magnitude +Q is shown moving through a uniform magnetic field of strength B with a velocity v. The magnetic field is directed upwards. In this configuration, the force F exerted on the positive charge is directed OUT of the page toward the reader (Figure 7b). If the particle was negatively charged, everything else being the same, the force on the negative charge would be directed IN to the page, away from the reader. This situation is pictured in Figure 7c and the lower half of Figure 7a. The magnetic force F for a charged particle Q moving with velocity v in a magnetic field B is always directed at right angles to both the direction of the field $\vec{B}$ and the velocity $\vec{v}$. The direction of force on a moving POSITIVE charge can be determined from the following simple rule:

(i) First, locate the plane determined by vectors $\vec{v}$ and $\vec{B}$ and arrange $\vec{v}$ and $\vec{B}$ with their tails at the same position P. Suppose $\vec{v}$ is up and $\vec{B}$ is to the right, as in Figure 8a.

(ii) Next, imagine a "right-handed" screwdriver oriented perpendicularly to the plane of $\vec{v}$ and $\vec{B}$, with its

FIG. 7

blade positioned at P. Now imagine turning the screwdriver in the same direction which is required to rotate vector $\vec{v}$ into $\vec{B}$ (with B held fixed) through the smaller angle separating $\vec{v}$ and $\vec{B}$. (See Figure 8b.)

(iii) The direction of advance (or recession) of a screw being driven by the rotating screwdriver gives the direction of force on the positive particle. In Figure 8b, the clockwise rotation of the screwdriver would cause the screw to advance;therefore, the direction of force is into the page.

(iv) If the particle had been negative, the rule would have been applied as explained in the three steps above, except that the direction obtained in (iii) simply would have been reversed; that is, if step (iii) had indicated a force IN for a positive particle, the direction would have been OUT for a negative particle.

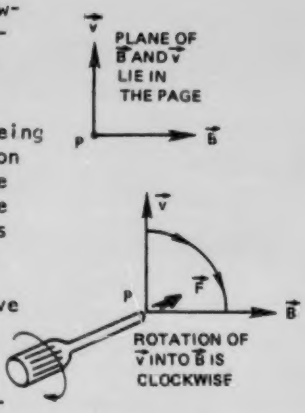

$\vec{v}$
PLANE OF $\vec{B}$ AND $\vec{v}$ LIE IN THE PAGE
P
$\vec{B}$
$\vec{v}$
P
$\vec{F}$
$\vec{B}$
ROTATION OF $\vec{v}$ INTO $\vec{B}$ IS CLOCKWISE

FIG. 8

The amount of force on the charged particle depends upon the velocity component perpendicular to the magnetic field. There is no force on a particle moving parallel to the magnetic field.

GENERATORS AND MOTORS

The force on a moving charge in a magnetic field lies at the heart of the principle of operation of the electric generator and electric motor. In the generator, external mechanical forces are used to push a conducting wire through a strong magnetic field, thereby inducing magnetic forces on the charge parallel to the wire and generating a current in the wire. In the motor, electric current (moving charge) is forced through a conducting wire located in a strong magnetic field. The magnetic forces in this case act on the charges in the wire, but in a direction perpendicular to the wire, thereby pushing the wire. In generators, external mechanical power is converted into electric power. In motors, external electric power is converted to mechanical power.

The basic idea behind the generator is illustrated in Figure 9.

In Figure 9a, a long, straight conducting wire filled with electrons and initially carrying no current, is pushed downward into a magnetic field B directed from left to right. When the wire is forced downward, each electron in the wire also moves downward with the wire: A negative charge moves downward in a magnetic field directed from left to right, representing electric charge moving perpendicularly in a magnetic field. In accordance with Figure 7, the moving charge (electron) must experience a magnetic force. Application of the screwdriver rule explained in connection with Figure 8 indicates that magnetic force upon each electron must be directed along the wire INTO the page, away from the reader. Motion of the wire in the magnetic field therefore

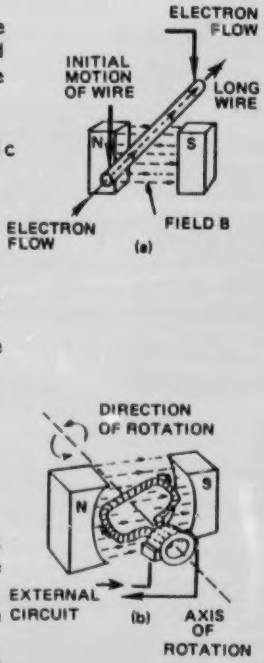


FIG. 9

generates an electron current flow in the wire.

If the wire shown in Figure 9a is part of a loop shown in Figure 9b and if the loop rotates as shown, electron current can be generated in the left and right portions of the loop. Current is generated in opposite directions as shown, to provide continuous current flow to an external circuit. This design, then, is the simplest example of a working generator.

The electric motor can be thought of as a generator working in reverse. In Figure 10, the wire is initially stationary, situated in the magnetic field as shown. Now a current (electron) is forced through the wire, out toward the reader. This process again constitutes electric charge moving in a magnetic field; in accordance with Figures 7 and 8, a magnetic force in a specified direction will act upon the moving charge. Application of the right-handed screwdriver rule shows that the force on the moving charge is directed downwards. Since this moving charge is perpendicular to the wire, the wire experiences a force and moves downward. Thus electrical power used initially to force current through the conductor therefore is transformed into the mechanical power in the moving wire. If the wire in Figure 10 is made part of a loop and fastened to a rotating shaft, somewhat as shown in Figure 9b, the mechanical power generated in the rotating loop can be used to do useful external work.

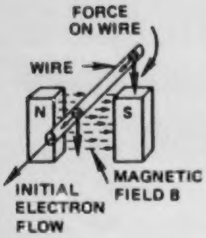


FIG. 10

SUMMARY

Magnetic forces and fields play an important role in electricity and in electronics. Electric motors, generators, coils, inductors, etc., depend upon such forces and fields. In the case of motors and generators, magnetic fields exert forces upon electric charges moving through them.

MAPPING MAGNETIC FIELDS

MATERIALS

2 bar magnets.
1 U magnet.
Non-magnetic metal base (see Figure 11).
Double-strength window glass plate, 8 x 11 inches.
Iron filings in sifter-type jar.

PROCEDURE

1. Refer to Figure 11. Slip glass plate under the metal clips in such a manner that it rests securely on non-magnetic metal base.

2. Slide bar magnets into slots shown with S pole next to N pole, as shown. Sprinkle iron filings onto glass plate near center, over the N-S pole pair. Tap glass plate several times until the magnetic field lines stand out. Compare with Figure 6a. Remove glass plate; carefully brush iron filings from glass plate; and replace glass plate.

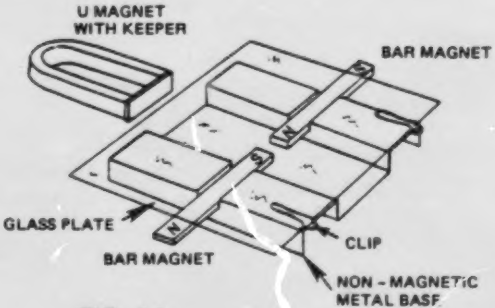


FIG. 11

3. Reverse orinentation of one of the bar magnets in (2) above so that a N-N or S-S pole pair exists under the center of glass plate. Sprinkle iron filings on glass plate and tap as necessary. Compare magnetic-field line pattern exhibited by iron filings with Figure 6b. Again, remove glass plate; brush off filings; and replace glass plate.

4. Remove bar magnets from side slots, and slide U magnet (without keeper) into large center slot with two poles underneath center section of plate. Sprinkle iron filings on plate and tap plate as needed. Examine magnetic field lines and compare with Figure 6c.

5. Why do the line patterns in steps 3, 4 and 5 differ?

FORCES BETWEEN CURRENTS END FIELDS

MATERIALS

1 wooden stand, 12 x 30 inches. (See Figure 12.)
2 flexible braid wires, 25 inches long x 1/16 inch diameter (use braided ground sheath in a coaxial cable if braid wires are not readily available).
1 double-pole, double-throw switch.
1 12-volt lead-acid auto battery.
Heavy-duty copper wire and battery clips.

PROCEDURE

1. Refer to Figure 12. In addition to two vertical braided wires stretched over 10-inch hole, as shown in Figure 12, all other connecting wires to the DPDT switch and 12-volt auto battery must be heavy gauge, capable of carrying large currents.

2. Arrange the apparatus - all home built - as shown in Figure 12a, wired as shown in Figure 12b. Caustion: Leave DPDT switch in open position. (Wires from battery to DPDT switch should be heavy battery cables with lugs on one end and spring clips on the other.) Wire W in Figure 12b should be connected from top of board to the DPDT switch in such a manner as not to be either parallel to or too near two vertical braided wires. The two braided wires, each about 25 inches long, should not be stretched too tightly between upper and lower connections on wooden board; otherwise, sideways attraction and repulsion will not be visible.

3. If electron current moves through each wire in same direction (say, up - this is position A of the DPDT switch) the two wires will be attracted and move towards each other. If electron current moves through each wire in opposite directions (say, up along the right wire and down along left wire - this is position B of DPDT switch), the two wires will repel and push away from each other. The attraction and repulsion easily can be observed near center of the wires where sideways movement is the largest.

4. Illustrate attraction and repulsion between the two wires by throwing the DPDT switch first to position A, then to position B. BE SURE NOT TO LEAVE DPDT SWITCH IN POSITION A OR B FOR MORE THAN SEVERAL SECONDS, SINCE VERY LARGE CURRENTS ARE DRAWN. LARGE CURRENTS DRAWN FOR EVEN A FEW MINUTES WILL DRAIN BATTERY OR OVERHEAT WIRES.

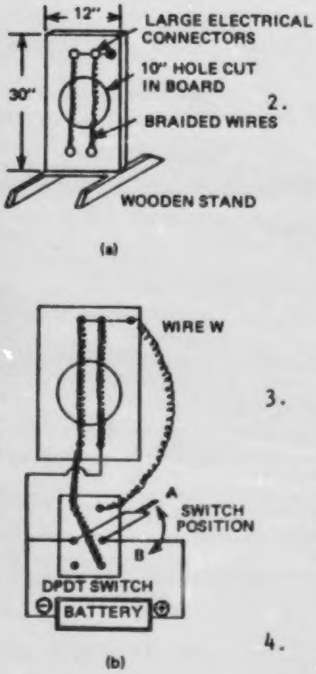


FIG. 12

5. Refer to Figure 6d; it shows direction of a magnetic field set up around a straight, current-carrying wire. Refer to Figures 8 and 10, which show how to determine the direction of force on a wire carrying a current when that wire is in the presence of a magnetic field. The wire on the right side in Figure 12a creates a magnetic field around the wire on the left side. After determining which way the electron current moves in the left wire for either position A or B of the DPDT switch, deduce (with the help of Figures 6d, 8 and 10) the direction of force on left wire, thereby verifying attractive or repulsive forces observed in the actual experiment.

MAKING A LIFTING ELECTROMAGNET

MATERIALS

20 x 1 cm bar of soft iron bent into U-shape. (See Figure 13.)
Several 1 1/2-volt laboratory dry cells.
About 125 ft of #25 cooper wire, single-layer, cotton-covered.
Armature bar with ring (see Figure 13).
2 plastic or wooden spools, 6 cm long x 1 cm bore x 1.5 cm wrapping radius.

PROCEDURE

Refer to Figure 13. The purpose of this experiment is to construct an electromagnet as shown in Figure 13 and to determine it lifting force.

1. Bend a piece of soft iron (20 cm long x 1 cm diameter) into a symmetrical U shape. Obtain or make two spools (6 cm long x 1 cm bore x 1.5 cm wrapping radius) that fit snugly over the ends of the U-shaped iron core.

2. Wrap the spools with #25 copper wire (single-layer cloth-cover), laying down about 45 turns per linear inch of spool. Wrap

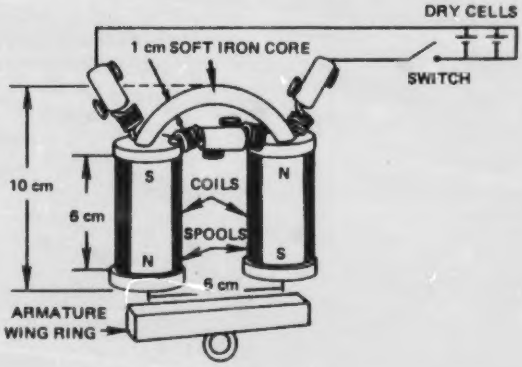


FIG. 13

each spool with two layers. Pay close attention to the direction of wrap around each coil, marking the direction, if necessary, on each coil. Slip spools on iron core, and connect two coils in such a manner that when current flows through the two spools, the ends of the spools near the ends of the iron core will be opposite poles, either N-S or S-N. Use the rule given in connection with Figure 6e to confirm that the poles are correct. For example, if the lower end of the left spool has a N pole because of direction of electron current flow through its windings, as shown in Figure 13, then the right spool must be placed on the iron core in such a manner that electron current through its windings will assure a S pole on its lower end.

3. Now connect a 1.5-V dry cell battery (or two 1.5-V dry cell batteries in parallel) across loose ends of wires on the two spools, as shown in Figure 13. Close the single-throw switch, and confirm that armature is pulled against faces of iron core spools. (If there is no magnetic attraction, carefully check the circuit and especially the spool connections.) If there is magnetic attraction, measure the lifting force of magnet by hanging known weights on the armature ring until the armature just breaks away. (Under the conditions given for construction of this electromagnet, there should not be much more than 0.5 amperes flowing through the dry cells and the wire. Nevertheless, check for overheating of spool wires; and do not leave the switch closed too long, since unnecessary battery drain will result.)

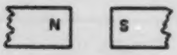
4. Discuss several possible applications of electromagnets.

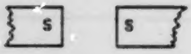
SELF CHECK

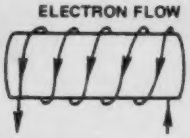
1. Explain with the use of diagrams how magnetic fields are produced by moving electrons in a permanent magnet.

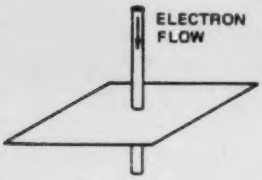
2. State the rules of attraction and repulsion between north and south magnetic poles.

3. Sketch the magnetic-field lines in each of the following figures:



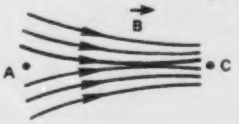


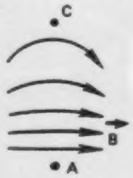

ELECTRON FLOW


ELECTRON
FLOW

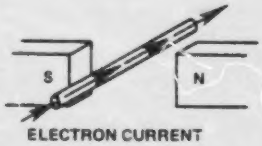
4. Indicate locations of north and south poles of solenoid in above figure.

5. An electron moves with constant speed from point A towards point C in the magnetic fields sketched below. In each case does the magnetic force on the electron increase or decrease along its path? In which direction is the electron deflected?

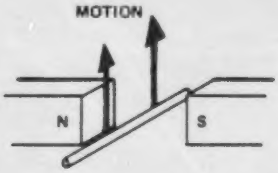




6. A long wire carrying an electron current, as shown in sketch, is located between a N-S pole pair. What is the direction of the magnetic force on the wire?


S
N
ELECTRON CURRENT

7. A long, straight conducting wire is forced upwards in the magnetic field between the N-S pair shown below. What is the direction of electron current flow in the conductor caused by the magnetic force?



8. Draw and label a diagram of an electric motor-generator.

1T1

THERMOMETERS

The thermometer is an instrument used to measure temperature. A portion of the thermometer is brought into thermal equilibrium with the body whose temperature is being measured. The indicated temperature is the equilibrium temperature of the sample and the thermometer sensor. For example, an ordinary medical thermometer consists of a bulb filled with mercury that expands into a tube as its temperature rises. This force of expansion produces a temperature reading as the mercury level indicates the temperature of the bulb and the patient on the calibrated tube. There are other types of thermometers, several of which are included in this module. In the laboratory, the student will calibrate thermometers and convert from Fahrenheit to Celsius scales.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the terms:

  Fahrenheit temperature scale.
  Celsius temperature scale.
  Absolute zero.
  Kelvin temperature scale.
  Rankine temperature scale.

- Draw the following devices and label the basic parts:

  Thermocouple.
  Pyrometer.
  Liquid-in-glass (thermometer).

- Convert a temperature given in one scale to its equivalent in any other scale without being given the conversion formula.
- Given two liquid-in-glass uncalibrated thermometers, calibrate one in Celsius and one in Fahrenheit and measure a given container of water within ±2°.

DISCUSSION

The most commonly-used type of thermometer is the liquid-in-glass thermometer, illustrated in Figure 1.

Volume is one physical property of a liquid that changes with temperature. The volume of mercury, for example, increases if the temperature is increased and decreases if the temperature is decreased. The increased volume of warmed mercury within the confined glass tube of the thermometer causes the level to rise. By selecting certain reference points (such as the freezing and boiling point of water) and uniformly subdividing the distance between them, a temperature scale is established.

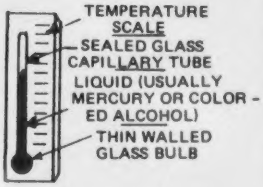

TEMPERATURE SCALE
SEALED GLASS CAPILLARY TUBE
LIQUID (USUALLY MERCURY OR COLORED ALCOHOL)
THIN WALLED GLASS BULB

FIG. 1

TEMPERATURE SCALES

Until recently the "Fahrenheit" temperature scale was used almost exclusively in the United States. There is now an intensive effort to adopt the "Centigrade," or "Celsius," scales used throughout the rest of the world. In defining either scale, it is necessary to determine two reference-point temperatures. Once these reference points are determined, the position of the zero point and the size of the temperature unit is obtained. One of these reference points is called "freezing point." This is the temperature at which water changes to ice. The other point is called the "boiling point." This is the temperature at which the water boils and changes to steam. On the Celsius scale, the freezing point is given a value of 0° and the boiling point, 100°. On the Fahrenheit scale these points are assigned the numbers 32° and 212° respectively.

When constructing a liquid-in-glass thermometer with a Celsius scale, first insert the thermometer in an ice-water mixture and carefully mark the tube at the height of the liquid column. Then immerse the thermometer in boiling water and again mark the height of the liquid column on the tube. The distance between these marks is then divided into one hundred equal parts and numbered from 0° to 100° as shown in Figure 2. If required, the scale can be extended above 100° and below 0°. The Fahrenheit scale, illustrated in Figure 2, is determined in a similar way by dividing the length of the column between the freezing and boiling points into 180 equal divisions and extending the scale in either direction. 0°C is the equiva-

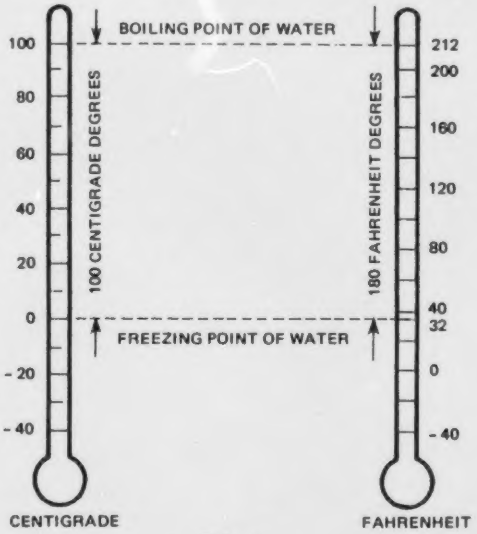

BOILING POINT OF WATER
100
80
60
40
20
0
-20
-40
100 CENTIGRADE DEGREES
180 FAHRENHEIT DEGREES
212
200
160
120
80
40
32
0
-40
FREEZING POINT OF WATER
CENTIGRADE
FAHRENHEIT

FIG. 2

lent temperature to 32°F and 100°C is the equivalent temperature to 212°F.

There is a simple method of recalling the relationship between Fahrenheit and Celsius temperature scales. If the temperature in Fahrenheit is known and is to be converted to Celsius, simply draw two simple thermometers and establish ratios as shown in Example A.

EXAMPLE A: SCALE RATIO BETWEEN CENTIGRADE AND FAHRENHEIT

Given: T_C or T_F.

Find: Their equivalent in the other system.

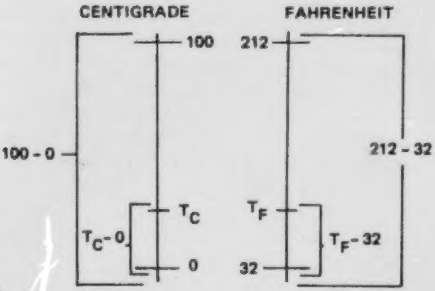
Solution: Equate the ratios from both sides:

$$\frac{T_C - 0}{100 - 0} = \frac{T_F - 32}{212 - 32}$$

Solving this ratio for T_C or T_F yields:

$$T_C = \frac{5}{9}(T_F - 32)$$

$$T_F = \frac{9}{5}\,T_C + 32$$



Example B shows a calculation for the temperature values when T(°F) = T(°C).

EXAMPLE B: CALCULATION OF VALUES WHEN T°F = T°C

Given: T(°F) = T(°C).

Find: The actual temperatures at this condition.

Solution: $T_C = \frac{5}{9}(T_F - 32)$

Since $T_C = T_F$, T_F is replaced with T_C thus:

$T_C = \frac{5}{9}(T_C - 32)$

$$9\ T_C = 5\ T_C - 160$$

$$4\ T_C = -160$$

$$T_C = -40° = T_F$$

At -40° the temperature scales are the same, i.e., -40°C = -40°F.

ABSOLUTE TEMPERATURE SCALES

There is an experimental and theoretical lowest temperature known as "absolute zero." Absolute zero is -273.2°C or rounded off, -273°C. Since it is sometimes inconvenient to work with negative temperatures, temperature scales have been defined whose zero point is absolute zero. These scales are called "absolute temperature scales." Scales using the Fahrenheit degree and the Celsius degree as their smallest divisions are both in common use and are illustrated in Figure 3. The scale using the Celsius degree is called the "kelvin" scale, and the scale using the Fahrenheit degree is called the "rankine" scale.

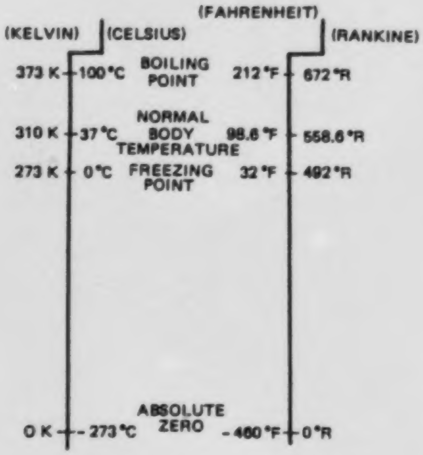

(KELVIN)
(CELSIUS)
(FAHRENHEIT)
(RANKINE)
373 K
100°C
BOILING POINT
212°F
672°R
310 K
37°C
NORMAL BODY TEMPERATURE
98.6°F
558.6°R
273 K
0°C
FREEZING POINT
32°F
492°R
0 K
-273°C
ABSOLUTE ZERO
-460°F
0°R

FIG. 3

Absolute temperatures can be obtained with the aid of Equations 1 and 2.

$$T(K) = T\ (°C) + 273 \qquad (1)$$

$$T(°R) = T\ (°F) + 460 \qquad (2)$$

Note: The symbol for degrees Kelvin in the SI systems of units is K and does not include the degree (°) symbol.

OTHER THERMOMETERS

The thermocouple shown in Figure 4 works on the principle that when wires of any two unlike metals are formed into a complete circuit (as shown in Figure 4), an emf is generated whenever the junctions (A and B) and the junction C are at different temperatures. In the circuit shown, the junctions A and B are kept at 0°C by ice and water in a Dewar flask (essentially a thermos bottle). The

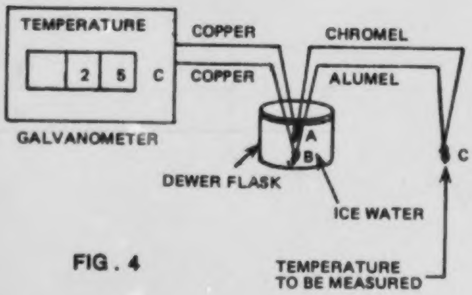

TEMPERATURE
2 5
C
GALVANOMETER
COPPER
COPPER
CHROMEL
ALUMEL
A
B
C
DEWER FLASK
ICE WATER
TEMPERATURE TO BE MEASURED

FIG. 4

junction C is placed in contact with the body whose temperature is to be measured. Since these junctions will be at different temperatures, a voltage is generated, read by the galvanometer G, and converted into a temperature read out.

PYROMETER

The optical pyrometer shown in Figure 5 is used primarily to measure hot temperatures from a distance, accomplished by looking at a hot furnace through a special telescope containing a lamp filament and a red filter A. By adjusting the rheostat D the current through the lamp filament B is increased and the filament starts to glow. The current through the filament is increased until the color of the filament just matches the background color of the oven. The temperature of the oven then can be read on meter E.

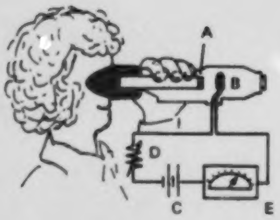


FIG. 5

SUMMARY

The most common thermometer is a device that measures temperatures by gaging the amount of expansion of mercury within a glass tube. Heat causes a thermal-expansion force used for measurement purposes. Other thermometers include the optical pryometer and the thermocouple used with a galvanometer.

THERMOMETER CALIBRATION

MATERIALS

Celsius thermometer.
Fahrenheit thermometer.
Ice and water and two beakers.
Source of heat (Bunsen burner).
Ring stand and ring.
Wire gauze.
Roll of duct tape or equivalent.
Masking tape.

PROCEDURE

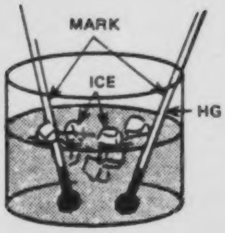


FIG. 6

1. Select one Fahrenheit and one Celsius thermometer.

2. Cover etched numbers on the thermometer with a tape, leaving a portion of glass uncovered to observe mercury or alcohol tube (capillary).

3. Pack crushed ice and water in a beaker; and mark lowest level of mercury on both tubes, as shown in Figure 6. Record in Data Table.

4. Place thermometers in beaker of water at room temperature.

5. Slowly bring water to a boil. Mark highest point to which mercury rises, as shown in Figure 7. Record.

6. Allow thermometers to cool. On one thermometer, divide space between freezing and boiling into 100 divisions, and on the other tube into 180 divisions. A strip of masking tape can be attached to the tube, and intervals can be marked on tape. Mark every 5 divisions or so. The first thermometer is calibrated in Celsius degrees; the second, in Fahrenheit degrees.

7. Place both thermometers in a container of water at room temperature, and record reading on each scale in Data Table.

8. Heat water slowly until Fahrenheit thermometer reads 170°F. Record the temperature of Celsius thermometer.

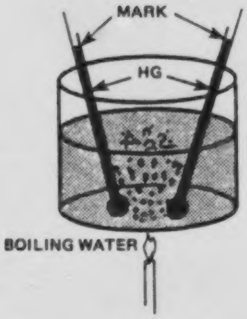


FIG. 7

9. Convert the "°F" temperature to "°C" by using equation in Data Table.

10. Compare it to actual "T_C" reading in Step 8.

DATA TABLE

Type Thermometer	Ice Water (Lowest Level)	Boiling Water (Highest Level)	Room Temp.	Temp. of Celsius Thermometer at 170°F	Temp. Conversion °F to °C
Celsius (100 divisions)			°C		(see below equation)
Fahrenheit (180 divisions)			°F	170°F	

Equation for Converting °F temperature to °C.

$$T_C = \frac{5}{9}\ (T_F - 32) = \frac{5}{9}\ (170°C - 32°C)$$

SELF CHECK

1. Define the following terms:

   Fahrenheit temperature scale.
   Celsius temperature scale.
   Absolute zero.
   Kelvin temperature scale.
   Rankine temperature scale.

2. Describe and illustrate with schematics the basic operation of a:

   Thermocouple.
   Pyrometer.
   Liquid-in-glass (thermometer).

3. The melting point of gold is 1063°C. What is its melting point in degrees Fahrenheit?

4. Normal room temperature is 72°F. What is room temperature in °C?

5. Convert each of the following to °C

   a. -32°F.
   b. 350 K.
   c. 28°F.
   d. 1300 K.

6. At what temperature is T(OR) = T(K)? Derive this temperature analytically as was done in Example B.

1T2

TEMPERATURE DIFFERENCE AND HEAT FLOW

The efficient movement, or transmission, of heat energy from one region to another is an important technology. From early times, people have had to seek ways to protect themselves from the intense heat and cold of the environment. With the development of civilization, problems involving heat transmission have become more complex.

In this module, the student will be introduced to the three ways by which heat is transmitted: conduction, convection and radiation, and will see that temperature difference causes the movement of heat energy, and acts, therefore, as the forcelike quantity in the thermal system. In the laboratory, the student will determine the conduction of heat through a common material.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define these terms:

  Heat conduction.
  Heat convection.
  Heat radiation.
  Thermal equilibrium.

- Calculate heat flow rate, given formulas and material properties.

- Measure, record, and graphically represent temperatures taken at equal time intervals for various positions within a slab of conducting material; and explain the characteristics of the curves obtained.

DISCUSSION

If one end of a metal rod is placed in a flame and the other end is held in the hand, that part of the rod being held soon feels warm, although the end being held is not itself in direct contact with the flame. In this case, heat reaches the cooler end of the rod by conduction. Heat is conducted through the metal rod from the warm end to the cool end.

Heat also can be transferred from one place to another by the actual motion of hot gases or liquids. Hot air rising over an open campfire or a candle is an example. Steam rising over a pot of boiling water shows how hot air moves, circulating heat throughout the room. Heat moved in this manner is due to a process called "convection." If the heated gas or liquid is forced to move by a fan or blower, the process is called "forced convection." An automobile uses forced convection to move heated water from engine to radiator and back. The hot water is cooled while passing through the radiator by forced convection of air provided by the fan and movement of the vehicle along the road.

A third way in which heat can be transmitted is by radiation. When one stands in front of a fireplace, heat is radiated through space from the fire to the person's body in the form of invisible infrared light. In this way, heat from the sun reaches the earth.

Whether heat is transferred by conduction, by natural convection, or by radiation, it always flows spontaneously from a region of high temperature to a region of low temperature. The larger the temperature difference between source and receiver, the more easily heat is moved. Temperature difference is the forcelike quantity that causes movement of heat energy in the thermal system.

On very hot days, an air conditioner removes heat that leaks into a building. On cold days, a furnace replaces heat that leaks out of the building.

Movement of heat in a building is similar to movement of water in a leaking ship. If a ship leaks, the bilge pump is turned on. If water is leaking faster than the bilge pump can remove it, the ship eventually will fill with water and sink. On the other hand, if the leak is slow enough, the bilge pump easily can remove the water.

When a building is designed, its "heat losses" and "heat gains" must be considered. Heating and cooling loads must be based on heat loss and heat gain. Engineers and architects can control, to a degree, heat losses and gains by choosing the proper materials to be used in constructing and insulating houses.

To show that heat flows from a region of high temperature to a region of low temperature, consider the situation illustrated

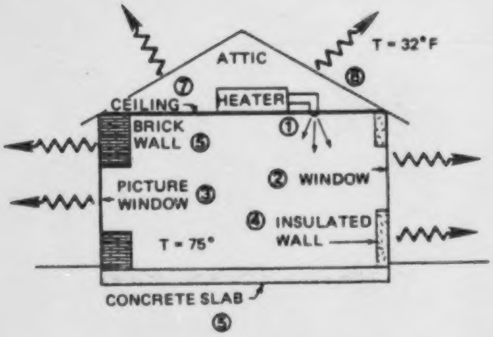


FIG. 1

in Figure 1. Figure 1 depicts a room maintained at a temperature of 75°F by an attic heater. The heater pumps heated air into the room through ducts located in the ceiling (1). Outside temperature is 32°F. Heat from the interior of the house "leaks" to the outside through many different paths. Some heat is conducted through windows (2 and 3), while some is conducted directly through the insulated wall (4). Other conduction paths are through the brick wall (5) and the concrete slab (6). Much heat escapes through the ceiling (7) and through the roof (8). No matter which path the heat follows, it always moves from the warmer area (room) to the cooler area (outside).

When temperature difference between the outside and inside of the house shown in Figure 1 is not large, heat losses will be small; and the heater will not be required to run continuously. On the other hand, if the outside temperature should drop to -40°F, the heat loss may be greater than heat provided by the heater. If the heater were turned off, the inside of the house would eventually reach outside temperature; and, at that point, temperature difference would equal to zero. When this occurs, the house and its environment would be in thermal equilibrium; and no heat would flow from one to the other.

In industries, equipment gets so hot that water or other liquids are required to flow over or around them to carry away heat. Examples are automobile engines, laser heads, nuclear reactors, cutting tools, and some machines and electrical components. In all of these examples, heat is transferred from the hot objects to the cooler fluids by convection, by conduction, or by radiation.

CONDUCTION EXPERIMENT

Different materials conduct heat at different rates, as can be demonstrated by touching iron and wood (at room temperature) with a hand (at body temperature). The iron is considerably cooler than the wood. This is because heat is conducted away from the warmer hand much more quickly by iron than by wood. Poor conductors (wood, felt, air, asbestos) are called "thermal insulators." The ability of a substance to transmit heat by conduction is a physical property of the particular material. Heat transferred by conduction through a material (Figure 2) can be calculated with Equation 1.

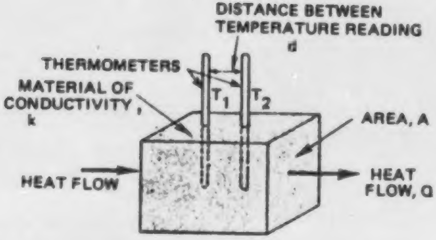

DISTANCE BETWEEN TEMPERATURE READING
d
THERMOMETERS
MATERIAL OF CONDUCTIVITY, k
T_1
T_2
AREA, A
HEAT FLOW
HEAT FLOW, Q

FIG. 2

$$Q_H = \frac{kA\ (T_2 - T_1)}{d} \qquad (1)$$

where

Q_H = heat flow rate, Btu/h or cal/s.

k = coefficient of thermal conductivity,

$$\frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2\text{F}^\circ} \quad \text{or}$$

$$\frac{\text{cal} \cdot \text{cm}}{\text{h} \cdot \text{cm}^2\text{C}^\circ}$$

A = area ft^2 or cm^2.

$(T_2 - T_1)$ = difference in temperature between the two surfaces "d" ft or cm apart.

d = thickness of material considered, in or cm.

Example A illustrates the use of this equation in solving a problem.

EXAMPLE A: HEAT FLOW THROUGH INSULATION

Given: An 80 ft wall is insulated with 3 in of glass wool having a thermal conductivity of 0.29 (Btu • in)/(h • ft^2 • F°). The inside temperature is 72°F, and the outside temperature is 18°F.

Find: The heat energy lost through the wall in 24 hours. (Neglect the insulating effects of other materials in the wall.)

Solution:

$$Q_H = \frac{kA\ (T_2 - T_1)}{d}$$

$$= \frac{\left(0.29\ \frac{\text{Btu} \cdot \text{in}}{\text{h}\cdot\text{ft}^2\cdot\text{F}^\circ}\right)\ (80\ \text{ft}^2)(72^\circ\text{F} - 18^\circ\text{F})}{(3\ \text{in})}$$

$$= \left(0.29\ \frac{\text{Btu} \cdot \cancel{\text{in}}}{\text{h}\cdot\cancel{\text{ft}^2}\cancel{\cdot\text{F}^\circ}}\right)\left(\frac{80\ \cancel{\text{ft}^2}}{3\ \cancel{\text{in}}}\right)(\ 54\cancel{^\circ\text{F}})$$

$$Q_H = 417.6\ \text{Btu/h}$$

Heat energy escapes through the wall at a rate of 417.6 Btu/h. Thus, in 24 hours the heat loss is —

$$H = (417.6\ \text{Btu/h})(24\ \text{h})$$

$$H = 10{,}022\ \text{Btu}$$

SUMMARY

In thermal systems, the forcelike quantity is temperature difference. Heat energy flows in any system in response to the temperature differences in the system, and the rate at which heat flows through any substance depends upon the temperature differences across that substance, as well as upon the characteristics of the substance. Thermal insulating materials are chosen for their low thermal conductivity.

HEAT FLOW BY CONDUCTION

MATERIALS

Electric temperature-measuring device such as a thermocouple or a calibrated thermistor.

Slab of conducting material (concrete, brick, etc). Slab can be made from a block of solid concrete (a 2" concrete patio block) with holds drilled at 3-cm intervals across the center of the block. Five holes are needed to hold temperature sensors. Holes should be just large enough to hold the sensors snugly to reduce heat losses.

Propane torch on a solid stand.

Graph paper.

PROCEDURE

1. Set up conductive slab and source of high temperature as shown in Figure 3. Be certain the experiment is positioned well away from any flammable material.

2. When everything is set up and fire safety is ensured, record initial block temperature (T°) in Data Table. Initial temperatures will correspond to room temperature and should be the same for all sensors.

3. Apply the constant heat source to end of insulated slab; record temperature readings at each sensor position at two-minute intervals.

4. When temperature of sensor nearest the hot end approaches upper limit of its calibration,

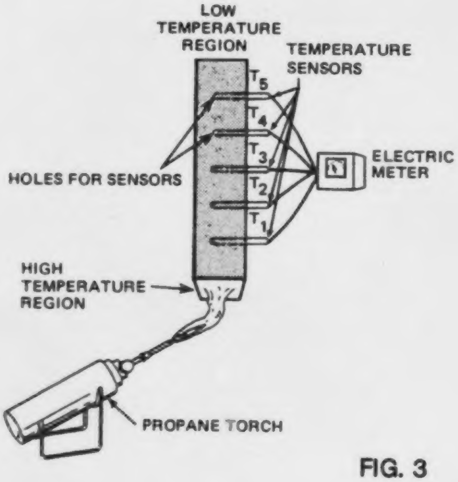

LOW TEMPERATURE REGION
TEMPERATURE SENSORS
T_5
T_4
T_3
T_2
T_1
ELECTRIC METER
HOLES FOR SENSORS
HIGH TEMPERATURE REGION
PROPANE TORCH

FIG. 3

remove heat source; and record all sensor readings in Data Table.

5. Plot data on a graph similar to Figure 4.

6. Explain what your graph shows about heat conduction through the material. How does this experiment demonstrate temperature difference as a forcelike quantity?

DATA TABLE

Thermometer Location		Time (2-minute intervals)								
		t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
1										
2										
3										
4										
5										

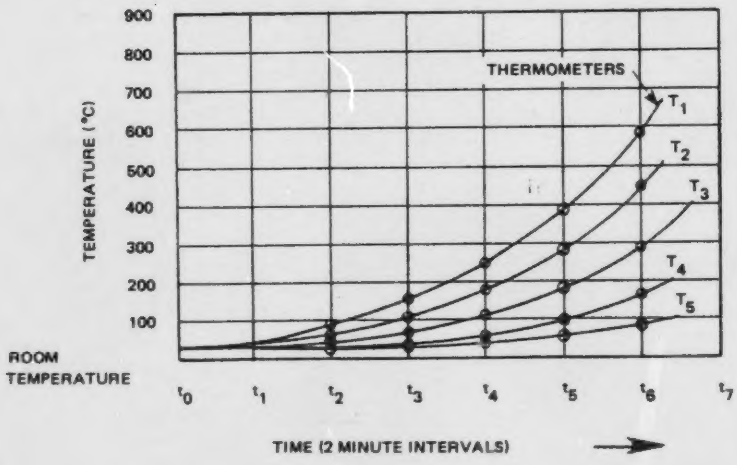


FIG. 4

SELF CHECK

1. Define the following terms:

   Heat conduction.
   Heat convection.
   Heat radiation.
   Thermal equilibrium.

2. An old window, 5 ft x 8 ft, was removed, and a wall of 4-inch face brick was laid in the opening. On a winter day, the outside surface of the brick was 18°F, and the inside surface was 58°F. What is heat flow rate in Btu/h through the bricked-up window?

$$K_{(brick)} = 9.0 \frac{\text{Btu} \cdot \text{in}}{\text{h} \cdot \text{ft}^2 \cdot \text{F}^\circ}$$

3. Given an explanation of the graph of experimental results, why do curves rise at different rates?

4. If a material with a higher conductivity had been used in the laboratory experiment, describe qualitatively how the graphs would change.

5. If a 40-ft^2 wall loses 9,000 Btu in 20 hours, what is the coefficient of thermal conductivity of the wall, if the average temperature difference between the inside and outside air is 60F° and wall is 2 inches thick?

6. How thick should a wall be if it is to lose no more than 1200 Btu/hr when it is 20°F outside and 70°F inside? The wall has an area of 48 ft^2 and a coefficient of thermal conductivity of $0.2 \frac{\text{Btu} \cdot \text{in}}{\text{ft}^2 \cdot \text{h} \cdot \text{F}^\circ}$.

7. What is the area of a wall if when the temperature difference on opposite sides of the wall is 20°F, the wall is designed to transfer no more than 300 Btu/hr. The wall is designed to be 3 inches thick and it has a coefficient of thermal conductivity of 0.33.

8. What is the average outside temperature if a wall 80 ft^2 in area with K = 0.27 transfers 20,000 Btu of heat in one day? The wall is 2 inches thick and inside the house the temperature is maintained at 70°F.

WORK

APPLICATION MODULES

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WORK DONE ON AN ELEVATOR

Since their invention, elevators and escalators have been used to lift loads against the force of gravity. Different types of machinery can provide the energy to do the lifting work. This work can be delivered as fluid energy in a hydraulic automobile lift and as rotational mechanical energy in a motor and winch attached to an elevator. In this module, the mechanical work required to lift objects against the force of gravity is defined and the inclined plane and its advantages in lifting discussed. In the laboratory, the student will determine the work required to lift objects vertically and on an incline.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define work in a mechanical system in terms of an equation that relates force and displacement.
- Calculate the work done on the body, given a force vector applied to a body and a displacement distance.
- Calculate the work done on an elevator, given an elevator weight, load, and vertical distance of travel.
- Given the appropriate equipment, determine (1) the work required to move a weight vertically and (2) the work necessary to move a weight up an inclined plane, neglecting friction.

WORK DONE ON AN ELEVATOR

Busy, multi-story business buildings usually contain one or more elevators. The elevators are used to transport people from one floor to another, saving them the effort of climbing long flights of stairs. The elevators travel in a vertical plane and are propelled by cables attached to winches (Figure 1). These winches usually are powered by electric motors housed at the top of the elevator shafts. The electric motor turning the winches causes the cables to exert a work-producing force on the elevators. Elevators are designed to stop and start smoothly as they travel the vertical shaft. If the passenger desired to travel several floors, the elevator accelerates smoothly as it lifts and maintains a constant speed until it decelerates smoothly to a stop at the desired floor. The safe maximum load is posted inside most elevators to prevent overloading. The work required to cause an elevator to be lifted through the vertical distance originates in the electric motor and winch-cable combination. The amount of work provided must be not only the work required to lift the elevator, but also the work required to overcome friction in all moving parts of the elevator.

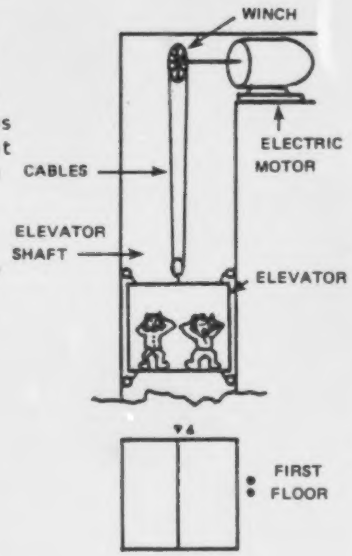


FIG. 1

WORK IN A MECHANICAL SYSTEM

In Concept Module 2-0, "Work," mechanical work was defined by Equation 1 and is repeated in this module as Equation 1.

$$W = F\ell \qquad (1)$$

where

W = mechanical work done by a force.
F = magnitude of component of force acting in the direction of motion.
ℓ = distance through which force acts.

Both force and displacement are vector quantities, and their vector direction is of fundamental importance. The force vector in Equation 1 must be in a direction parallel to that of the displacement vector. This relationship is illustrated in Figure 2.

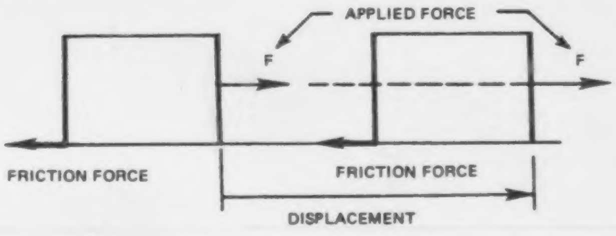


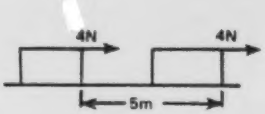
FIG. 2

Note also that in Figure 2 the object being moved has a friction force whose vector sense is opposite to the sense of the displacement vector. Its line of action still is parallel to the displacement vector and, therefore, does work on the object. Often, one must consider the work done by each force on an object separately. This concept will be applied to a discussion of the elevator later in this module.

Although work is the product of two vector quantities, it is a scalar quantity without direction, like other scalar quantities of volume and area. Work often is graphically depicted as the area under a force-displacement curve. For applications in which the force may vary over the displacement, the graphical method of determining work on an object is very useful. Example A illustrates this principle.

EXAMPLE A: WORK AS AREA UNDER A CURVE

Given: An object having a single constant force of 4 Newtons applied to it moves through a distance of 5 meters in the direction of the force.



Find:
(a) The mechanical work done.
(b) How the mechanical work equals the area underneath a force-distance curve.

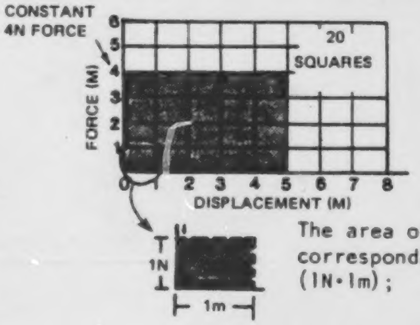
Solution: (a) $W = F\ell$

$= (4N)(5m)$

$= 20\ N\cdot m$

$W = 20\ J$

(b) Draw a force-displacement graph:



The area of each square of graph corresponds to 1 Joule of work (1N·1m);

therefore, the mechanical work done by the force is

$$(20 \text{ squares})\left(\frac{1 \text{ Joule}}{\text{square}}\right) = 20 \text{ J}$$

WORK AGAINST GRAVITY ON AN ELEVATOR

Figure 3 is a diagram of an elevator's motion from the first to the tenth floor. In the following discussion, frictional energy losses will be neglected. As the elevator leaves the first floor, the applied force must be greater than the weight of the elevator for the acceleration-to-speed to occur. Therefore, the work done by the lift machinery supplying the applied force through the cables must exceed the work done by the gravitational force through the acceleration region. The difference goes into the increasing kinetic energy of the elevator. As the elevator reaches its maximum allowable speed, the applied force in the cables equals the weight of the elevator. When this happens, the elevator moves at a constant speed; and the work done by the lift machinery is equal to the work done by gravity. In this constant-speed region of motion, the net force on the elevator is zero; and the net work on the elevator is zero, although it is moving at constant speed.

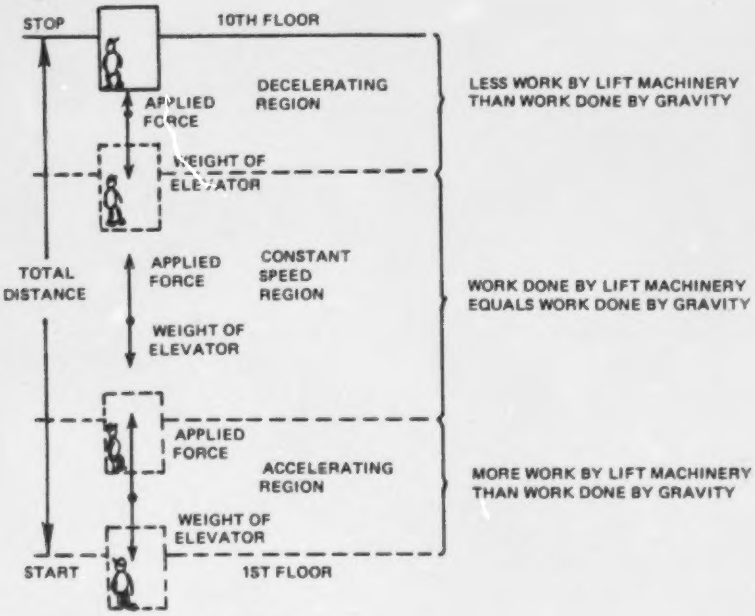

STOP
10TH FLOOR
APPLIED FORCE
DECELERATING REGION
WEIGHT OF ELEVATOR
LESS WORK BY LIFT MACHINERY THAN WORK DONE BY GRAVITY
TOTAL DISTANCE
APPLIED FORCE
CONSTANT SPEED REGION
WEIGHT OF ELEVATOR
WORK DONE BY LIFT MACHINERY EQUALS WORK DONE BY GRAVITY
APPLIED FORCE
ACCELERATING REGION
WEIGHT OF ELEVATOR
MORE WORK BY LIFT MACHINERY THAN WORK DONE BY GRAVITY
START
1ST FLOOR

FIG. 3

This may seem strange, yet Newton's first law of motion states, "In the absence of a net force acting on it, ... a body in motion will remain in motion at constant velocity."

As the elevator approaches the tenth floor, it begins to decelerate. Here, less force is applied to the elevator cables, and less work is done by the lift machinery than the work done by gravity.

While stopping at the tenth floor, the lift machinery must reapply a force through the cables equal to the weight of the elevator. Since the elevator is not moving while at the tenth floor, no work is done on it either by gravity or by the cables.

Calculating the total work done on the elevator by the lift, machinery is not as difficult as it seems. To understand this, observe Figure 4, which shows the force-displacement graph of the elevator's motion. Notice that the "hump" area in the acceleration region of the graph could be chopped off smoothly, flipped upside down, and fitted into the hole of the graph in the deceleration region. The resulting graph is shown in Figure 5.

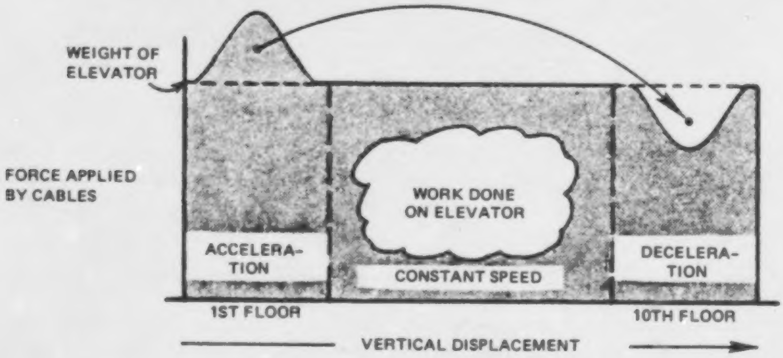

WEIGHT OF ELEVATOR
FORCE APPLIED BY CABLES
WORK DONE ON ELEVATOR
ACCELERA-TION
CONSTANT SPEED
DECELERA-TION
1ST FLOOR
10TH FLOOR
VERTICAL DISPLACEMENT

FIG. 4

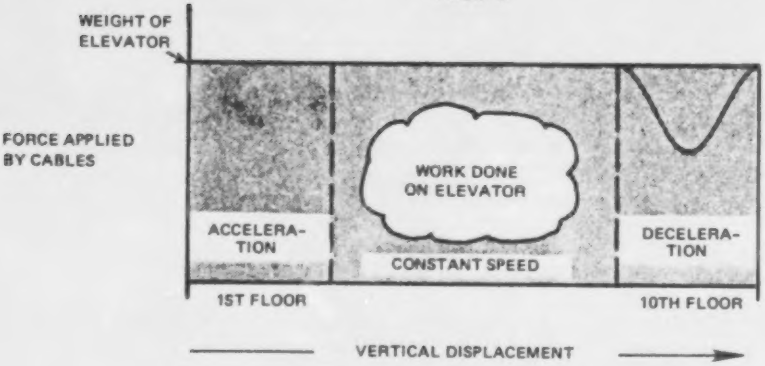

WEIGHT OF ELEVATOR
FORCE APPLIED BY CABLES
WORK DONE ON ELEVATOR
ACCELERA-TION
CONSTANT SPEED
DECELERA-TION
1ST FLOOR
10TH FLOOR
VERTICAL DISPLACEMENT

FIG. 5

The graph shows that the work done on an elevator by the lift machinery can be determined because the force applied to the elevator equals the weight of the elevator throughout the vertical displacement. Therefore, the work done on an elevator can be determined by Equation 2.

$$W = W_T h \qquad (2)$$

where

W = work done on an elevator.
W_T = weight of elevator.
h = vertical displacement of elevator.

Actually, this concept can be used to determine the work done against gravity while lifting any body through a vertical displacement. Equation 3 applies for any body being lifted through a vertical displacement.

$$W_g = W_T h \qquad (3)$$

where

W_g = work done against gravity.
W_T = weight of body being lifted.
h = vertical displacement.

Example B illustrates the use of Equation 3 in solving a problem.

EXAMPLE B: WORK DONE ON AN ELEVATOR

Given: A 10,000-Newton elevator plus a 4000-Newton load is lifted through a vertical displacement of 5 floors (7 meters between floors).

Find: The work done by the lifting machinery on the elevator.

Solution: (1) The total weight of the elevator must include dry weight plus load

Total Weight = 10,000 N + 4000 N

Total Weight = 14,000 N

(2) The total vertical displacement

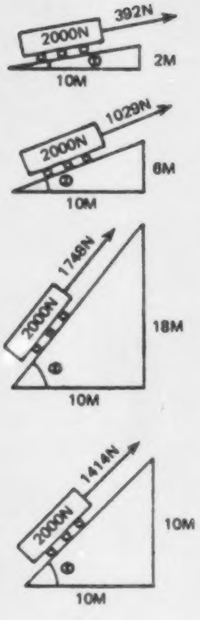
$$\text{Vertical displacement} = (5 \text{ floors})\left(\frac{7 \text{ meters}}{\text{floor}}\right)$$

Vertical displacement = 35 meters

(3) $$\text{Work done on the Elevator} = \left(\begin{array}{c}\text{Weight of}\\ \text{Elevator}\end{array}\right)(\text{vertical displacement})$$

$$= (14{,}000\ \text{N})(35\ \text{m})$$

$$\text{Work done on the Elevator} = 490{,}000\ \text{J}$$



392N
2000N
2M
10M
1029N
2000N
6M
10M
1748N
2000N
Φ
18M
10M
1414N
2000N
Φ
10M
10M

FIG. 6

WORK DONE AGAINST GRAVITY BY USING AN INCLINED PLANE

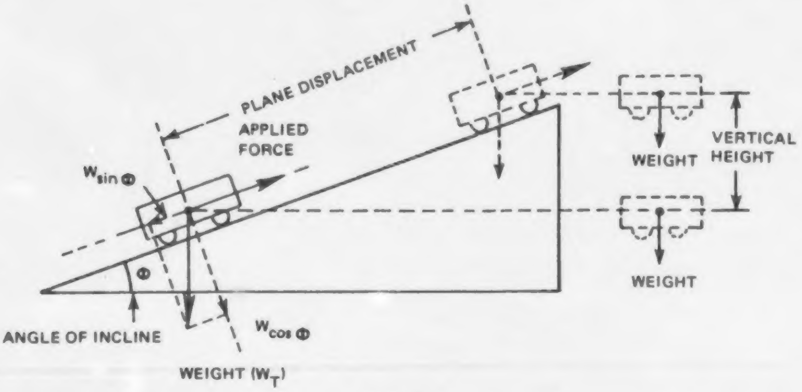
An inclined plane is a simple machine that allows less force than its weight to be exerted on an object as it travels through a longer distance than the actual vertical displacement. Figure 6 shows an object being moved through a vertical distance by traveling along inclined planes at different angles of incline. The object being moved (Figure 6) has a weight of 2000 N and is pulled on rollers to reduce friction to a minimum. The forces shown would be the forces required to pull the object up the plane at a constant velocity. The effect of friction is not considered.

In Figure 7, an object being rolled up an inclined plane is compared to the same object being lifted through a vertical height. The work done in lifting the object is expressed by Equation 4.

$$W_L = W_T h \tag{4}$$

where

W_L = work done in lifting.
W_T = weight.
h = vertical height.



PLANE DISPLACEMENT
APPLIED FORCE
W sin Φ
Φ
VERTICAL HEIGHT
WEIGHT
WEIGHT
ANGLE OF INCLINE
W cos Φ
WEIGHT (W_T)

FIG. 7

In order to determine the work exerted on the object along the plane, the weight vector is resolved into components. In Figure 7, one component, W cos θ, perpendicular to the plane does no work. The other component, W sin θ, must equal the applied force, F, as the object is moved in a displacement along the plane. Therefore, the work done along the plane is given by Equation 5.

$$W_p = F\ell \tag{5}$$

where

W_p = work along the plane.
F = applied force.
ℓ = plane displacement.

or,

$$W_p = W_T \sin\theta \ \ell \tag{6}$$

By the definition of sin θ, this work is —

$$W_p = W_T\left(\frac{h}{\ell}\right)\ell.$$

According to Equation 4, however, the following is true:

$$W_p = W_L \tag{7}$$

The inclined-plane principle is applied in lifting people in busy department stores by an escalator. The unit powering the escalator can apply less force along the plane of the escalator, and more people can be moved than with the elevator.

SUMMARY

Work done by an elevator's lift machinery to supply force through the cables must exceed the work done by gravitational force through the acceleration region. The work done on the elevator by the lift machinery can be determined because the force applied to the elevator equals its weight throughout the vertical displacement.

LABORATORY

WORK DONE IN LIFTING A MASS THROUGH A DISTANCE

MATERIALS

Hanging spring scale (calibrated in Newtons).
1000-gm mass with hook.
Meter stick.

PROCEDURE

In this experiment, the student calculates the work done in lifting a mass through a distance, notes the effect on the applied force while starting and stopping the mass, and compares these effects to the applied force on the mass, if the mass remains motionless.

1. Hang a 1000-gm mass on the spring scale. Record in Data Table the force indicated on scale. Lower mass to floor.

2. Using a meter stick as a guide, raise mass at an extremely slow, constant rate (Figure 8). Record distance traveled in meters.

3. Calculate work done lifting mass, and record in Data Table 1.

4. Lower mass to floor, while carefully observing the scale. Raise mass through one-meter distance at a rapid rate, stopping mass at end of meter distance.

5. At the beginning of the lifting motion, is the scale reading more, or less, than the reading when the scales are motionless?

6. At the top near the end of the lifting motion, is the scale reading more or less than the reading when the scales are motionless?

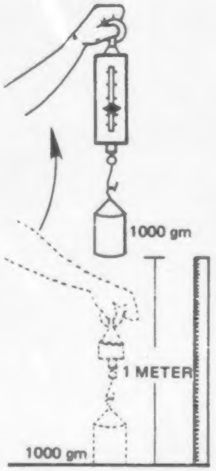


FIG. 8

WORK NECESSARY TO MOVE A WEIGHT UP AN INCLINED PLANE

MATERIALS

Inclined plane with adjustable pulley.
Meter stick.
Hall's car (small metal car with cavity for loading masses;
car must have low-friction wheels).
Metric mass set and hanger.
Mass weights of 100, 300, and 500 gram.
String (stout).
Protractor.

PROCEDURE

1. Using a beam balance, obtain the mass of the Hall's car in kilograms, and record it in Data Table 2.

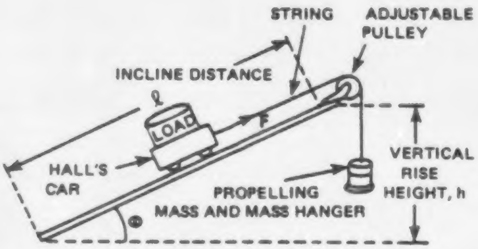


FIG. 9

2. Arrange Hall's car on inclined plane as in Figure 9. Be sure that the adjustable pulley is positioned in such a manner that string attached to Hall's car is parallel to plane.

3. Measure angle of incline θ with a protractor, and record it in Data Table 2.

4. Place a 500-g load mass in Hall's car, and add enough mass to mass hanger so that with a gentle push to overcome friction, the car will move up the plane at a relatively constant speed. If car stops or moves erratically, add slightly more mass to propelling mass hanger. If car accelerates, decrease mass on mass hanger. The purpose of this propelling mass is to provide a constant applied force F to Hall's car throughout its travel up plane. The applied force is provided by gravity, producing constant gravitational force on propelling mass. This force is transmitted to the car by the string, and acts parallel to the incline.

5. Measure an inclined distance ℓ along the plane and record in Data Table 2. Measure vertical rise height h, and record in Data Table 2.

6. Calculate work done on car along plane, and record in Data Table 3.

7. Calculate work done to lift Hall's car plus load through the vertical rise height. Record result in Data Table 3.

8. Find difference in work done on car along plane and work required to lift car through vertical rise height. Record this result in Data Table 3. Explain this difference.

9. Repeat Steps 1 through 8 twice more, changing the loaded mass by 200-gram increments. Record all data in tables as previously instructed.

DATA TABLE 1

Scale Reading (Force Applied) = ________ Newtons

Distance Traveled = ________ meter

Work Done Lifting Mass = ________ Joule

DATA TABLE 2

Mass of Car + Load (kgm)	Distance (ℓ) Along Plane (m)	Angle of Incline (degrees)	Vertical Rise (h) (m)	Weight of Car* plus load (Newtons)

$$\text{*Weight of Car in Newtons} = \left[\begin{array}{c}\text{Mass of Car}\\ \text{plus load}\\ \text{in kgm}\end{array}\right] \times 9.8\ \frac{m}{s^2}$$

DATA TABLE 3

Work Done on Car Up The Plane (Joules) Ⓐ	Work to Lift Car Through Vertical Rise (Joules) Ⓑ	Difference in Work (A - B) (Joules)

SELF CHECK

1. Define work in a mechanical system in terms of an equation that relates force applied to an object and the displacement through which an object moves.

2. A force of 35 Newtons acts on a body while moving the body through a displacement distance of 15 meters. Calculate the work done on the body by the force.

3. An elevator car weighs 3000 lb. Twelve people, averaging 150 lb each, step on the elevator. The elevator travels 20 floors (assume 25 ft between floors). Calculate the mechanical work required on the elevator car and load in ft•lb. Convert your answer to joules.

4. An inclined plane is used to move an 800-lb piano to the tailgate of a moving van, 4 ft from the street. The plane is 20 ft in length.

   a. How much force is required to move the piano up the plane at a constant speed?
   b. How much work is necessary to move the piano up the plane?
   c. How much work is required for 4 men to lift the piano to the tailgate?

5. In problem 4, how much work would be done if the 4 men held the 800-lb piano 2 ft off the ground for 1 minute?

6. In problem 4, how much work is done by the force of the plane on the piano?

WORK DONE BY A WINCH

The winch is a simple machine which has been used to expand the capacity of man to move heavy objects in the world today. The principle of the winch is applied to many areas of technology and of everyday life, such as the movement of people in elevators and escalators, lifting heavy machinery in industry, and the operation of bicycles and automobile and truck accessories.

This module presents an application of the concept of work in rotation systems, and describes in particular the rotation work into and out of a winch. Other topics discussed include the efficiency of winches, the use of geared winches and the effects of torque variations upon them. In the laboratory, the student will determine the work and efficiency of a winch.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe the main design features of a winch and list at least two applications.
- Given a force, a moment arm, and an angle of travel, calculate the rotational work done on a winch and by a winch.
- Calculate the efficiency of a winch, given the output work and the input work.
- Describe the effects of varying the direction of the input force on a winch from directions other than a tangent.
- Given the appropriate equipment, determine the work and efficiency of a winch.

DISCUSSION

A machine is a force-multiplying device. Its purpose is not to convert one form of energy to another, but merely to exert an output force on an object which is different from (usually larger than) the input force which is applied on the machine. One such machine is the winch. The winch plays an important role in our everyday life.

The winch is characterized by a drum, or axle, about which a rope or cable is wrapped securely. Attached to the drum is a hand crank or a motor by which a force is applied. Figure 1 illustrates several types of winches. A simple hand winch is frequently used by fishermen to pull boats from the water onto boat trailers. The fisherman may also use an electric-powered winch to raise and lower his boat anchor. Even his fishing reel is a simple, hand-powered winch. Some winches magnify the input force through the use of gears. Gears may magnify the input force in such a manner that a 30 lb effort by a man on the handle of a winch pulls a load of 3500 lbs. Winches are used widely in industry for many types of lifting jobs employing floor cranes, portable hoists, power eleveators and escalators.

WORK IN A ROTATIONAL SYSTEM

Mechanical work done on a rotating body is defined by Equation 1.

$$W_{ro} = \tau\theta \qquad (1)$$

where

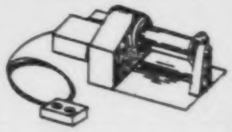
W_{ro} = mechanical work in rotational system.

τ = torque.

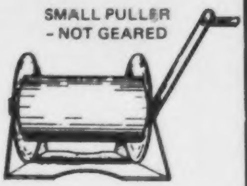
θ = angle of rotation.

Mechanical work is measured in units of ft·lbs or Joules (Newton x meter). Figure 2 shows a comparison of mechanical work done in a linear system and mechanical work in rotation system.

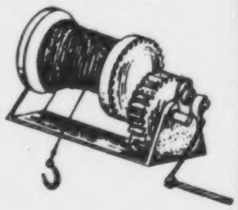
"PULL POWER"
ELECTRIC WINCHES



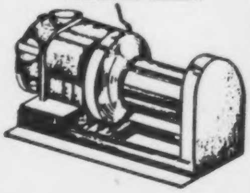
SMALL PULLER
- NOT GEARED



HOIST WINCH



POWERED WINCH



GEARED WINCH

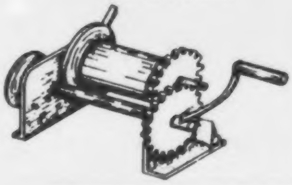


FIG. 1

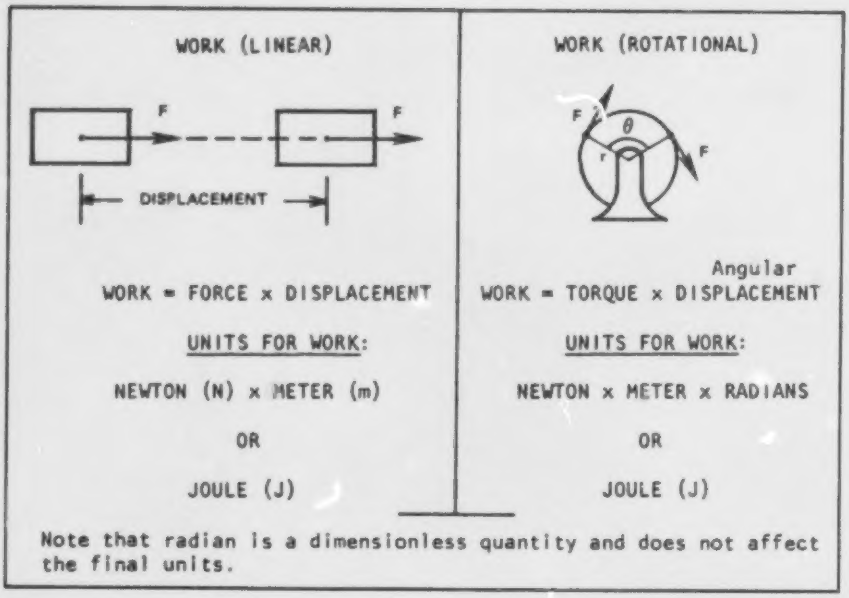

WORK (LINEAR)
F
F
DISPLACEMENT
WORK = FORCE x DISPLACEMENT
UNITS FOR WORK:
NEWTON (N) x METER (m)
OR
JOULE (J)
WORK (ROTATIONAL)
F
θ
r
F
Angular
WORK = TORQUE x DISPLACEMENT
UNITS FOR WORK:
NEWTON x METER x RADIANS
OR
JOULE (J)
Note that radian is a dimensionless quantity and does not affect the final units.

FIG. 2

In Figure 2, force in the linear system is in the same direction as the displacement vector throughout the motion of the body. In the rotational system, force F always remains perpendicular to the radius as the wheel moves through the angular displacement, θ.

OPERATING PRINCIPLES OF A SIMPLE HAND WINCH

In Figure 3, a simple hand-operated winch is shown with a force being applied to the handle. This force is normally much smaller than the weight being lifted. The handle moves through the same angular displacement, θ, as does the winch drum. The torque produced by the hand on the handle about the axis of the drum is called input torque and is given by Equation 2:

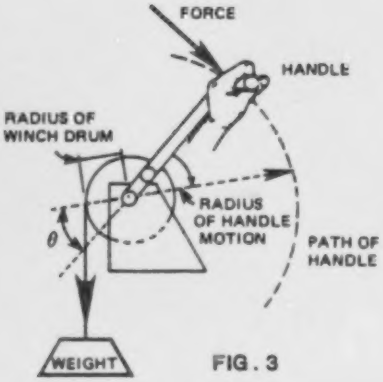

FORCE
HANDLE
RADIUS OF WINCH DRUM
RADIUS OF HANDLE MOTION
θ
PATH OF HANDLE
WEIGHT

FIG. 3

$$\tau_{in} = F_{in}\ r_{in} \qquad (2)$$

where

τ_{in} = input torque.
F_{in} = input force (of hand).
r_{in} = radius of handle.

The longer the radius of the handle, the smaller the force required to produce a given amount of torque about the drum axis. Equation 2 is valid only if the force exerted by the hand is applied perpendicularly to the radius of the handle. The input work on the winch is given by Equation 3.

$$W_{in} = F_{in} r_{in} \theta_{in} \tag{3}$$

where

W_{in} = work input.

Example A shows how Equation 1 is used in solving a problem.

EXAMPLE A: WORK INPUT TO A WINCH

Given: A hand winch whose handle radius is 30 cm long has a force of 150 N of force applied perpendicularly, through an angle of travel of 5 complete revolutions.

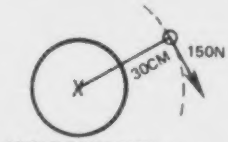
Find: Input work to the winch.

Solution: The angle of travel in radians is

$$\theta = t \text{ revolutions} \times \frac{2\pi \text{ radians}}{\text{revolution}}$$

$$= 10\pi \text{ radians}$$

$$\theta = 31.4 \text{ radians}$$



Calculating the WORK INPUT

$$W_{in} = F \times r \times \theta$$

$$W = (150 \text{ N}) \times (0.3 \text{ m}) \times (31.4)$$

$$W_{in} = 1413 \text{ J}$$

The load, or weight, acting upon the cable of the winch shown in Figure 3 is the output force, which produces an output torque acting around the radius of the drum cable, r. As the load or weight is lifted, the cable always wraps around the drum in a manner that this load force is always perpendicular to the radius of the drum. The output torque produced by the winch is given in Equation 4.

$$\tau_{out} = F_{out} r_{out} \tag{4}$$

where

τ_{out} = output torque.

F_{out} = output force load.

r_{out} = radius of winch drum with cable.

The work done by a winch is given by Equation 5.

$$W_{out} = F_{out}\ r_{out}\ \theta_{out} \tag{5}$$

where

W_{out} = work output.
θ_{out} = output angular displacement.

Example B shows the use of Equation 5 in calculating the work done by a winch.

EXAMPLE B: WORK OUTPUT BY A WINCH

Given: A hand winch whose drum radius is 3 cm. A thickness of wrapped cable of 2 cm is attached to the drum. A load force of 900 N is applied to the winch cable. The winch drum turns 5 complete revolutions.

Find: The work output by the winch.

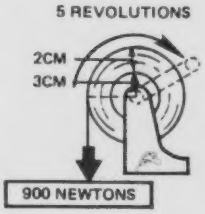
Solution:

1) The angle of travel, θ, as in Example A is 31.4 radians.

2) The total winch drum radius must equal the cable-wrap thickness plus the drum radius, or 5 cm.

3) Calculating the WORK OUTPUT,

$$\begin{aligned} W_{out} &= F\ r\ \theta \\ &= (900\ N) \times (0.05\ m) \times 31.4 \\ W_{out} &= 1413\ J \end{aligned}$$



Examples A and B refer to the same hand winch. Input work equals the output work; this situation occurs whenever friction is negligible. Actually, the input work must be slightly greater to overcome the frictional energy losses in the winch. The winch is a very simple machine. Friction encountered in the winch can be very low; thus, the efficiency of the winch can be quite high.

The efficiency of machinery, in general, is defined as the ratio of the OUTPUT WORK to the INPUT WORK, as shown in Equation 6.

$$\eta = \frac{W_{out}}{W_{in}}\ (100) \tag{6}$$

where

η = % efficiency.

W_{in} = work input.

W_{out} = work output.

Efficiency usually is expressed as a percent. Therefore, the decimal fraction which results from a calculation with Equation 6 is mulitplied by 100 to give the percent efficiency. Example C illustrates the use of Equation 6 in the calculation of the efficiency of a winch.

EXAMPLE C: EFFICIENCY OF A WINCH

Given: A winch has 5600 J of work done on it (input work). The output work done by the winch is 5200 J (400 J of work done to overcome friction).

Find: Efficiency of the winch in percent.

Solution:

$$\eta = \frac{W_{out}}{W_{in}} (100)$$

$$= \frac{5200 \text{ J}}{5600 \text{ J}} \times 100$$

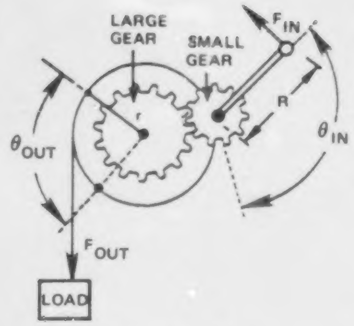
$$= 0.93 \times 100$$

$$\eta = 93\%$$

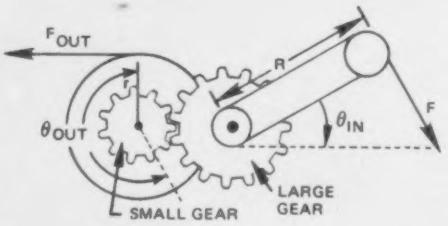
Normally winches are highly efficient. The efficiency of the ideal winch illustrated in Examples A and B is 100%, representing the maximum limit of efficiency for any machine. A 100% efficient machine would have no friction losses.

WINCHES WITH GEARS

Often modern-day winches contain gears between the winch drum and the crank mechanism. The purpose of the gears is to require less applied force to the cranking mechanism at the expense of a greater amount of angular travel. With the simple hand crank, the angular displacement of the cranking mechanism is the same as the angular displacement of the winch drum. With a winch having gears, the angular displacement of the winch drum is much different from the angular displacement of the crank mechanism, as shown in Figure 4. Figure 4 illustrates two types of gearing for different winch applications. The geared lift winch in Figure 4a has a small gear on the crank mechanism, while the larger gear is on the winch drum, which causes the angular displacement of the winch drum, θ_{out}, to be much smaller than angular displacement of the crank mechanism θ_{in}. The speed winch (Figure 4b) has a different purpose. Its



(a) GEARED LIFT WINCH



(b) A SPEED WINCH

FIG. 4

major purpose is to take up line or cable at a fast rate; therefore, the crank handle is attached to the larger gear, and the small gear is attached to the winch drum. This arrangement causes the angular displacement of the winch drum to be much larger than the angular displacement of the crank mechanism. No matter what the angular displacement relationship caused by gears is, the WORK INPUT to the winch is still only slightly greater than the WORK OUTPUT; and the winch efficiency is still high. The WORK INPUT and the WORK OUTPUT are still calculated from Equations 3 and 5, respectively. The use of gears in a winch causes additional friction that must be overcome by input force.

VARIATION OF TORQUE ON A WINCH

As previously stated, the input force to a winch should be applied perpendicularly to the radius of the crank mechanism. In practice, this is not always the case. Frequently, force is applied in a different direction, causing torque on the crank to be less than expected. How this relationship happens is illustrated in Figure 5, in which a bicycle crank is shown.

The thrust force of the foot is not always directed perpendicularly to the radius of the crank. If the thrust force is resolved into two components - one along the radius of the crank, the other in a direction perpendicular to the crank radius - a physical significance can be given to these components. The radial portion (component) of the thrust force causes a force on the bearings at the crank center, causing a reduced force to be available for the production of torque. The torque-producing force is that portion of the force perpendicular to the radius. As the crank is pushed, the torque produced is not constant; and in order to calculate the work done per revolution, an average value for torque must be found.

A graphical method can be employed in rotation to find the average torque produced. If the torque versus angular displacement is plotted, as shown in Figure 6, the area under the torque-displacement curve equals the work done. This method is used frequently in complete analysis of work done on rotating machinery.

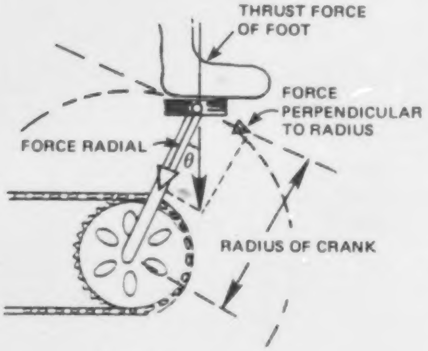


FIG. 5

The same effect of non-constant torque applies to hand-crank winches. Electric motors acting through gears provide a more constant source of torque to the crank mechanism of a winch, thereby making an electric-powered winch useful in providing the smooth, constant work needed to move elevators and escalators.

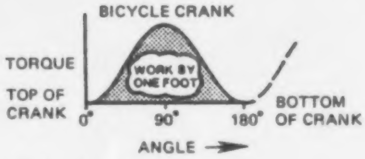


FIG. 6

SUMMARY

The winch is a mechanical device that produces rotational work through pulleys, gears, chains or other components. Some winches are designed for lift, or power applications; others, for speed. They are among the more efficient mechanical devices, finding wide and varied use.

WORK OUTPUT AND EFFICIENCY OF A WINCH

MATERIALS

Baitcasting fishing reel (similar to Shakespeare Thirfy Model 1902).
20 or more yards of nylon fishing line (20 lb-Test).
Set of metric slotted weights.
2 - Weight hangers.
Fishing rod (suggest a 2 ft crappie-rod).
Vernier caliper.
Circle cut of 1/4" plywood with a 5" diameter.
0-180, 1° protractor.
Support board with C-clamps.
2 - Meter sticks.

PROCEDURE

1. Prepare fishing reel and plywood circle, as shown in Figure 7. Drill holes to fit handle and handle nut of fishing reel. Use a rasp to make a groove around the circumference of the plywood circle.

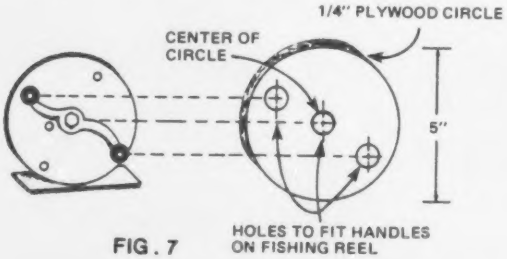

1/4" PLYWOOD CIRCLE
CENTER OF CIRCLE
5"
HOLES TO FIT HANDLES ON FISHING REEL

FIG. 7

2. Mount plywood circle on handle to fit, as shown in Figure 8. Wrap rubber bands around handle to hold plywood circle snug and firm.

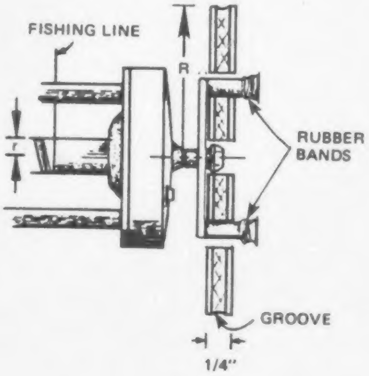

FISHING LINE
R
r
RUBBER BANDS
GROOVE
1/4"

FIG.8

3. Mount fishing reel to fishing rod, and support rod and reel, as shown in Figure 9.

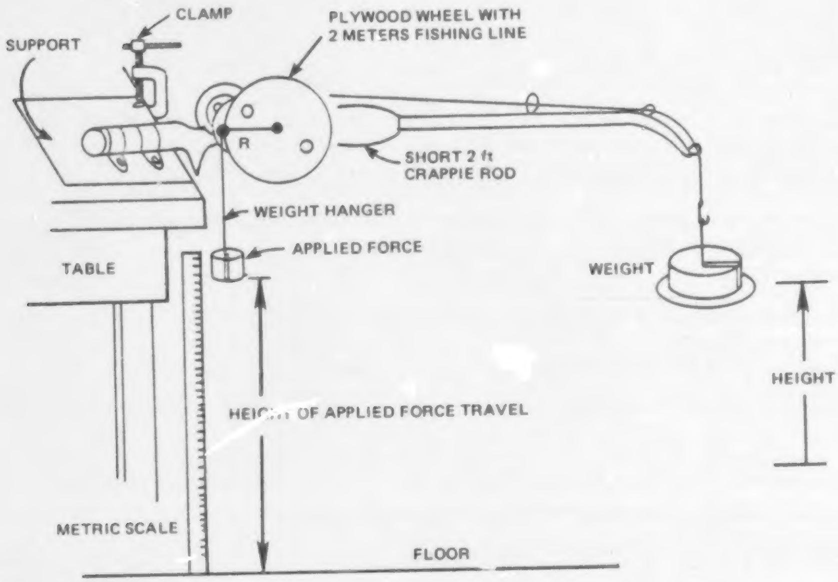

CLAMP
SUPPORT
PLYWOOD WHEEL WITH 2 METERS FISHING LINE
R
SHORT 2 ft CRAPPIE ROD
WEIGHT HANGER
APPLIED FORCE
TABLE
WEIGHT
HEIGHT
HEIGHT OF APPLIED FORCE TRAVEL
METRIC SCALE
FLOOR

FIG. 9

4. Measure radius r of inner fishing line drum, shown in Figure 8, and measure radius of the applied force, R, shown in Figure 8. Record these radii in Data Table.

5. Place a 50 gram mass on the 50 gram mass hanger, and a 50 gram mass hanger on the line wrapped around the plywood wheel, while holding the system to prevent movement.

6. Adjust applied-force acting at edge of plywood wheel by adding masses in such a manner that system will be balanced.

7. By hand, begin motion by pulling downward slightly (to overcome friction) in order that applied-force hanger travels at approximately a constant speed. If applied-force hanger fluctuates and stops before hitting floor, add small masses to produce a smooth movement to floor.

8. Move applied force hanger up near reel, and mark on wheel where line meets plywood wheel.

9. Release hanger and observe the angular travel (i.e., complete revolutions plus fraction parts of revolution) of plywood

wheel. Record this angle in degrees in Data Table. (Use protractor to measure fraction of revolution.)

10. Mark also where fishing line on reel drum meets edge of drum.

11. Record all masses on each hanger and vertical distance traveled by the 100 gram mass in Data Table.

12. After the trial, mark point at which fishing line meets drum. Measure distance along fishing line from mark-to-mark, and record in Data Table.

13. Perform work and efficiency calculations as shown in Data Table.

14. How does the efficiency of this winch compare with that of the winch in Example C?

DATA TABLE

OUTPUT WORK	INPUT WORK
Radius of line drum, r = ______ cm	Radius of APPLIED FORCE, R = ______ cm
Distance (d) from MARK-TO-MARK ON FISH LINE, ______ cm	ANGULAR TRAVEL, θin = ______ degree = ______ radians
$\theta_{out} = \frac{d}{r} = \frac{\text{\_\_\_\_\_\_ cm}}{\text{cm}}$ θ_{out} = ______ radian	
Mass = ______ gram (include weight hanger)	Applied Mass = ______ gram (include weight hanger)
Weight = ______ Newtons	Weight = ______ Newtons

CALCULATION RESULTS

WORK OUTPUT = [Mass x g] x r x θ_{out}

= [______kg x 9.8 $\frac{m}{s^2}$] x ______m x ______rad

WORK OUTPUT = ______ Joule

WORK INPUT = [Mass x g] x R x θ_{in}

= [______kg x 9.8 $\frac{m}{s^2}$] x ______m x ______rad

WORK INPUT = ______ Joule

EFFICIENCY (%) $\eta = \frac{W_{out}}{W_{in}}$ (100)

EFFICIENCY of Fishing Reel (%) = ______

SELF CHECK

1. Explain the design features of a winch, and give at least two applications.

2. A lift winch crank of 25 cm radius has a force of 100 N applied perpendicularly to the radius as it rotates through 15 revolutions. The winch drum measures 5 cm in diameter and the winch lifts 6825 N weight, rotating through one revolution. Calculate the following values:

   a) Calculate WORK INPUT to the winch.
   b) Calculate WORK OUTPUT done by the winch.
   c) Calculate efficiency of the winch.
   d) How much work is used in overcoming friction?

3. A simple hand winch designed for boat trailers has a hand crank radius of 35 cm, and a winch drum of 5 cm radius. A 3 cm thickness of rope extends the radius of the winch drum. When a force of 50 N is applied to the crank for one revolution, find the following:

   a) How much force is applied to the boat assuming a 100% efficient winch?
   b) How much rope is taken up?

4. What is the output work of a winch which is 57% efficient and has a crank radius of 25 cm when a force of 40 N is applied perpendicularly to the crank handle for 3 revolutions?

5. What is the ratio, θ_{in}/θ_{out}, for a 50% efficient winch when a crank handle radius of 20 cm and a drum radius of 4 cm is used with an applied force of 40 N to lift an 800 N weight? Assume the thickness of the cord does not add a significant amount to the radius of the drum.

6. A manufacturer is designing a simple hand winch with a hand crank radius of 40 cm. What should the drum radius be if the winch is to lift a 700 N weight by using an applied force of 30 N? Assume a 100% efficient winch.

2F1

WORK DONE BY A WATER PUMP

Energy is defined as the ability to do work. Energy also can be defined as the ability to produce action or to accomplish change. Science and technology are built upon action and change; consequently, upon energy. Today's energy shortage, caused by the limited energy resources of the earth, could limit mankind's ability to do work or accomplish change.

In this module, the student will review principles of work and energy as they apply to fluids and electricity. Values for work and energy involving fluids and electricity will be derived experimentally and in problems. In the laboratory, the student will measure current and voltage supplied to a pump, and will calculate the work and efficiency of the pump.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:
    - Mechanical work.
    - Fluid work (done by a pump).
    - Electrical work.
    - Efficiency.
- Define the forcelike and displacement quantities in both fluid and electrical work system.
- Given the weight of the fluid and the height lifted, calculate the work done by a pump.
- Given the appropriate equipment, measure current and voltage supplied to a pump, and calculate the pump's work and efficiency.

DISCUSSION

In Concept Module 2-0, a general formula for work was given as shown in Equation 1.

WORK = FORCELIKE QUANTITY x DISPLACEMENTLIKE QUANTITY. (1)

Special versions of this formula were derived for mechanical, fluid and electrical systems.

FLUID WORK

In this module, the student will determine the amount of work done by a pump in lifting a quantity of water to a known height. Just as a force can be applied to a book to cause it to move across a table, a force must be applied to water to cause it to move up a tube to a greater height.

Use of Equation 1, the formula for mechanical work, makes possible the calculation of fluid work done by a pump lifting water. But in dealing with fluids, the "force" is equal to the weight of the liquid moved; and the "displacement" is the height to which the water is lifted. The mechanical work formula can be rewritten, therefore, in the form shown in Equation 2 for fluid work.

$$W = \rho_W hV \qquad (2)$$

where

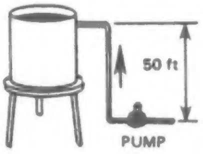
ρ_W = weight density in lb/ft^3.
h = height in feet.
V = volume of fluid moved (ft^3).

Example A illustrates the use of Equation 1 in solving a problem.

EXAMPLE A: FLUID WORK IN LIFTING WATER

Given: 500 gallons of water are pumped to a height of 50 feet. (Recall: 1 ft^3 = 7.48 gal; 1 ft^3 = 62.4 lb.)

Find: Amount of work done.



Solution: $W = \rho_W hV$

$$= (500\text{ gal})\left(\frac{1\text{ ft}^3}{7.48\text{ gal}}\right)\left(\frac{62.4\text{ lb}}{1\text{ ft}^3}\right)(50\text{ ft})$$

$$= 4170\text{ lb} \times 50\text{ ft}$$

$W = 209{,}000\ \text{ft}\cdot\text{lb} = 2.09 \times 10^5\ \text{ft}\cdot\text{lb}$

Note: Since 2.09×10^5 ft·lb of work was done in lifting the water, the tank of water is now storing 2.09×10^5 ft·lb of energy in the form of potential energy.

ELECTRICAL WORK

Electrical work is defined by the formula given in Equation 3.

$$W = VQ \tag{3}$$

where

V = potential difference through which charge is moved.
Q = quantity of charge moved.

Measurement of charge moved is not always experimentally practical. However, using the definition of current, namely $I = Q/t$, charge can be defined as $Q = It$. (I = current; t = time elapsed.) This expression for "charge" in Equation 3 yields the formula shown in Equation 4 for electrical work:

$$W = VIt \tag{4}$$

where

I = current in amps.
t = time in seconds.

Potential difference, current and time are measurable quantities. Potential difference is measured by a voltmeter; current, by an ammeter; and time, by the clock. Equation 4 is used to solve a problem in Example B. Example C equates energy stored to work done.

In Examples B and C the electrical work needed to drive the pump was considerably greater than the energy stored in the water. In other words, the input work was greater than the output work. The ratio of output work to input work is called efficiency and it is usually expressed as a percentage. Efficiency can be calculated by the formula shown in Equation 5 and Example D.

$$\eta = \frac{W_{out}}{W_{in}} (100) \qquad (5)$$

where

η = % efficiency.
W_{out} = work output.
W_{in} = work input.

EXAMPLE B: ELECTRICAL WORK TO DRIVE A PUMP

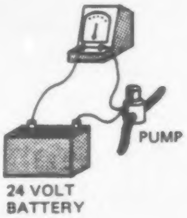
Given: A water pump connected to a 24-volt power supply for a period of 30 seconds; the current to the pump measures 10 amperes.

Find: Amount of electrical work done.

Solution:

$$W = VIt$$
$$= (24\ V)(10\ A)(30\ s)$$
$$W = 7200\ J$$

7200 joules of work are needed to drive the pump.



EXAMPLE C: ENERGY STORED BY LIFTING WATER

Given: The pump in Example B lifts 50 kilograms of water to a height of 10 meters. (Recall: Weight of a 1 kg mass is 9.8 newtons.)

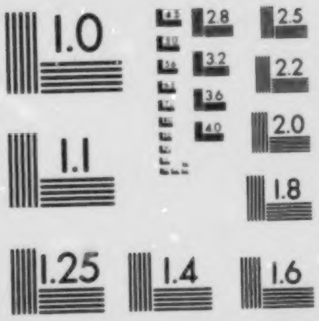
Find: Amount of energy stored in the water.

Solution:

$$\text{Energy stored} = W = \rho_W h\ V$$
$$= (50\ kg)\left(\frac{9.8\ N}{1\ kg}\right)(10\ m)$$
$$= (490\ N)(10\ m)$$
$$W = 4900\ N\cdot m = 4900\ J$$

4900 joules of potential energy are stored in the water because of its higher position above the reservoir.

2


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

EXAMPLE D: EFFICIENCY OF A WATER PUMP

Given: The pump described in Examples B and C.

Find: Its efficiency.

Solution:

$$\eta = \frac{W_{out}}{W_{in}} (100)$$

$$= (\frac{4900\ J}{7200\ J})(100\%)$$

$$= (0.68)(100\%)$$

$$\eta = 68\%$$

The pump is 68% efficient. This figure indicates that 32% of the energy supplied is lost to friction in the pump and fluid resistance in the pipes.

SUMMARY

The work done by a water pump consists of the force which must be applied to water to displace it to a given height (or against other resistance). Weight density of the fluid being pumped and height to which pumped determine how much work the pump does.

PUMP WORK AND EFFICIENCY

MATERIALS

VOM (volt-ohm-milliammeter).
d.c. ammeter (0-10 amp range).
d.c. power supply.
d.c. water pump.
Connecting wire.
Timer or stopwatch.
Meter sticks and metric tape.
Plastic hose to fit pump outlet.
Two bathroom scales.
Large container such as a garbage can.

PROCEDURE

1. Set up pump and other equipment as shown in Figure 1.

2. Connect all wiring as shown in Figure 2.

3. Fill container with water.

4. Turn on power supply, and adjust voltage to value listed in the Data Table for trial 1.

5. Turn off power supply, and record in Data Table total of readings on the platform scales for the start of trial.

6. Measure height from top of water in container to highest point in outlet hose (h).

7. Turn on power supply and timer simultaneously, and allow the pump to run for time specified in

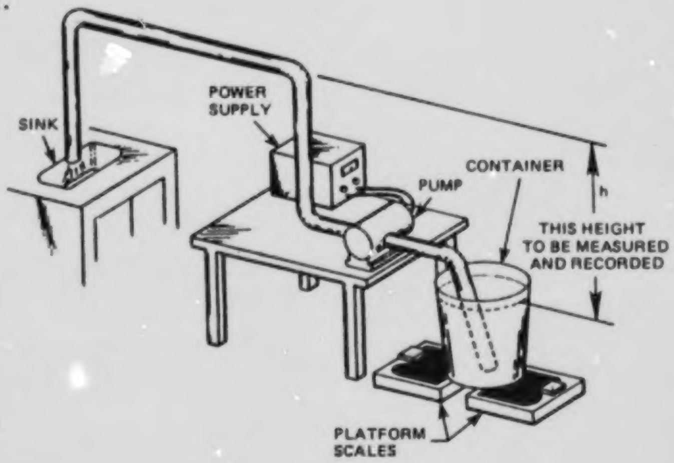

SINK
POWER SUPPLY
CONTAINER
PUMP
h
THIS HEIGHT TO BE MEASURED AND RECORDED
PLATFORM SCALES

FIG. 1

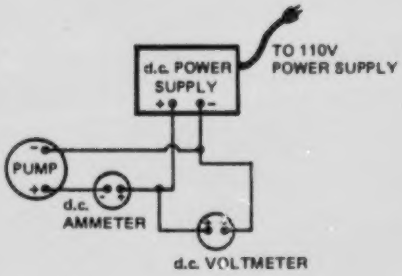

d.c. POWER SUPPLY
+ -
TO 110V POWER SUPPLY
PUMP
- +
d.c. AMMETER
d.c. VOLTMETER

FIG. 2

the Data Table for trial 1. Measure and record in Data Table current to pump motor. Make sure pump is turned off exactly at end of specified time.

8. Record in Data Table total of readings on platform scales for the end of trial.

9. Measure height (h) from top of water remaining in the container to highest point in outlet hose. Add this to measurement taken in step 6, and divide by 2 in order to obtain an average value for the height the water is lifted. Record this average value in Data Table.

10. Refill container.

11. Repeat steps 5 through 10 for trials 2, 3 and 4.

12. Calculate input electrical work, output mechanical work and efficiency of system for each trial run, and complete Data Table.

13. How would the items calculated in step 12 change for a liquid having less density than water?

DATA TABLE

Trial	Voltage (Volts)	Current (Amps)	Time (s)	Total Scale Readings		Input Electrical Work* (J)	Output Mechanical Work**	% Efficiency ***
				Start of Trial (kg)	End of Trial (kg)			
1	8		120					
2	10		120					
3	10		60					
4	10		60					

*Electrical work: $W = VIt$

**Mechanical work (fluids): $W = \rho_W h$

(Recall that 1 kg mass weighs 9.8 newtons.)

***Efficiency: $\eta = \frac{W_{out}}{W_{in}} (100)$

Height to which water is lifted _______ m

SELF CHECK

1. Define:

   Fluid work (done by a pump).
   Electrical work.
   Efficiency.

2. What are the forcelike and displacementlike quantities in fluid work?

3. What are the forcelike and displacementlike quantities in electrical work?

4. A pump operating at 24 volts while drawing 5 amps current fills a tank of 1000-gallon capacity. The water is raised 30 ft before it can be discharged into the tank. How much work is done by the pump? How much time will be required to fill the tank if the efficiency of the system is 75%?

5. A pump operating on 110 V with an efficiency of 80% lifts 200 kg of oil to a height of 40 m in 2 minutes. How much work is done? What current is required?

6. What is the efficiency of a pump which has filled a 2000 m^3 tank to capacity by raising the water 40 m above a reservoir before discharging it into the tank. The pump operates on 110 volts while drawing an 8 amp current. It takes the pump 300 hours to fill the tank.

7. What is the proper operating voltage of a water pump if it is designed to fill the tank of Problem 6 in 5 days while drawing a current of 12A? The pump is 75% efficient.

8. How high can a water pump lift 300 kg of water in a period of 5 minutes if the pump is 80% efficient and operates on 110 volts while drawing a current of 8 amps?

WORK DONE BY A HYDRAULIC LIFT

Hydraulic tools are commonly found in today's world. In the local garage, a mechanic raises a car on a hydraulic lift to get under the chassis. With a few light strokes of a lever, dentists and barbers use hydraulics to lift their chairs to a convenient working height.

In this module, work as it applies to mechanical output, work of a hydraulic lift system and work done on and by hydraulic fluids as they exert pressure within the hydraulic system of the lift are described. A hands-on procedure using a simple automotive hydraulic jack provides an opportunity to observe and calculate work done by the lift. It should be noted that the hydraulic jack is used as equipment for Modules 1F3, 2F2, and 8F1. Each of these modules addresses a different concept: force, work and force transformers, respectively.

OBJECTIVES

To complete this module, the student should be able to:

- Describe a hydraulic lift in two or three sentences, including a labeled diagram.
- List four hydraulic lift applications.
- Calculate output work of a hydraulic lift working against gravity, given appropriate unknowns.
- Calculate work done by fluid system in a hydraulic lift, given piston area, stroke length and fluid pressure.
- Given appropriate equipment, determine input, output work and efficiency of a hydraulic jack.

DISCUSSION

A hydraulic lift can be defined generally as a device designed to lift heavy loads. Specifically, the hydraulic lift transforms work done on its fluid system into mechanical work done against gravity to lift the heavy load. The element that accomplishes this transformation is called the hydraulic (working) cylinder. The simplest kind of hydraulic lift positions the working cylinder directly under the load, and the piston stroke of the cylinder equals distance traveled by the load. This type of lift is known as a hydraulic jack. Other hydraulic lifts apply mechanical work of the working cylinder to a system of levers to gain increased distance in which the heavy load travels. Figure 1 shows these two types of cylinder positions.

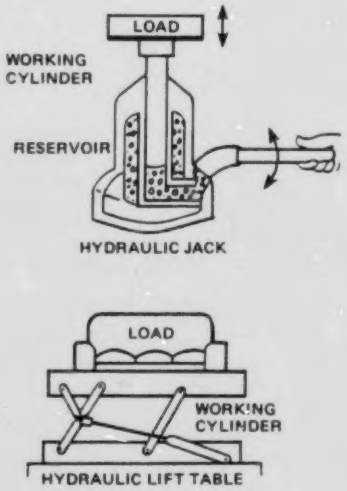


FIG. 1

Another important element of a hydraulic lift is the fluid system, which provides leverage to amplify an input force and a medium by which pressure is transmitted equally throughout. The hydraulic fluid moves through a circuit, sometimes composed of high-pressure hoses. A reservoir of fluid ensures that an adequate supply of fluid is always available. Many hydraulic lift systems contain a pump at the heart of the fluid system. The pump must maintain a high pressure in the fluid and supply changes in fluid volume. Various relief and control valves ensure that the system behaves as desired and meets safety requirements. A hydraulic tailgate lift on a truck is an example of a hydraulic lift system that incorporates all of the above mentioned elements, most of which are shown in Figure 2.

Every hydraulic lift system is powered by some source of input work. In Figure 2, an electric motor provides input work to drive the hydraulic pump. In the simple hydraulic jack lift, input work is provided by force exerted on the handle by the user.

In the case of the hydraulic car lift shown in Figure 3, input work is provided by high air pressure. The oil reservoir is pressurized and the oil valve opened to allow oil to flow under the plunger and raise the car.

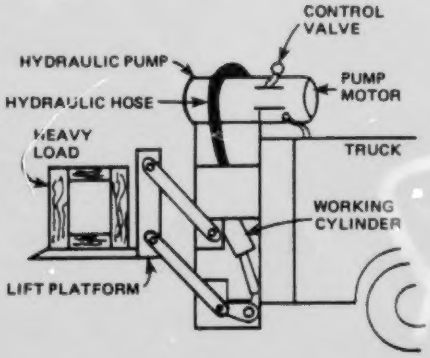


FIG. 2

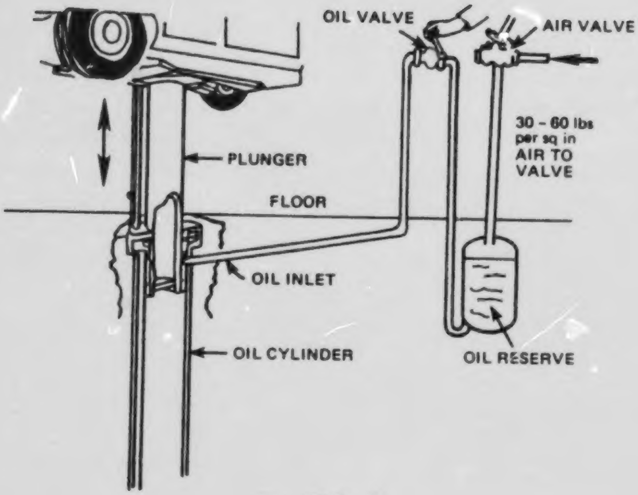

OIL VALVE
AIR VALVE
30 - 60 lbs per sq in AIR TO VALVE
PLUNGER
FLOOR
OIL INLET
OIL CYLINDER
OIL RESERVE

FIG. 3

HYDRAULIC LIFT OUTPUT WORK

The output work of a hydraulic lift is mechanical work done against gravity to raise a heavy load. As shown in Figure 4, direction of load weight (a gravitational force) is opposite to the displacement. The magnitude of the displacement is the height traveled by the load. Lifting force applied to the load must be at least equal to weight of the load through the travel duration.

Work done by the lift is given by Equation 1:

$$\text{Work done by hydraulic lift} = \text{load weight} \times \text{height traveled} \quad (1)$$

Example A illustrates a simple application of Equation 1.

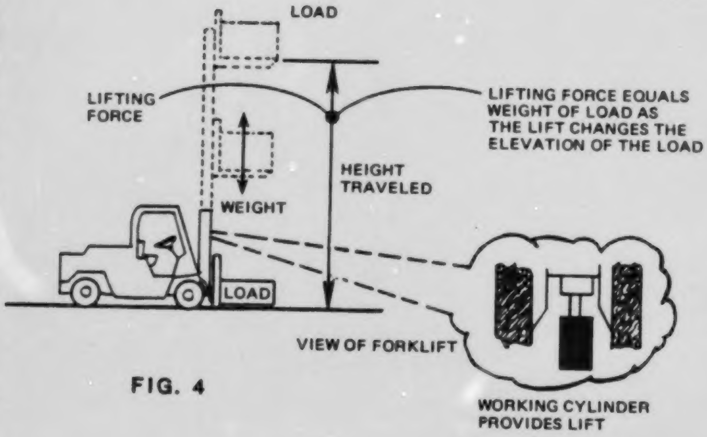

LOAD
LIFTING FORCE
LIFTING FORCE EQUALS WEIGHT OF LOAD AS THE LIFT CHANGES THE ELEVATION OF THE LOAD
HEIGHT TRAVELED
WEIGHT
LOAD
VIEW OF FORKLIFT
WORKING CYLINDER PROVIDES LIFT

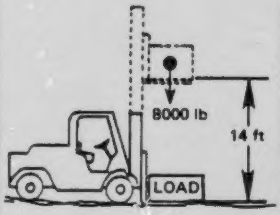
FIG. 4

EXAMPLE A: HYDRAULIC FORK LIFT WORK CALCULATION

Given: A forklift hydraulic system lifts a load weighing 8000 lb through a distance of 14 feet vertically.

Find: Output work.

Solution:
$$\begin{aligned}\text{Work} &= \text{weight} \times \text{height}\\ &= (8000\ \text{lb}) \times (14\ \text{ft})\\ \text{Work} &= 112{,}000\ \text{ft} \times \text{lb}.\end{aligned}$$



WORK DONE BY FLUID SYSTEM

In Concept Module 3-0, work in fluid systems was given by Equation 2:

$$\text{Work (fluids)} = \text{pressure} \times \text{volume change}.$$
$$W = P \times \Delta V. \tag{2}$$

To illustrate this concept, a simple hydraulic cylinder is shown in Figure 5.

Fluid volume change can be determined by Equation 3:

$$\text{Fluid volume change} = (\text{cross-sectional area of piston}) \times (\text{length of piston travel}).$$
$$\Delta V = A \times L. \tag{3}$$

Combining Equations 2 and 3, Equation 4 is obtained:

$$\text{Work done by fluid system in hydraulic cylinder} = \text{fluid pressure} \times \text{cross-sectional area of piston} \times \text{length of piston travel}.$$
$$W = P \times A \times L. \tag{4}$$

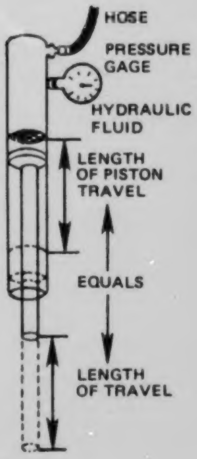
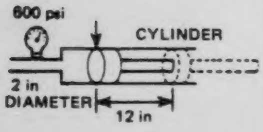


FIG. 5

Example B illustrates the use of Equation 4.

EXAMPLE B: WORK DONE IN A FLUID SYSTEM


600 psi
CYLINDER
2 in
DIAMETER
12 in

Given: A fluid system in which fluid pressure is 600 lb/in^2, hydraulic cylinder is 2 inches in diameter and piston travels 12 inches.

Find: Work done by the fluid system.

Solution:

$$\text{Work} = \text{pressure} \times \begin{matrix}\text{piston}\\ \text{area}\end{matrix} \times \begin{matrix}\text{piston}\\ \text{travel}\end{matrix}$$

$$= (600\ lb/in^2) \times (\pi\ in^2) \times (1\ ft)$$

$$\text{Work} = 1885\ ft \times lb.$$

Cross-Sectional Area

$$A = \frac{\pi}{4} D^2$$

$$= \frac{\pi}{4} (2\ in)^2$$

$$A = \pi\ in^2.$$

In the conversion process from work done by fluid in the cylinder to output mechanical work against gravity in a lift, the process is about 97% efficient (almost all fluid work is converted to usable mechanical lifting work). Only a small percentage of energy is lost to friction. The example in Figure 6 illustrates the energy (work) in a hydraulic lift system with a mechanical energy input.

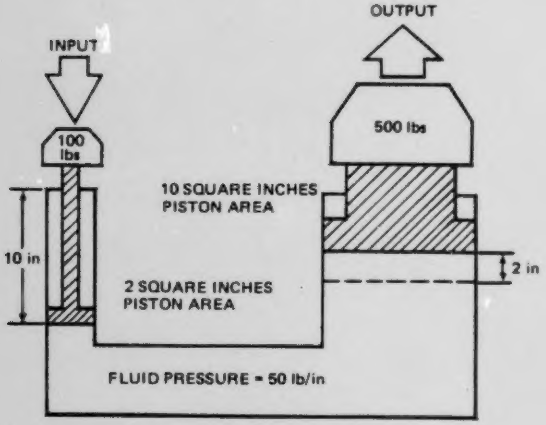

INPUT
OUTPUT
100 lbs
500 lbs
10 SQUARE INCHES PISTON AREA
10 in
2 in
2 SQUARE INCHES PISTON AREA
FLUID PRESSURE = 50 lb/in

FIG. 6

Input Work	Work on Fluid by Input Piston
$W_{inp} = F_{in} \times d_{in}$ $= (100\ \text{lb})(10\ \text{in})$ $W_{inp} = 1000\ \text{lb-in}$	$W = PAd$ $= (50\ \text{lb/in}^2) \times (2\ \text{in}^2) \times (10\ \text{in})$ $W = 1000\ \text{lb-in}$

Work by Fluid on Output Piston	Output Work
$W = PAd$ $= (50\ \text{lb/in}^2)(10\ \text{in}^2) \times (2\ \text{in})$ $W = 1000\ \text{lb-in}$	$W_{out} = F_{out} \times d_{out}$ $= (500\ \text{lb}) \times (2\ \text{in})$ $W_{out} = 1000\ \text{lb-in}$

By examination of numerical results shown in Figure 6, the statement can be made that, while force is multiplied by the lift, work done is transmitted undiminished throughout the system. The increase in force supplied by the lift is achieved at the expense of load distance moved. Note clearly that the applied force of 100 pounds must move through a distance of ten inches to lift the 500 pound load through a reduced distance of 2 inches.

LABORATORY

MATERIALS

One hydraulic jack (1 1/2 ton or 3 ton) tapped for a pressure gage, with lever.
Pressure gage (0 - 100 psi). (Should be sufficient if suggested weights are used.)
Vernier caliper.
Metal scribe.
Set of heavy weights (1 kg, 2 kg, 5 lb).

PROCEDURE

1. Arrange experiment equipment as shown in Figure 7, making certain that weights are loaded on the output piston in such a manner that they will not fall.
2. Measure diameter of output piston with vernier caliper; record in data table.
3. Measure diameter of small input piston with vernier caliper; record in data table.
4. Place two large weights on output piston; grasp lever and press input piston down as far as it will go. Mark piston where it enters body of the jack, as shown in Figure 8. (Mark A with a metal scribe.)
5. Raise lever as far as possible; mark piston where it enters body of the jack. (Mark B in Figure 8 with metal scribe.)
6. Using vernier caliper, measure length from mark A to mark B; record this length in data table. This length is called input piston stroke travel.
7. Mark output piston where it enters and leaves the body; raise weights up through about a 6-inch distance. Count number of complete full strokes; note pressure of hydraulic fluid while raising weights.

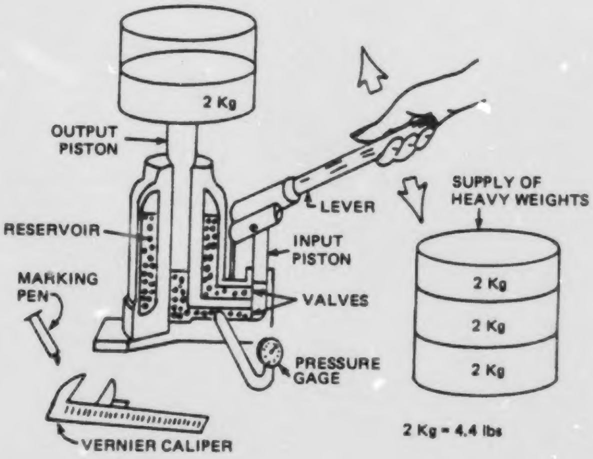

2 Kg
OUTPUT PISTON
LEVER
SUPPLY OF HEAVY WEIGHTS
RESERVOIR
INPUT PISTON
MARKING PEN
VALVES
2 Kg
2 Kg
2 Kg
PRESSURE GAGE
VERNIER CALIPER
2 Kg = 4.4 lbs

FIG. 7

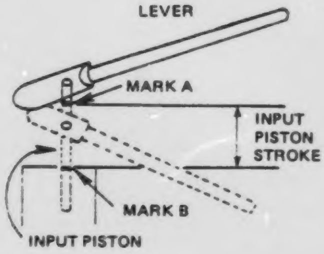

LEVER
MARK A
INPUT PISTON STROKE
MARK B
INPUT PISTON

FIG. 8

Record number of strokes, fluid pressure, height that large weights were raised and value of weights (masses) raised.

8. Lower output piston to mark again; add additional load to output piston and repeat steps 4 through 7. Repeat step 8 until all rows in data table are completed.
9. Complete the data table by performing the following calculations.

$$\underset{\text{(ft-lb)}}{\text{Work output}} = \underset{\text{(lb)}}{\text{force}} \times \underset{\text{(ft)}}{\text{load travel}}.$$

$$\underset{\text{(ft-lb)}}{\text{Work done by fluid on output piston}} = \underset{(\text{lb/in}^2)}{\text{fluid pressure}} \times \underset{(\text{in}^2)}{\text{area of output piston}} \times \underset{\text{(ft)}}{\text{load travel}}.$$

$$\underset{\text{(ft-lb)}}{\text{Work done on fluid by input piston}} = \underset{(\text{lb/in}^2)}{\text{fluid pressure}} \times \underset{(\text{in}^2)}{\text{area of input piston}} \times \underset{\text{(ft)}}{\text{travel of input piston per stroke}} \times \text{number of strokes}.$$

$$\underset{(\%)}{\text{Efficiency}} = \frac{\text{work output (ft-lb)}}{\text{work done by input piston (ft-lb)}} \times 100.$$

DATA TABLE

1. Output piston diameter _____ in - output piston cross-sectional area _____ in^2. ($A = \frac{\pi}{4} D^2$.)
2. Input piston diameter _____ in - input piston cross-sectional area _____ in^2.
3. Input piston travel per stroke _____ ft.

MEASUREMENTS			
Load on Output Piston (lb)	Height of Load Travel (ft)	Hydraulic Fluid Pressure (lb/in^2)	No. of Input Strokes

CALCULATION RESULTS			
Work Output (ft x lb)	Work Done by Fluid on Output Piston (ft x lb)	Work Done on Fluid by Input Piston (ft x lb)	Efficiency %

SELF CHECK

1. Draw and label a sketch of a hydraulic lift.

2. List four applications of hydraulic lifts.

3. A hydraulic jack is used to lift a weight of 1200 lb a distance of 8 inches. How much work is done by the jack?

4. The input piston of the above jack has an area of 0.25 in^2 and a stroke length of 1.5 in. The fluid pressure is 692 lb/in^2 and 38 strokes are required to raise the weight. Calculate the input work.

5. Calculate the efficiency of the above jack.

6. How many strokes of an input piston of radius 0.5 inches are required to lift a weight of 2000 lb through a distance of 1 ft if the fluid pressure in the lift is 500 lb/in^2 and the stroke length is 2 inches? Assume the lift is 100% efficient.

7. What is the efficiency of the lift in Problem 6 if it requires 35 strokes to raise the 2000 lb load through a distance of 1 foot?

2T1

HEAT ENERGY REQUIRED FOR A TEMPERATURE INCREASE

Engineers and technicians need to know thermal properties of materials, such as specific heat, heat of fusion, and heat of vaporization, that are vital for heating and cooling applications. In industrial chemical processes, the necessity often arises to determine how much energy is needed to heat, cool, melt, or boil the chemical reactants. Thermal properties of materials often are measured with the aid of a device called a "calorimeter."

This module introduces the use of the calorimeter for measurement of the thermal properties of materials. In the laboratory, the student will use a calorimeter (or experimental apparatus similar to a calorimeter) to measure some thermal properties of materials.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Sketch and label the eight parts of a common metallic-can calorimeter.
- Define the "water equivalent" of a calorimeter.
- Describe the use of the principle of conservation of energy used in solving calorimetry problems.
- Given six of the seven following factors, calculate the seventh:

    Mass of water in calorimeter.
    Water equivalent of calorimeter.
    Initial temperature of sample.
    Mass of sample.
    Initial temperature of water.
    Final temperature of water.
    Specific heat of sample.

- Given the mass of water in a calorimeter, the water equivalent of the calorimeter, the initial temperature and mass of a liquid sample, the specific heat of the sample as a liquid and a solid, the freezing point of the sample, and the initial and final temperature of the water, calculate the heat of fusion of the sample.
- Given the appropriate equipment, determine the specific heat of a metal sample and heat of fusion of water.

DISCUSSION

Energy is required to heat, to melt, or to boil materials. Materials differ from one another in the amount of heat required to produce a certain temperature rise and in amount of energy required to melt or to boil a specific amount of the material. These thermal properties — specific heat, heat of fusion, and heat of vaporization — have been defined in preceding modules. Knowledge of these terms is important for many scientific and engineering applications.

Measurement of these important quantities often is carried out with a device called a calorimeter. The word "calorimeter" comes from the Greek word "calor," meaning heat, and from the Latin word "meter," meaning measure. Thus, a calorimeter literally is a device for measuring heat.

There are many types of calorimeters; the most common design in a metallic can containing water. The can contains a measured quantity of water and has a cover through which a thermometer and a stirrer are inserted. An insulating jacket, or pocket of air, surrounds the can to reduce heat losses. Basic structure is sketched schematically in Figure 1.

The thermometer is used to measure temperature of the water before and after the unknown quantity of heat is introduced into the calorimeter. The amount of heat may be found from the measured rise in temperature of the water.

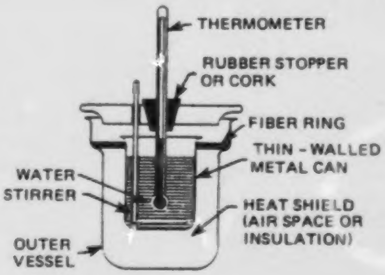


FIG. 1

MEASUREMENT OF SPECIFIC HEAT

A sample of material whose specific heat c_s is to be found is measured to have a mass m_s. Water is poured into a calorimeter, stirred, and its temperature measured. The sample is heated to a known temperature T_s and transferred into the calorimeter. When the mixture has had time for the temperature of the sample and the water to equalize, temperature of the water is measured. T_1 and T_2 are the initial and final temperatures of the water, respectively; and m_w is the mass of the water. Since the inner calorimeter can and stirrer also are heated, heat gained by calorimeter and stirrer must be included. m_c is the mass of the inner calorimeter can and stirrer, and c_c its specific heat. Assuming that no heat is lost from the calorimeter during the experiment, heat given up by the hot sample in cooling must equal heat gained by the water and by the calorimeter can. The basic equation, Equation 1, was introduced in Concept Module 2-0, "Work," and is repeated here:

$$W = mc\ (T_2 - T_1) \tag{1}$$

where

W = thermal work.
m = mass.
c = specific heat of substance.
T_1 = initial temperature
T_2 = final temperature.

Since T_2 is the final temperature of the sample, energy given up by the sample is indicated by Equation 2.

$$H_L = m_s c_s\ (T_s - T_2) \tag{2}$$

where

H_L = heat energy lost by sample.
m_s = mass of sample.
c_s = specific heat of sample.
T_s = temperature of sample.

Thermal energy lost by the sample is gained by the water and the calorimeter can (and stirrer). Energy gained by the water is found by use of Equation 3.

$$H_G = (m_w)(1)(T_2 - T_1) \tag{3}$$

where

m_w = mass of water.
H_G = heat energy gained by water.
1 = specific heat of water.

Energy gained by the calorimeter can is found by use of Equation 4.

$$H_G = m_c c_c\ (T_2 - T_1) \tag{4}$$

where

m_c = mass of calorimeter and stirrer.
c_c = specific heat of calorimeter can and stirrer.

If heat energy is to be conserved, the total energy gained by the calorimeter and water is equal to the energy lost by the sample. This equality can be stated mathematically in the form shown in Equation 5.

$$m_s c_s (T_s - T_2) = m_w (T_2 - T_1) + m_c c_c (T_2 - T_1) \qquad (5)$$

The right-hand side of the equation represents heat gained by the water and calorimeter. Heat gained by the can is equal to that gained by a mass of water equal to the product $m_c c_c$, heated through the temperature difference $T_2 - T_1$. The product $m_c c_c$, therefore, is called the "water equivalent" of the calorimeter. For many commercial calorimeters, value of the water equivalent will be specified by the manufacturer or marked on the calorimeter. The water equivalent has units of energy per unit temperature (cal/°C). If water equivalent of the calorimeter is added to the mass of water present, the calorimeter in all succeeding calculations can be ignored. When the water equivalent of a calorimeter is known, the solution of calorimeter problems is easier. Equation 5 can be rewritten in the shortened form shown as Equation 6.

$$m_s c_s (T_s - T_2) = (m_w + m_c c_c)(T_2 - T_1) \qquad (6)$$

Note that the specific heat c_s of the sample now can be determined from Equation 6, since all other quantities — namely m_s, T_s, T_2, T_1, m_w, and $m_c c_c$ — are known, or have been measured.

In a similar way, the calorimeter can be used to measure heat associated with a change of phase (melting, freezing, vaporization, or condensation) of a material. Change of phase is not accompanied by a change in temperature, but it does require energy. Changes from the solid to the liquid phase (melting) and from the liquid to the vapor phase (boiling) causes a substance to absorb energy. In reverse processes (condensing and freezing), the substance liberates energy.

Latent heat of fusion can be measured with a calorimeter when the sample is allowed to freeze. The amount of heat liberated during freezing will raise temperature of the calorimeter water by an amount measured. This relationship is given by Equation 7.

$$H_F = L_f m_s \qquad (7)$$

where

H_F = heat energy liberated by freezing.
L_f = latent heat of fusion.
m_s = mass of substance.

Energy given off by the substance during the freezing process is absorbed by the water and by the calorimeter. If water temperature increases from T_1 to T_2, the energy balance is given in Equation 8:

Heat liberated = heat absorbed

$$L_f m_s = m_w\,(T_2 - T_1) + m_c c_c\,(T_2 - T_1)$$

or

$$L_f m_s = (m_w + m_c c_c)(T_2 - T_1) \qquad (8)$$

To determine the latent heat of freezing experimentally, begin with temperature of the sample somewhat above, and end somewhat below, its freezing point. This process causes one or more additional contributions of heat energy caused by temperature change of the sample before and after freezing.

Example A combines all the above elements and illustrates how a calorimeter problem is solved.

EXAMPLE A: HEAT LOSS BY SAMPLE COOLED FROM ABOVE MELTING POINT

Given: 100-g sample of liquid lead at an initial temperature of 500°C has been inserted into a calorimeter containing 200 g of water at 5°C and having a water equivalent of 150 cal/°C. Melting point of lead is 327°C, and its heat of fusion is [illegible]6 cal/g. Specific heat of liquid lead is 0.035 and of solid lead is 0.031.

Find: Final temperature of the water.

Solution:

(1) Heat energy loss by the lead in cooling from 500°C to the melting point (327°C):

$$m_{\substack{\text{liquid}\\\text{lead}}}\;c_{\substack{\text{liquid}\\\text{lead}}}\;(T_2 - T_1) = (100\text{ g})\left(0.035\ \frac{\text{cal}}{\text{g }^\circ\text{C}}\right)(500^\circ\text{C} - 327^\circ\text{C})$$

$$\text{Loss (1)} = 605\text{ cal}$$

(2) Heat energy loss by the lead while freezing (no change in temperature of lead occurs; that is, T remains at 327°C):

$$\begin{aligned} H_F &= L_f m_s \\ &= \left(5.86\ \frac{\text{cal}}{\text{g}}\right)(100\text{ g}) \end{aligned}$$

$$\text{Loss (2)} = 586\text{ cal}$$

(3) Heat energy loss by solid lead in cooling from the freezing point, 327°C, to the final temperature (T):

$$m_{\substack{\text{solid}\\\text{lead}}}\, c_{\substack{\text{solid}\\\text{lead}}}\, (T_2 - T_1) = (100\ \cancel{g})\left(0.031\ \frac{\text{cal}}{\cancel{g}\ °\text{C}}\right)(327 - T)$$

$$\text{Loss (3)} = \left(3.1\ \frac{\text{cal}}{°\text{C}}\right)(327 - T)$$

(4) Heat energy gained by water:

$$m_{\text{water}}\, c_{\text{water}}\, (T - T_1) = (200\ \text{g})\left(1\ \frac{\text{cal}}{\text{g}\ °\text{C}}\right)(T - 5°\text{C})$$

$$\text{Gained (1)} = \left(200\ \frac{\text{cal}}{°\text{C}}\right)(T - 5°\text{C})$$

(5) Heat gain by calorimeter:

$$\left(\substack{\text{water}\\\text{equivalent}}\right)(T - T_1) = \left(150\ \frac{\text{cal}}{°\text{C}}\right)(T - 5°\text{C})$$

$$\text{Gained (2)} = \left(150\ \frac{\text{cal}}{°\text{C}}\right)(T - 5°\text{C})$$

Since heat gain must equal heat loss, these separate factors can be added together to give an energy-balanced equation.

Heat energy gain = heat energy loss

$$200\ \frac{\text{cal}}{°\text{C}}\ (T - 5°\text{C}) + 150\ \frac{\text{cal}}{°\text{C}}\ (T - 5°\text{C}) = 605\ \text{cal} + 586\ \text{cal} + 3.1\ \frac{\text{cal}}{°\text{C}}(327 - T)$$

$$350\ (T - 5) = 1191 + 3.1\ (327 - T$$

$$353.1\ T = 1191 + 1014 + 1750$$

$$= 3955$$

$$\therefore \quad T_2 = 11.2°\text{C}$$

Final temperature of the water is 11.2°C.

Notice that in Example A, the final temperature T_2 was found. In actual practice, the equilibrium temperature of 11.2°C could have been measured, and the equation above used to determine latent heat for lead. (See, for example, Problem 6 in the Self Check at the end of this module.)

In Example A, specific heat was assumed to be a constant, independent of temperature. This assumption is not strictly true, because specific heat does vary slightly as temperature changes. However, at ordinary temperatures and over reasonably small temperature ranges, specific heat can be considered as constant. The error introduced by this assumption will not be great.

Another source of error involves heat exchanged between the calorimeter and the surroundings. Therefore, a quality calorimeter should be well-insulated to minimize this loss, as any heat losses may seriously affect measurements. If possible to arrange the experiment in such a manner that initial temperature of the water in the calorimeter is slightly below that of the surroundings, and the final temperature slightly above, this error can be reduced. In such a case, the heat lost to, and gained from, the surroundings will tend to cancel out.

SUMMARY

An accurate determination of the heat energy necessary to raise the temperature of a body a certain amount requires that the mass and specific heat of the body be known. The specific heat of a substance can be determined experimentally with a calorimeter. Careful procedures and measurements are required, and the heat energy transferred to the calorimeter also must be accounted for. Calorimeters also can be used to determine the latent heat of a material.

DETERMINATION OF SPECIFIC HEAT OF METAL SAMPLE

MATERIALS

Water calorimeter. (An acceptable calorimeter can be made from styrofoam cups or from other materials.)
Thermometer.
Balance.
Samples of copper and aluminum. (Other metals can be substituted.)
Ice.
Bunsen burner and ring stand with wire gage; or hot plate.
Beaker.
Tongs (insulated jaws).

PROCEDURE

1. Determine mass of a small quantity of water, m_w, with a balance. Record in Data Table 1. Pour water into a calorimeter.

2. Determine and record mass (m_s) of metal sample with balance. Immerse metal sample in a beaker of water; heat it on hot plate until water has boiled. Measure and record this temperature (T_s). At the same time, stir and measure temperature T_1 of water in the calorimeter. With tongs, quickly remove metal from boiling-water bath and insert it into calorimeter. Be careful not to transfer water with metal. Stir thoroughly; allow time for temperature to come to equilibrium. Measure and record final temperature T_2 of water in calorimeter.

3. From measurements taken, calculate specific heat, using formula in Equation 5. If a commercial calorimeter is used, use manufacturer's stated water equivalent of the calorimeter. If a home-made calorimeter, from a can of known metal, is used, weigh can, and use a table of specific heats to determine water equivalent. (Table 4 in Concept Module 2-0 lists specific heats of several common metals.)

4. When the specific heat of metal has been determined, compare results to tabulated value (Table 4 of Module 2-0). Complete Data Table 1. Discuss possible sources of error with instructor.

5. Repeat Steps 1 through 4 with other sample provided.

MEASUREMENT OF HEAT OF FUSION OF ICE

MATERIALS

Same materials as in first procedure.

PROCEDURE

1. Measure mass m_w of a small quantity of water; pour into calorimeter.

2. From an ice-water bath (a well-stirred mixture of water and ice at 0°C), dip out a piece of ice, dry it with a paper towel. Mass m_i of ice dipped out may be determined later by weighing entire calorimeter can containing original water and melted ice. Record in Data Table 2 the temperature of water (T_1) in calorimeter just before adding ice.

3. Drop piece of ice into calorimeter, and stir until all ice melts. (If all ice does not melt, start again with somewhat warmer water in calorimeter.)

4. Measure and record final (minimum) temperature T_2.

5. Reweigh calorimeter can to determine mass of ice added; record ice mass.

6. Calculate heat of fusion using Equation 8, adding a factor that takes into account the increase in temperature of mass of melted ice following melting process. The equation becomes --

   $$L_f m_i + m_i (1)(T_2 - 0°\text{C}) = (m_w + m_c c_c)(T_1 - T_2).$$

   Two terms on left side of the equation will be amount of energy required to melt ice and then raise melted ice to final temperature T_2. Note that, on the right side $(T_1 - T_2)$ is used, since final temperature T_2 is lower than initial temperature T_1.

7. Compare heat of fusion of water determined here with known value of 80 calories per gram. Complete Data Table 2. Discuss possible sources of error in this experiment with instructor.

Sample	m_w	m_s	T_s (Sample)	T_1	T_2	c_s (Calculated)	c_s (Standard)	% Error
1 ____								
2 ____								

m_w	m_i (Ice)	T (Ice)	T_1	T_2	L_f (Calculated)	L_f (Standard)	% Error
		0°C				$80\ \frac{\text{cal}}{\text{g}}$	

SELF CHECK

1. Describe the construction of a calorimeter; sketch and label its parts.

2. Define "water equivalent" of a calorimeter.

3. Describe how to use a calorimeter to measure the following quantities:

   Specific heat.
   Heat of fusion.

4. Show how Equations 5 or 6 and 8 demonstrate that energy is conserved, in this case, heat energy.

5. Suppose that one adds a piece of aluminum weighing 100 g and heated to a temperature of 180°C to a water calorimeter containing 200 g of water at 10°C and with a water equivalent of 152 cal/deg. What will be the final temperature of the well-stirred mixture? (Specific heat of aluminum is 0.22 cal/g-C°.)

6. Suppose that one drops some liquid gallium metal (which melts near 30°C) at a temperature of 50°C and a weight of 100 g into a water calorimeter containing 100 g of water at 5°C. After the mixture is stirred, final temperature is 20°C, and the gallium metal has solidified. The water equivalent of the calorimeter is 46.35 cal/deg. Specific heat of the liquid gallium is 0.0954, and that of solid gallium is 0.0895 cal/g-C°. What is the latent heat of fusion of gallium metal? (Remember to include separate contributions for cooling of liquid gallium, heat of fusion, and cooling of solid gallium.)

7. What is the water equivalent of a steel calorimeter weighing 100 g? (Use Table 4 of Concept Module 2-0.)

8. Since no calorimeter is a perfect insulator, there is always an exchange of heat energy between the calorimeter and its environment. Devise an experimental method to determine the effective (i.e., one that takes into account exchanges with the environment) water equivalent of a calorimeter. Assume that you are given a known quantity of a metal whose specific heat is known. Write down the necessary heat equation and solve it for the effective water equivalent of the calorimeter.

9. When dropping hot metal into the calorimeter, water is carelessly splashed out of the calorimeter. Assuming that the mass of water which the student uses in Equation 5 to determine the specific heat of the metal is the original mass of the water (before spillage), how would the spillage of water affect the calculated value of the metal's specific heat? Would the calculated value be too high or too low? Justify your answer.

2T4

FUNDAMENTALS OF AIR CONDITIONING

The process of cooling an object or an area is accomplished by work, which removes heat from either. Heat is energy; therefore, heating and cooling basically are processes of moving energy from one place to another. Fans, pumps and motors are used to move energy through a system. If a fan is placed in a window and used to pull warm air from the house, while another window fan pulls in cooler evening air to replace it, the fans are moving heat energy. When a fan is directed at somebody, that person experiences a cooling effect, since evaporation of body moisture carries away excess body heat. More complex systems that are capable of moving larger quantities of heat energy have been constructed, creating the air-conditioning or refrigeration effects. This module introduces the basic principles of air conditioning, including energy and cost efficiency. In the laboratory, the student will determine energy and cost efficiency, and will measure temperature of an air-conditioning system.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

  Calorie.
  Kilocalorie.
  Btu.
  Mechanical equivalent of heat.
  EER (energy efficiency ratio).
  Btu/h.

- Determine the EER (energy efficiency ratio) of an air conditioning unit, given the Btu/h and watts input.

- Given the necessary data, calculate the cost difference per day of air conditioners with different EERs.
- Draw a schematic diagram, illustrating the parts of an air conditioning unit; and describe the temperature and pressure changes that take place within the system.
- Given the appropriate equipment, determine cost and energy efficiency of an air-conditioning unit, and measure temperatures of an air-conditioning system.

DISCUSSION

Heat (energy) applied to a body causes the temperature of the body to rise. In fact, this relationship is how the basic unit of heat is defined. There are three common units in use today for measuring heat energy:

The Calorie.
The Kilocalorie.
The British thermal unit (Btu).

The heat unit most commonly used in refrigeration and air conditioning theory is the Btu, which is the quantity of heat that must be supplied to one pound of water to raise its temperature one degree Fahrenheit (Figure 1).

A calorie is the quantity of heat required to raise 1 gram of water through one Celsius degree; therefore, a kilocalorie is the quantity of heat required to raise 1 kilogram of water through one Celsius degree.

In all three of these definitions, heat itself is not directly observed; rather, the change (temperature) that it produces. The conversion between units is given by the following relations:

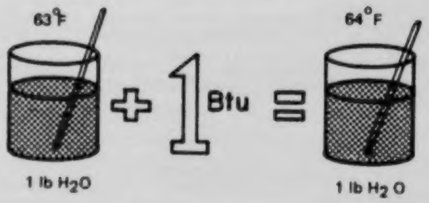


FIG. 1

1 kilocalorie = 1000 calories.
1 Btu = 252 cal = 0.252 kilocalorie.

Before mechanical refrigeration was invented, ice was used to refrigerate. A ton of ice melting in 24 hours absorbs heat at the rate of 12,000 Btu per hour. This figure has carried over into the field of mechanical refrigeration in such a manner that a refrigeration system which removes heat at the rate of 12,000 Btu per hour is said to have a capacity of "one ton." The Btu rating of an air conditioner needed to cool a house of a given size can be calculated as shown in Example A.

Energy in mechanical systems usually is expressed in ft·lbs, ergs or joules. Energy in the form of heat is expressed in calories or Btu's. The relative magnitudes of the "mechanical units" and the "heat units" of energy are known quite accurately and are given as follows:

1 Btu = 778 ft·lb.
1 calorie = 4.186 joules.
1 kilocalorie = 4186 joules.

EXAMPLE A: CONVERTING TON CAPACITY TO BTUs

Given: A 2700 ft^2 home, for which there is an estimated requirement for a 5-ton air-conditioning unit.

Find: The number of Btu per hour (Btu/h) removed from the home by this system.

Solution:

$$H\ (\text{Btu/h}) = \text{Tonnage} \times \frac{12{,}000\ \text{Btu/h}}{1\ \text{ton}}$$

$$= 5\ \text{ton} \times \frac{12{,}000\ \text{Btu/h}}{1\ \text{ton}}$$

$$H\ (\text{Btu/h}) = 60{,}000\ \text{Btu/h}$$

These relations are often expressed in ratio form, and are called "the mechanical equivalent of heat." Thus, 4.186 joules/calorie or 778 ft·lb/Btu are mechanical equivalents of heat.

Mechanical cooling by use of an air conditioner may seem complex; however, the process is merely an application of several principles that already should have been studied. These principles can be summarized in a series of four statements, which should be kept in mind as the functioning of an air conditioner is described. They are as follows:

- As a gas is compressed, its temperature increases.
- Removing sufficient heat energy from a gas causes it to liquefy.
- A liquid under pressure boils at a higher temperature.
- Boiling liquids absorb energy (heat).

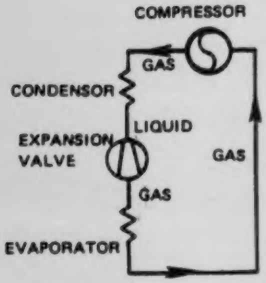


FIG. 2

Freon-12 is a gas vapor at room temperature and pressure. Many air-conditioning systems use this gas or a similar one. Freon is put into an air-conditioning system at a certain pressure. The continual change of the freon back and forth between its gas and liquid phases forms the basis for the air-conditioning system. Figure 2 illustrates the changing state of freon through the air conditioner. A detailed explanation follows.

Freon is introduced into a system and maintained there at a minimum pressure of about 70 psi. The freon starts at the compressor (Figure 2) at about room temperature and at 70 psi. Figure 3 shows that the compressor increases the pressure of the freon in the system. As a result of the increased pressure, there also is an increase in gas temperature. At 200 psi, freon turns to a liquid at about 110°F. Since the gas is now at about 140°F, it will liquefy if cooled by 30 degrees. Liquefaction will be the next step in the process.

The gas is passed through the condenser coils, which function like an automobile radiator. A fan draws outside air past the coils, removing heat from the hot, circulating gas. The freon thus is cooled sufficiently to liquefy (Figure 4).

When liquid freon reaches the expansion valve, it is allowed to drop in pressure to about 70 psi. The lower pressure allows the liquid freon to boil as it rushes into the evaporator coils. The evaporator, acting as a "reverse radiator," supplies the heat energy needed for the freon to boil or vaporize. Necessary heat is supplied by the warm room air as it passes over the evaporator. The air passing over the evaporator is cooled (Figure 5) and is circulated back into the room to lower the ambient temperature.

The gaseous freon now is returned to the compressor, and the cycle repeats itself. Figure 6 summarizes the entire system.

The efficiency of air conditioners becomes increasingly important because of dwindling energy supplies and increasing costs. The energy efficiency ratio (EER) of a central air-conditioning unit is an important factor that determines cost of operation. The energy efficiency ratio measures the amount of cooling an air-conditioning system accomplishes per watt of electricity used. It is arrived at by dividing the Btu/h capacity by the wattage input. The more efficiently an air conditioner operates, the less electricity it uses, and the more money and energy are saved.

The EER of several air conditioning systems are listed in Figure 7.

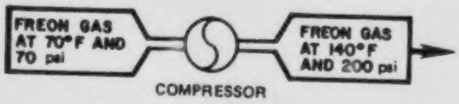

FREON GAS AT 70°F AND 70 psi
COMPRESSOR
FREON GAS AT 140°F AND 200 psi

FIG. 3

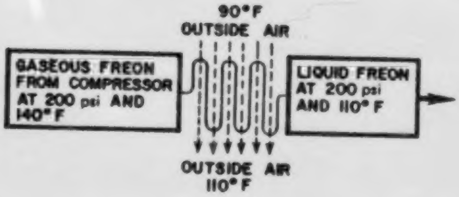

90°F OUTSIDE AIR
GASEOUS FREON FROM COMPRESSOR AT 200 psi AND 140°F
LIQUID FREON AT 200 psi AND 110°F
OUTSIDE AIR 110°F

FIG. 4

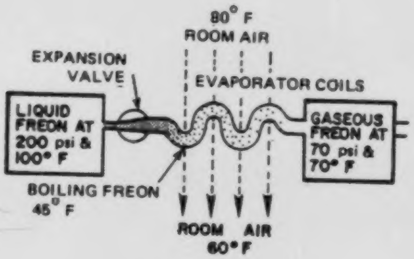

80° F ROOM AIR
EXPANSION VALVE
EVAPORATOR COILS
LIQUID FREON AT 200 psi & 100° F
GASEOUS FREON AT 70 psi & 70° F
BOILING FREON 45° F
ROOM AIR 60° F

FIG. 5

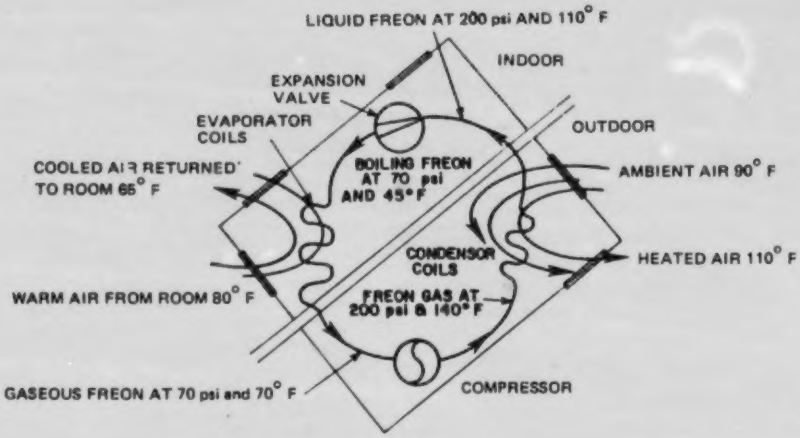
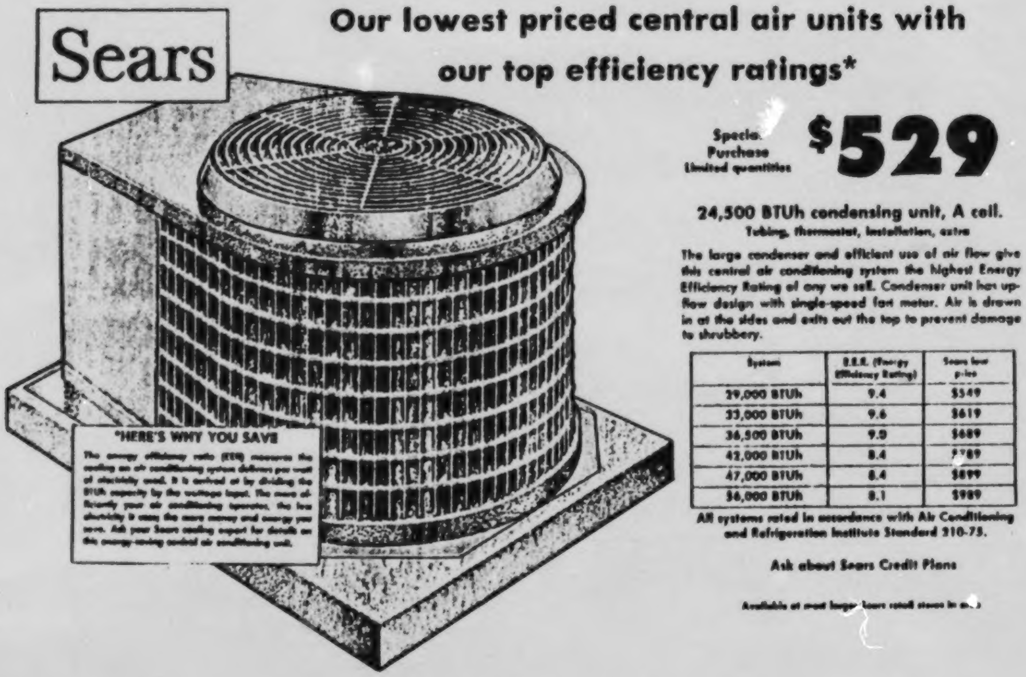

LIQUID FREON AT 200 psi AND 110° F
INDOOR
EXPANSION VALVE
EVAPORATOR COILS
OUTDOOR
COOLED AIR RETURNED TO ROOM 65° F
BOILING FREON AT 70 psi AND 45° F
AMBIENT AIR 90° F
CONDENSOR COILS
HEATED AIR 110° F
WARM AIR FROM ROOM 80° F
FREON GAS AT 200 psi & 140° F
GASEOUS FREON AT 70 psi and 70° F
COMPRESSOR

FIG 6



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The large condenser and efficient use of air flow give this central air conditioning system the highest Energy Efficiency Rating of any we sell. Condenser unit has upflow design with single-speed fan motor. Air is drawn in at the sides and exits out the top to prevent damage to shrubbery.

System	E.E.R. (Energy Efficiency Rating)	Sears low price
29,000 BTUh	9.4	$549
33,000 BTUh	9.6	$619
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47,000 BTUh	8.4	$899
56,000 BTUh	8.1	$989

All systems rated in accordance with Air Conditioning and Refrigeration Institute Standard 210-75.

Ask about Sears Credit Plans

Available at most larger Sears retail stores in area

*HERE'S WHY YOU SAVE

The energy efficiency ratio (EER) measures the cooling an air conditioning system delivers per watt of electricity used. It is arrived at by dividing the BTUh capacity by the wattage input. The more efficiently your air conditioning operates, the less electricity it uses the more money and energy you save. Ask your Sears cooling expert for details on this energy-saving central air conditioning unit.

FIG 7

EER and cost-effectiveness calculations for specific units are shown in Example B.

EXAMPLE B: COMPARING UNITS WITH DIFFERENT EERs

Given: An air-conditioning system with an EER of 10 as compared to a system with an EER of 5.

Find: The amount of money saved, using this system.

Solution: Assume that 30,000 Btu/h are to be removed from the house by either unit.

$$\text{EER} = \frac{\text{Btu/h}}{\text{wattage}} = 10 \text{ (Consider the more efficient system first.)}$$

$$\text{Wattage} = \frac{\text{Btu/hr}}{10} = \frac{30{,}000}{10} = 3000 \text{ W or } 3.0 \text{ kilowatts}$$

If electricity costs 7¢ per kilowatthour, and if the system is run continuously 24 hours per day, the cost for one day's use of this system would be as follows:

$$\text{Cost} = 3.0 \text{ kilowatts} \times 24 \text{ hours} \times \frac{7¢}{\text{kilowatthour}}$$

$$= \$5.04 \text{ per day}$$

$$\text{EER} = \frac{\text{Btu/h}}{\text{wattage}} = 5 \text{ (Consider the less efficient system next.)}$$

$$\text{Wattage} = \frac{30{,}000}{5} = 6000 \text{ watts} = 6 \text{ kilowatts}$$

$$\text{Cost} = 6 \text{ kilowatts} \times 24 \text{ hours} \times \frac{7¢}{\text{kilowatthour}}$$

$$= \$10.08 \text{ day}$$

If the EER is doubled, the efficiency is doubled; and the costs are then cut in half.

SUMMARY

Through mechanical work, the air conditioner produces a temperature difference that results in the cooling of objects or areas. Freon (or similar gas) is compressed by the compressor and is vaporized at the expansion valve, at which point cooling results.

COST AND ENERGY EFFICIENCY OF AN AIR CONDITIONER

MATERIALS

Air conditioning unit or system.
Electrical and thermal specifications for the air conditioning system.

PROCEDURE

1. From an installed air conditioner or a manufacturer's specification sheet, obtain the following published information:

   Power consumption _______ watts.
   Cooling capacity (H) _______ Btu/h.
   Energy efficiency ratio _______ (EER).

   Note: The information on an installed unit is located on a metal plate located on the housing of the outside compressor unit.

2. Calculate the EER of the unit by dividing the cooling capacity in Btu/h by the power consumption:

$$\text{EER} = \frac{\text{Btu/h}}{\text{wattage}}$$

   Compare this value to the published EER rating.

3. From an air conditioning dealer, obtain the following information:

   Installed cost of several air conditioning units with different EERs.
   Average operating cost (utility bills) per year for each unit.

4. Calculate the total cost over 10 years for each unit according to this formula: Total cost = cost of unit + 10 x (annual

operating cost). Plot this data as a function of EER (Figure 8).

5. Which air conditioner (EER) is more economical to own?

6. Repeat step 4, assuming that the cost of electrical power has doubled. Plot the data as a function of EER (Figure 9).

7. Under this condition, which air conditioner is more economical to own?

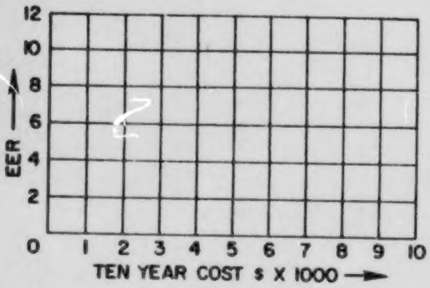

FIG. 8

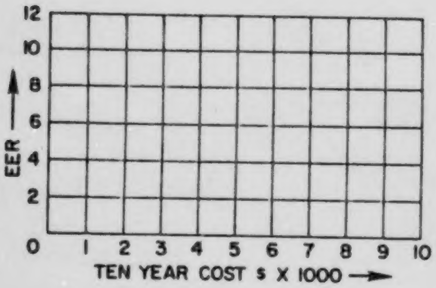

FIG. 9

MEASURING TEMPERATURES OF AN AIR-CONDITIONING SYSTEM

MATERIALS

Contact thermometer (thermocouple).
Air conditioning unit or system.
Electrical and thermal specifications for the air conditioning system.
Graph paper.

PROCEDURE

Note: This procedure should be considered optional (at the discretion of the instructor), since it can only be performed at certain times of the year.

1. Use standard mercury thermometer to measure air temperature at the following locations in or near operating air conditioner:

   Inside a building at return-air register.
   Inside a building at cool-air outlet to a room.
   Ambient temperature of outside air.
   Temperature of outside air as it leaves condenser coils (while fan is blowing).

2. If possible, measure temperature of two copper pipes containing freon at point outside the building where they enter compressor unit. (Use contact thermocouple thermometer.)

   - The cooler pipe is carrying freon entering compressor.
   - The hotter pipe is carrying freon leaving condenser.

3. Record data from steps 1 and 2. Compare these six temperature readings to the typical values shown in Figure 6.

DATA TABLE

Temperatures °F	Warm Air From Room	Cooled Air Returned To Room	Outside Ambient Air	Outside Hot Air From Condenser	Freon Entering Condenser	Freon Leaving Condenser
From Figure 6						
Measured value						

SELF CHECK

1. Define the terms:

   Calorie.
   Kilocalorie.
   Btu.
   Mechanical equivalent of heat.
   EER.
   Btu/h.

2. Determine the EER from a specification plate on a window air conditioner listing the following information:

   18,000 Btu/h.
   220 V.
   2250 watts.

3. Calculate the 24-hour cost of each of these air conditioners with the following specifications: (electricity cost 6¢/kilowatthour)

   12,500 Btu/h, 1559 watts.
   18,000 Btu/h, 2250 watts.
   24,000 Btu/h, 2665 watts.

4. Draw a schematic diagram illustrating the parts of an air-conditioning unit, and describe the temperature and pressure changes that take place within the system.

5. Find the tonnage of an air conditioning unit if the unit can remove 87,000 Btu/hr from a room.

6. How many Btu/in can an air conditioner with an EER of 15 and power rating of 2250 watts remove from a house?

7. Convert 45,000 Btu/h to calories/second.

RATE

APPLICATION MODULES

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* Identifies concept support modules that introduce new physics.

3M2

VELOCITY AT CONSTANT ACCELERATION

Velocity, or speed, is the basic rate-like quantity of the mechanical system. Since linear acceleration is the rate at which speed changes, speed of an object at any later time can be determined if its initial speed and acceleration are known. In this module the student will solve problems involving linear velocity and acceleration. Since the solutions to problems involving variable acceleration involve mathematical techniques beyond the scope of this course, only constant acceleration is used. In the laboratory the student will determine acceleration by measuring the speed of an object at the beginning and at the end of a time interval.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate final speed, given initial speed, acceleration and duration of acceleration.
- Calculate acceleration, given speed at two different times.
- Calculate time required to accelerate an object from one speed to another, at a constant acceleration.
- Given the appropriate equipment, determine experimentally the acceleration of a moving body by measurements of speed and time.

Figure 1 shows the graph of speed versus time for a moving object. At time t_1 the speed was v_1, and at time t_2 it was v_2. Acceleration is the slope of the line, which can be found by Equation 1.

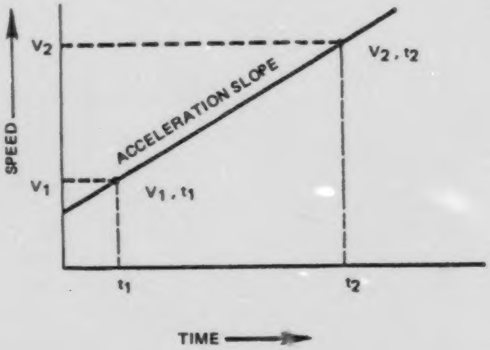


FIG. 1

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

or

$$a = \frac{v_2 - v_1}{t} \qquad (1)$$

where

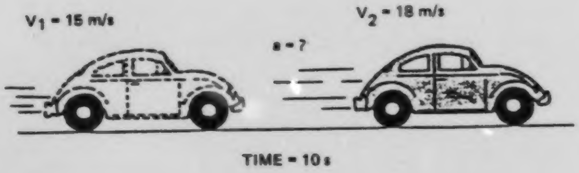
a = acceleration.
v_1 = initial speed (speed before change).
v_2 = final speed (speed after change).
t_1 = time at which speed was v_1.
t_2 = time at which speed was v_2.
t = time elapsed between the motion at v_1 and the motion at v_2.

Example A shows the use of Equation 1 in solving a problem.

EXAMPLE A: ACCELERATION OF AN AUTOMOBILE

Given: An automobile accelerates from 15 m/s to 18 m/s in 10 s.

Find: Its acceleration.



Solution:

$$a = \frac{v_2 - v_1}{t}$$

$$= \frac{(18\ \text{m/s} - 15\ \text{m/s})}{10\ \text{s}}$$

$$= \frac{3\ \text{m/s}}{10\ \text{s}}$$

$$a = 0.3\ \frac{\text{m}}{\text{s}^2}$$

The automobile was accelerated 0.3 m/s every second.

Equation 1 can be rearranged into another useful form. When both sides are multiplied by t, $a = \frac{v_2 - v_1}{t}$ becomes $at = v_2 - v_1$. Next, v_1 can be added to both sides of the equation, and the sides reversed, to produce Equation 2 as follows:

$$at + v_1 = v_2 - v_1 + v_1$$

$$at + v_1 = v_2$$

$$v_2 = v_1 + at \qquad (2)$$

where

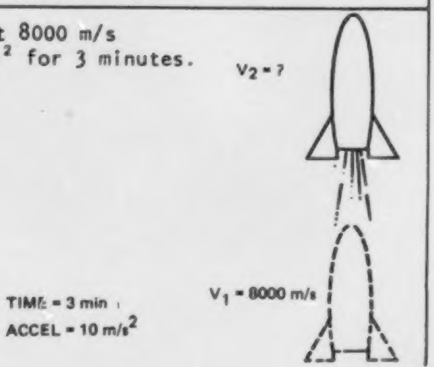
v_2 = final speed.
v_1 = initial speed.
a = acceleration.
t = time.

Example B demonstrates the use of Equation 2 in solving a problem.

EXAMPLE B: FINAL SPEED OF A ROCKET

Given: A rocket traveling at 8000 m/s accelerates at 10 m/s² for 3 minutes.

Find: Its new speed.



Solution:
$$\begin{aligned} v_2 &= v_1 + at \\ &= 8000 \text{ m/s} + (10 \text{ m/s}^2)(180 \text{ s}) \\ &= 8000 \text{ m/s} + 1800 \text{ m/s} \\ v_2 &= 9800 \text{ m/s} \end{aligned}$$

After 3 minutes the speed is 9800 m/s.

Another form for the acceleration formula is easily derived for time in Equation 3 as follows:

$$\begin{aligned} a &= \frac{v_2 - v_1}{t} \\ at &= v_2 - v_1 \\ t &= \frac{v_2 - v_1}{a} \qquad (3) \end{aligned}$$

where

t = time.
v_2 = final speed.
v_1 = initial speed.
a = acceleration.

Example C shows the use of Equation 3 in solving a problem.

EXAMPLE C: TIME REQUIRED TO REACH A GIVEN SPEED

Given: An object is accelerating at 0.5 m/s².

Find: Time required to produce a speed of 27 m/s, beginning from rest.

Solution:
$$\begin{aligned} t &= \frac{v_2 - v_1}{a} \\ &= \frac{27 \text{ m/s} - 0 \text{ m/s}}{0.5 \text{ m/s}^2} \\ &= \frac{27 \text{ m/s}}{0.5 \text{ m/s}^2} \\ t &= 54 \text{ s} \end{aligned}$$

The object has accelerated to 27 m/s in 54 seconds.

GRAVITY

The most common source of constant acceleration is due to the pull of the Earth's gravity. The acceleration of gravity near the surface of the earth is a constant, having a value of 32 ft/s^2 or 9.8 m/s^2. This acceleration is independent of the mass and shape of an object, which indicates that in a vacuum, where there is no air resistance, a feather will fall just as rapidly as a silver dollar. In the presence of air, however, a feather, a leaf, or a sheet of paper will flutter and fall more slowly because its shape and air resistance produce drag forces. The falling motion of heavier objects, such as stones and balls, is not affected significantly by the presence of air.

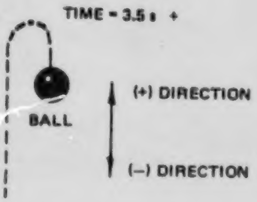
The value for acceleration of gravity decreases for an object very slowly, as it moves further in altitude from the earth's surface. There are also very small local variations at different points on the earth's surface, due to local variations in density of the earth below. The values given above are suitable in all cases dealt with in this module.

Problems in which velocity and acceleration are in different directions must be solved. This type of problem is solved in Example D, in which a ball is thrown vertically upward and is decelerated (or accelerated downwards) by the pull of gravity until it stops its upward motion and begins to fall, being accelerated downward.

EXAMPLE D: SPEED OF A BALL THROWN UPWARD

Given: A ball is thrown vertically upward with an initial speed of 18.6 m/s.

Find: Its speed after 3.5 seconds.



Solution:

$$\begin{aligned} v_2 &= v_1 + at \\ &= 18.6 \text{ m/s} + (-9.8 \text{ m/s}^2)(3.5\text{s}) \\ &= 18.6 \text{ m/s} - 34.3 \text{ m/s} \\ v_2 &= -15.7 \text{ m/s} \end{aligned}$$

Since the final speed is negative, the ball is descending at 15.7 m/s after 3.5 seconds.

Notice that the upward direction was chosen as the positive direction; therefore the downward direction becomes the negative direction. The initial speed was upward (positive), and the acceleration downward (negative). Since the final speed was negative, the ball travels downward. The direction chosen as positive was unimportant because the same sign convention was maintained throughout the solution.

SPECIAL CASES

If in Equation 1, 2 and 3 the initial speed is zero (that is, the body starts from rest), the equations can be shortened, in order that there will be only a single speed to consider. The equations become: $a = \frac{v}{t}$, $v = at$, and $t = \frac{v}{a}$. Memorization of these special cases as separate formulas is unnecessary, since $v_1 = 0$ can be substituted into the original formulas to achieve the same result.

SUMMARY

Velocity, or speed, of an object at any time during its motion can be calculated if its initial speed and acceleration are known. Acceleration refers to the rate at which speed changes, and can be plotted graphically as the slope of velocity versus time. Velocity, however, can be plotted as the slope of distance versus time, or the distance traveled within a specific time.

DETERMINATION OF ACCELERATION

MATERIALS

Two pieces of aluminum angle (or angle iron) slightly over 2 m long.
One-inch diameter steel ball.
Two timers with at least 0.1 second accuracy.

PROCEDURE

In this experiement, a ball will be allowed to roll down an inclined track where it is accelerating, and then over a level track where it continues to move at constant velocity. (See Figure 2.) From data taken during the level portion of its motion, the velocity at the bottom of the incline can be determined. Once this data has been found, the constant acceleration of the ball during the inclined portion of its motion can be determined.

1. On each piece of aluminum angle, mark a distance of one meter from one end.

2. Position the two pieces of angle, as shown in Figure 2. One of the pieces is horizontal and the other rests inside it with its right end (B) one meter from the end of the horizontal piece (C). The other end of second length of angle is raised to form an angle of approximately twenty-five degrees with the horizontal.

3. Place steel ball at one meter mark on elevated angle (point A), as shown in Figure 2.

4. Perform in the following sequence:

   a. Release ball and start Timer One simultaneously.
   b. When ball reaches point B, stop

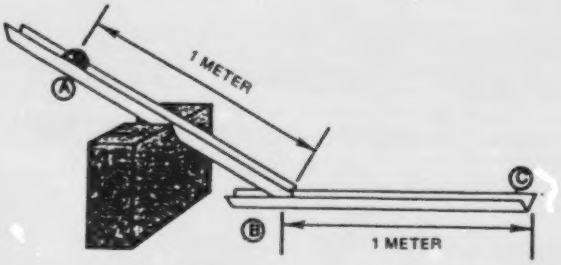


FIG. 2

Timer One and start Timer Two. (Suggestion: Use two people to start and stop timers.)
c. When the ball reaches point C, stop Timer Two.

5. Record times in Data Table 1.

6. Repeat experiment two more times and average times recorded.

7. Perform calculations necessary to complete Data Table 1.

8. Repeat experiment using 2 meter lengths for AB and BC, and increase the angle to about thirty degrees for AB. Record data on Data Table 2.

9. Complete Data Table 2.

10. How does this laboratory procedure confirm the principles in the Discussion?

DATA TABLE 1

DATA

Distance AB: d_1 = ________ m
Distance BC: d_2 = ________ m

		Trial 1	Trial 2	Trial 3	AVERAGE
Time from A to B	t_1				
Time from B to C	t_2				

Initial speed $v_A = 0$ m/s.

CALCULATIONS

To determine speed v_B:

$$v_B = v_C = \frac{d_2}{t_2} = \underline{\hspace{3em}} \text{ m/s}$$

To determine acceleration during descent:

$$a = \frac{v_B - v_A}{t_1} = \underline{\hspace{3em}} \text{ m/s}^2$$

DATA TABLE 2

DATA

Distance AB: $d_1 =$ __________ m.
Distance BC: $d_2 =$ __________ m

		Trial 1	Trial 2	Trial 3	AVERAGE
Time from A to B	t_1				
Time from B to C	t_2				

Initial speed: $v_A = 0$ m/s

CALCULATIONS

To determine speed v_B:

$$v_B = v_C = \frac{d_2}{t_2} = \_\_\_\_\_\_\_\_\_\_ \text{ m/s}$$

To determine acceleration during descent:

$$a = \frac{v_B - v_A}{t_1} = \_\_\_\_\_\_\_\_\_\_ \text{ m/s}^2$$

SELF CHECK

1. An object is dropped from the top of a cliff. What is its speed after 5 seconds (neglect air resistance)?

2. A train accelerates from 40 km/hr to 50 km/hr in 30 seconds. What is its acceleration?

3. With a deceleration of 0.8 m/s^2, how much time is required to stop an object traveling at 16 m/s?

4. A constant acceleration is applied to an object starting from rest. After 8 seconds its speed is 12.6 m/s. How much longer must the same acceleration be applied to increase its speed to 28 m/s?

5. An object is thrown vertically upward with an initial velocity of 6.2 m/s. What is its speed after 1.2 seconds?

6. An object initially traveling to the right at 2.3 m/s was observed 4.8 seconds later traveling to the left at a speed of 12.8 m/s. What is the acceleration (magnitude and direction), assuming it has constant value? (Suggestion: Establish a sign convention for left and right directions.)

3M3

DISPLACEMENT AT CONSTANT ACCELERATION

Everything in the physical world is in motion. In most cases, the velocity, or speed, of a moving object changes as motion continues. This change in velocity is called "accelerated motion." The simplest kind of acceleration is motion in a straight line, in which speed changes at a constant rate. This special kind of motion is called "uniformly-accelerated motion," or motion with constant acceleration. Displacement is the distance through which a body moves.

Acceleration is a "second-order" rate because it is the rate of a rate. More exactly, velocity is the rate at which displacement changes, and acceleration is the rate at which velocity changes. Hence, acceleration is a ratelike quantity of a more complex type than those explained in Concept Module 3-0, "Rate."

In this module, the student will learn that, given any three of the five motion parameters (displacement, initial velocity, final velocity, acceleration and time), the remaining two can be found. In the laboratory, the student will measure initial velocity, distance traveled and time traveled by an object to determine its acceleration.

Explanatory note: "Velocity" is a vector quantity whose magnitude is "speed." Although this module is confined to motion along a straight line, motion may be forward or backward. In certain instances, close attention must be given to this simple choice of directions, which is indicated by (+) and (-) signs attached to the speed. For this reason, the term "velocity" is preferable in most of the following discussion. At other times, the term "speed" will be employed.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given three of the following, determine the remaining two:

  Initial velocity, v_1.
  Final velocity, v_2.
  Constant acceleration, a.
  Elapsed time, t.
  Displacement, s.

- Define average velocity and displacement.
- Given the appropriate equipment, determine acceleration by measuring initial velocity and time required to travel a specific distance.

The definition of average velocity is the net displacement during a time interval divided by the length of that interval. This definition is expressed in Equation 1.

$$\bar{v} = \frac{s}{t} \qquad (1)$$

where
$\bar{v}$ = average velocity; bar over v indicates "average value."
s = net displacement, distance from starting point to final point.
t = time elapsed.

In the case of constant acceleration, average velocity also can be defined as the numerical average of the initial and final velocities, as shown in Equation 2.

$$\bar{v} = \frac{v_1 + v_2}{2} \qquad (2)$$

where
v_1 = initial velocity.
v_2 = final velocity.

Equations 1 and 2 are used to obtain Equation 3, which solves for net displacements at constant acceleration when v_1, v_2 and t are known.

$$s = \frac{(v_1 + v_2)\ t}{2} \qquad (3)$$

The final velocity of an object moving at constant acceleration is given by Equation 4.

$$v_2 = v_1 + at \qquad (4)$$

where
a = acceleration.

Equation 4 is substituted into Equation 3 as follows:

$$\begin{aligned} s &= \frac{(v_1 + v_2)\ t}{2} \\ &= \frac{[v_1 + (v_1 + at)]\ t}{2} \end{aligned}$$

$$s = v_1 t + \frac{1}{2} at^2$$

This substitution yields the formula for displacement at a constant acceleration as given by Equation 5.

$$s = v_1 t + \frac{1}{2} at^2 \qquad (5)$$

Example A shows how Equation 5 can be used to solve a problem.

EXAMPLE A: DISPLACEMENT AT A CONSTANT ACCELERATION

Given: An object with an initial speed of 2 meters per second and an acceleration of 1 meter per second per second.

Find: How far it will travel in 5 seconds.

Solution:

$$\begin{aligned} s &= v_1 t + \frac{1}{2} at^2 \\ &= (2\ \tfrac{m}{s})(5\ s) + \frac{1}{2}(1\ \tfrac{m}{s^2})(5\ s)^2 \\ &= 10\ m + \frac{1}{2}(25\ m) \\ s &= 22.5\ m \end{aligned}$$

The object will travel 22.5 meters in 5 seconds.

There are two terms in Equation 5. The first term, $v_1 t$, is the displacement during time t that would occur if the initial speed v_1 remained constant. The second term, $1/2\ at^2$, is the additional displacement due to speed gained through a constant acceleration a. Both of these terms are shown as dashed lines in Figure 1. Displacement at any time t is the sum of these two terms at that time. The graph of the $v_1 t$ term is a straight line, and that of the $1/2\ at^2$ term is curved (parabolic). The uppermost curve representing the sum, therefore, is also curved (again, parabolic).

In Figure 2, sample calculations are shown for the terms in Equation 5. Each term is calculated at the end of each second over a 5-second span. The body moves with an initial velocity of 2 m/s and a constant acceleration of 1 m/s^2.

Equation 4 solved for t gives $t = (v_2 - v_1)/a$. Substituting this into Equation 3 yields the following equation.

$$s = \frac{(v_2 + v_1)\ t}{2}$$

$$s = \frac{(v_2 + v_1)}{2} \times \frac{(v_2 - v_1)}{a} = t$$

$$s = \frac{v_2^{\,2} - v_1^{\,2}}{2a}$$

$$2\,as = v_2^{\,2} - v_1^{\,2}$$

This useful equation often is expressed as shown in Equation 6 for final velocity.

$$v_2^{\,2} = v_1^{\,2} + 2\,as. \qquad (6)$$

Example B shows the use of Equation 6 in solving a problem.

Example C shows the use of Equations 5 and 6 to solve a problem.

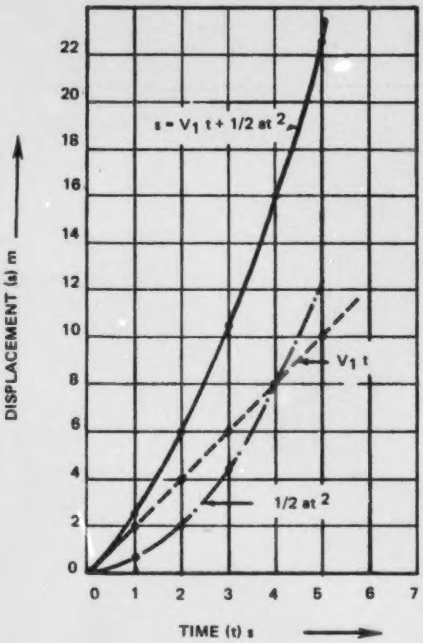


FIG. 1

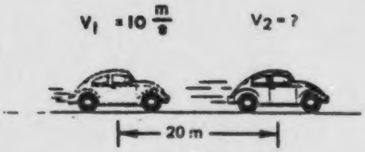
$v_1 = 2$ m/s.
$a = 1$ m/s².

t	$v_1 t$	$\frac{1}{2}at^2$	s
0	0	0	0
1	2	0.5	2.5
2	4	2	6
3	6	4.5	10.5
4	8	8	16
5	10	12.5	22.5

FIG. 2

EXAMPLE B: FINAL SPEED OF AUTOMOBILE

Given: An automobile with an initial speed of 10 m/s experiences an acceleration of 1.1 m/s² for a distance of 20 m.



Find: Its final speed.

Solution:

$$
\begin{aligned}
v_2{}^2 &= v_1{}^2 + 2\ as \\
&= (10\ m/s)^2 + 2\ (1.1\ m/s^2)(20\ m) \\
&= 100\ m^2/s^2 + 44\ m^2/s^2 \\
&= 144\ m^2/s^2 \\
&= \sqrt{144\ m^2/s^2} \\
v_2 &= 12\ m/s
\end{aligned}
$$

The car reaches a final speed of 12 m/s.

EXAMPLE C: ACCELERATION AND SPEED OF AN AUTOMOBILE

Given: An automobile undergoes a constant acceleration from an initial speed of 25 m/s for a period of 10 seconds and covers a distance of 500 m.

Find:
(a) Acceleration of the car.
(b) Final speed.

Solution:
(a) $s = v_1 t + \frac{1}{2} at^2$; Solve for a.

$$
\begin{aligned}
a &= \frac{2s}{t^2} - \frac{2\ v_1}{t} \\
&= \frac{2\ (500\ m)}{(10\ s)^2} - \frac{2\ (25\ m/s)}{10\ s} \\
&= \frac{1000\ m}{100\ s^2} - \frac{50\ m/s}{10\ s} \\
&= 10\ m/s^2 - 5\ m/s^2 \\
a &= 5\ m/s^2
\end{aligned}
$$

(b)

$$
\begin{aligned}
v_2{}^2 &= v_1{}^2 + 2\ as \\
&= (25\ m/s)^2 + 2\ (5\ m/s^2)(500\ m) \\
&= 625\ m^2/s^2 + 5000\ m^2/s^2 \\
v_2{}^2 &= 5625\ m^2/s^2
\end{aligned}
$$

$$v_2 = \sqrt{5625\ m^2/s^2}$$
$$v_2 = 75\ m/s$$

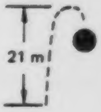
The automobile had a constant acceleration of 5 m/s^2 and attained a final speed of 75 m/s.

Example D shows two approaches to solving a problem involving the acceleration of gravity.

EXAMPLE D: TIME TO REACH MAXIMUM HEIGHT

Given: A ball thrown vertically upward reaches a maximum height of 21 m.

Find: How long it takes to reach that height.



Solution: (a) Known: $s = 21\ m$

$v_2 = 0\ m/s$

$a = -9.8\ m/s^2$ (Upward was chosen as positive direction.)

$$v_2^2 = v_1^2 + 2\,as$$
$$v_1^2 = v_2^2 - 2\,as$$
$$= (0\ m/s)^2 - 2\,(-9.8\ m/s^2)(21\ m)$$
$$v_1^2 = 411.6\ m^2/s^2$$
$$v_1 = 20.29\ m/s$$

Ball is thrown upward at an initial velocity of 20.29 m/s.

$$v_2 = v_1 + at$$
$$t = \frac{v_2 - v_1}{a}$$
$$= \frac{20.29\ m/s - 0\ m/s}{9.8\ m/s^2}$$
$$t = 2.07\ s$$

It takes 2.07 seconds to reach a height of 21 meters.

(b) Assume that the ball requires the same time to travel the 21 meters in a downward direction as it does going upward. Therefore, starting at the top, $v_1 = 0\ m/s$.

$$s = v_1 t + \frac{1}{2} at^2$$

$$s = \frac{1}{2} at^2$$

$$\frac{2s}{a} = t^2$$

$$t = \sqrt{\frac{2s}{a}}$$

$$= \sqrt{\frac{2\ (21\ m)}{9.8\ m/s^2}}$$

$$= \sqrt{\frac{42}{9.8}\ s^2}$$

$$= \sqrt{4.29\ s^2}$$

$$t = 2.07\ s$$

In this case, the downward direction is chosen as positive (+), so $a = +9.8\ m/s^2$, the same as in solution a.

SUMMARY

Displacement, the distance through which a body moves, is simple to calculate when the velocity of the moving body changes at a constant rate (at a constant acceleration). Displacement often must be determined for moving objects at a given speed and time. If any three of the following motion parameters are known, the remaining two can be found: displacement, initial velocity, final velocity, acceleration and time.

DETERMINING ACCELERATION AND DISPLACEMENT

MATERIALS

Two pieces of aluminum angle slightly over 2 m long or other suitable track on which a steel ball can roll.
One-inch diameter steel ball.
Two timers with an accuracy of one tenth second or better.

PROCEDURE

Acceleration of a ball rolling down an incline will be determined experimentally and then used to determine the distance the ball will roll down the incline in a given time.

1. On one piece of track, mark a distance of one meter from one end. On longer piece, mark a distance of 2 m from one end. Make another mark approximately 20 cm from 2-m mark.

2. Position two pieces of track as shown in Figure 3. Horizontal piece should be elevated so that slanted piece will make an angle of approximately 10 degress with horizontal.

3. Place ball at point B on end of slanted piece of track. Release ball; let it roll down incline. Measure and record in Data Table amount of time required for ball to travel 2 m. Repeat this three times; record average time in Data Table.

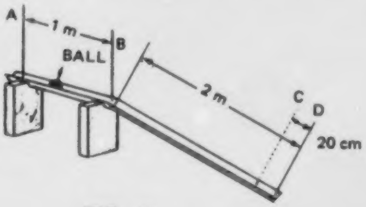


FIG. 3

4. Calculate ball acceleration.

5. Place steel ball at left end of horizontal piece; roll it toward other end. (Be careful not to cause any acceleration after point A.)

6. Perform following steps as ball rolls:
    When ball passes point A, start timer one.
    When ball passes point B, stop timer one; start timer two.
    When ball passes point C, stop timer two.

7. Record results in Data Table.
8. Perform all calculations necessary to complete that section of Data Table.
9. Measure distance BC; record in Data Table.
10. How does this laboratory procedure confirm the explanation of displacement in the Discussion?

DATA TABLE

Acceleration Calculation

Distance traveled: $s = 2\text{m}$
Initial velocity: $v_1 = 0$ m/s

Time required: $t =$ _______ s

Acceleration: $s = v_1 t + \frac{1}{2} a t^2$ $(v_1 t = 0)$

$a = \frac{2\ s}{t^2}$

$= 2 \times 2$ m ÷ (_______ s)^2

$=$ _______ m/s^2

Displacement Calculation

Distance AB: $d_1 =$ _______ m

Time for A to B: $t_1 =$ _______ s

Time for B to C: $t_2 =$ _______ s

Velocity at B: $v_1 = \frac{d_1}{t_1}$

$=$ _______ m ÷ _______ s $=$ _______ m/s

Distance traveled (BC): $s = v_1 t_2 + \frac{1}{2} a t_2^2$

$=$ _______ m/s × _______ s $+ \frac{1}{2} \times$ _______ m/s^2 × (_______ s)^2

$=$ _______ m

Distance BC (measured): $d_2 =$ _______ m

SELF CHECK

1. A ball is dropped from a window. How far will it fall in 3 seconds?

2. A rock is dropped over a cliff. What is its velocity after it has fallen 5.7 m?

3. An arrow was shot vertically upward. As it fell back to the ground, it had a velocity of 8.6 m/s at a height of 5.1 m.

   What was the initial velocity of the arrow?
   How high did it go?
   How much time was required for the arrow to fall from its maximum height to the ground?

4. A car traveling at 30 m/s stops with a constant deceleration applied for 10 seconds. How far did it travel in that 10 seconds?

5. A rock is thrown vertically upwards. If it takes the rock a total of 5 seconds to fall back to the earth, what is the maximum height the rock reaches?

3M5

ROTATIONAL MOTION

The study of motion is incomplete without consideration of rotational motion, since the most general type of motion is a combination of translation and rotation. Everything on the earth, whether moving or standing still with respect to the earth, undergoes rotation because the earth rotates on its axis and revolves about the sun. Since the discovery of the wheel, man has been concerned with the problem of rotational motion from a practical standpoint.

In this module, purely-rotational motion will be analyzed. The equations of motion governing rotation related angular displacement, angular velocity, and angular acceleration are analogous to those which describe rectilinear, or translational, motion. In the laboratory, the student will use the equations and principles of rotation to determine the angular velocity and angular acceleration of a phonograph.

OBJECTIVES

Upon completion of this module, the student should be able to:

- State and define the angular analogue of each of the following:

  Initial velocity, v_1.
  Final velocity, v_2.
  Constant acceleration, a.
  Elapsed time, t.
  Displacement, s.
  $v_2 = v_1 + at$.
  $s = v_1 t + \frac{1}{2} at^2$.
  $v_2^2 = v_1^2 + 2\,as$.

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- Given linear variables s and v_1 and the radius of rotation r, calculate angular counterparts θ and ω.

- Given three of the following, determine the remaining two:

  Initial angular velocity, ω_1.
  Final angular velocity, ω_2.
  Constant angular acceleration, α.
  Elapsed time, t.
  Angular displacement, θ.

- Given the appropriate equipment, determine the angular velocity and acceleration of a phonograph.

DISCUSSION

Figure 1 represents a rigid body, of arbitrary shape, that rotates about a fixed axis perpendicular to the plane of the figure at point Q. The position of the rotating body at any instant of time t is specified by the angle θ made by its radius vector QR with respect to a reference axis QX in the diagram. The coordinates r and θ are analogous to the rectangular coordinates x and y. The angle θ is defined conveniently in radians by Equation 1.

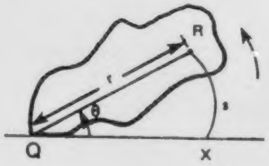


FIG. 1

$$\theta = \frac{s}{r} \qquad (1)$$

where

θ = any arbitrary angle subtended by an arc of length s.
s = arc length on circumference of circle of radius r.
r = radius.

As shown in Figure 2, the radian is a unit that has no physical dimensions, since it is the ratio of two lengths. For a complete revolution s = 2πr, and θ = 2π; therefore, there are 2π radians in a circle, or 2π radians = 360°.

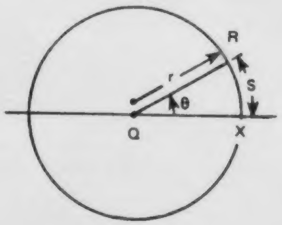


FIG. 2

If the rotating body requires time t to move from point X to point R, the arc length s is the linear displacement in the time interval t. Linear velocity is stated by Equation 2.

$$v = \frac{s}{t} \qquad (2)$$

where

v = linear velocity.
s = arc length — in this case, linear displacement.
t = elapsed time for displacement s.

Equation 1 relates the linear displacement to the angle θ, which is the angle through which the body rotates in the time interval t, or the angular displacement:

$$\theta = \frac{s}{r} \qquad (1)$$

$$s = r\theta \qquad \text{(Both sides multiplied by } r\text{)}$$

$$v = \frac{r\theta}{t} \qquad \text{(}r\theta \text{ substituted for } s \text{ in Equation 2)}$$

The quantity $\frac{\theta}{t}$ is the angular displacement θ divided by the elapsed time t, and is defined as the "angular velocity." The relationship between angular velocity and linear velocity is shown by Equation 3, where ω is substituted for θ/t.

$$v = r\omega \qquad (3)$$

where

v = linear velocity.

ω = angular velocity.

In this equation, angular velocity must be expressed in radians divided by time units, usually seconds. A more common unit for angular velocity is revolutions per minute, abbreviated r/min or rpm. The conversion from rpm to rad/s is —

$$1 \text{ r/min} = 2\pi \text{ rad}/60 \text{ s},$$

since one revolution is one trip around a circle or 2π radians, and since one minute is 60 seconds. Example A illustrates the use of Equations 1 and 3 in analyzing the rotation of automobile tires.

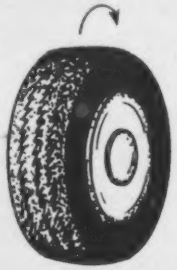
EXAMPLE A: ROTATION OF AUTOMOBILE TIRES

Given: An automobile with tires of 0.35 m radius moves at a speed of 18.0 m/s and travels a distance of 1.6 km. The rim speed of the tire must equal the translational speed of the car — in this case, 18.0 m/s.

Find:

(a) The angular velocity of the tires in rad/s and in r/min.

(b) The number of revolutions made by the tires.



Solution:

(a) The angular velocity in rad/s is found from Equation 3:

$$\omega = \frac{v}{r} = \frac{\text{rim speed of tire}}{\text{radius of tire}}$$

$$= (18.0 \text{ m/s})/(0.35 \text{ m})$$

$$\omega = 51.4 \text{ rad/s}$$

This result is converted to rpm (r/min):

$$\omega = (51.4)(60/2\pi)$$

$$\omega = 490.8 \text{ r/min}$$

(b) The angular displacement θ is found from Equation 1:

$$\theta = \frac{s}{r}$$

$$= (1.6 \times 10^3 \text{ m})/(0.35 \text{ m})$$

$$\theta = 4.57 \times 10^3 \text{ rad}$$

There are 2π radians in one revolution. The number of revolutions made in 4.57×10^3 radians is then $4.57 \times 10^3/2\pi$, or about 727.

ROTATION AT CONSTANT ANGULAR ACCELERATION

If the angular velocity is not constant, but changes from initial value to final value during a time interval, the rotating body undergoes angular acceleration. Angular acceleration is defined as the change in the angular velocity divided by the elapsed time, as indicated by Equation 4.

$$\alpha = \frac{(\omega_2 - \omega_1)}{t} \qquad (4)$$

where

α = angular acceleration.

ω_1 = initial angular velocity.

ω_2 = final angular velocity after time interval t.

Equation 4 can be rearranged to solve for final angular velocity as shown by the following process that results in Equation 5:

$$\alpha t = \omega_2 - \omega_1 \quad \text{(Both sides multiplied by t)} \qquad (4)$$

$$\omega_1 + \alpha t = \omega_2 - \omega_1 + \omega_1 \quad (\omega_1 \text{ added to both sides})$$

$$\omega_2 = \omega_1 + \alpha t \qquad (5)$$

Equation 5 is analogous to the relation for the final velocity for pure translation at constant acceleration: $v_2 = v_1 + at$.

A rotating body undergoes constant angular acceleration; for example, if the direction of rotation is changed. If the direction of rotation is initially counterclockwise, and the magnitude is initially ω_1, and the final angular velocity is ω_1 clockwise, then the angular acceleration is nonzero.

$$\alpha = \frac{(\omega_2 - \omega_1)}{t}$$

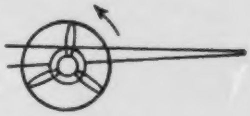
$$\alpha = \frac{(-\omega_1 - \omega_1)}{t} \quad (\text{Since } \omega_2 = -\omega_1)$$

$$\alpha = \frac{-2\omega_1}{t}$$

Of course, the clockwise direction can be chosen as positive, and counterclockwise is then negative. Angular acceleration has units of radians per second per second, or rad/s^2. Example B demonstrates the use of Equation 5 in solving a problem.

EXAMPLE B: ANGULAR ACCELERATION OF AN AIRPLANE PROPELLER

Given: An airplane propeller initially at rest accelerates at a constant angular acceleration of 420.0 rad/s^2 for 0.5 seconds.



Find: Its final angular velocity.

Solution: The initial angular velocity ω_1 is zero. From Equation 5 —

$$\omega_2 = \omega_1 + \alpha t$$

$$= 0 + (420.0 \text{ rad/s}^2)(0.5 \text{ s})$$

$$\omega_2 = 210.0 \text{ rad/s}$$

or in r/min —

$$\omega_2 = 210.0 \times 60/2\pi$$

$$\omega_2 = 2005 \text{ r/min}$$

An expression relating the angular displacement θ to the angular velocities ω_1 and ω_2 can be obtained from Equation 6 (the corresponding equation borrowed from rectilinear motion).

$$s = \frac{1}{2}(v_1 + v_2)t \qquad (6)$$

where

s = arc length (as in Figure 2).
v_1 = initial linear velocity.
v_2 = final linear velocity.

Angular displacement is found as follows from Equation 6:

$$s/r = (v_1 + v_2)t/2r \quad \text{(Both sides divided by r)}$$

$$\theta = \frac{1}{2}(v_1/r + v_2/r)t \quad (\theta \text{ substituted for } s/r)$$

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t \qquad (\text{Since } \omega_1 = v_1/r \text{ and } \omega_2 = v_2/r.) \quad (7)$$

The final angular velocity ω_2 can be eliminated from Equation 7 by use of Equation 5:

$$\theta = \frac{1}{2}(\omega_1 + \omega_1 + \alpha t)t \quad (\omega_1 + \alpha t \text{ substituted for } \omega_2.)$$

$$\theta = \frac{1}{2}(2\omega_1 + \alpha t)t$$

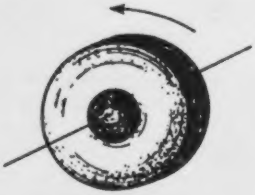
$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2 \qquad \text{(Multiplied by t/2 inside)} \qquad (8)$$

Equation 8 is the counterpart in rotation to the expression $s = v_1 t + 1/2\ at^2$ for linear displacement. Its use is demonstrated in Example C.

EXAMPLE C: ROTATION OF A GRINDSTONE

Given: The angular acceleration of a grindstone is 2.5 rad/s^2. The grindstone starts from rest and attains final velocity after 4.0 seconds.

Find: Its (a) angular displacement and its (b) final angular velocity after 4.0 seconds.



Solution: (a) Equation 8 is used to find θ; the initial angular velocity ω_1 is zero.

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$= 0 + \frac{1}{2}(2.5 \text{ rad/s}^2)(4.0 \text{ s})^2$$

$$\theta = 20.0 \text{ radians}$$

(b) The final velocity is found from Equation 7:

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t$$

$$20.0 \text{ rad} = \frac{1}{2}(0 + \omega_2)(4.0 \text{ s})$$

$$\omega_2 = 2\ (20 \text{ rad})/(4.0 \text{ s})$$

$$\omega_2 = 10.0 \text{ rad/s}$$

This result can also be found by using Equation 5.

Sometimes, the necessity arises to calculate ω_2 from ω_1, α, and θ without knowledge of the elapsed time t. This relation is derived as follows to result in Equation 9.

$$\omega_2 = \omega_1 + \alpha t \quad \text{(Equation 5)}$$

$$t = (\omega_2 - \omega_1)/\alpha \quad \text{(Solving Equation 5 for t)}$$

$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t \quad \text{(Equation 7)}$$

$$\theta = \frac{1}{2}(\omega_2 + \omega_1)(\omega_2 - \omega_1)/\alpha \quad \text{(Substituting } \omega_2\text{-}\omega_1/\alpha \text{ for t in Equation 7)}$$

$$\theta = \frac{1}{2}(\omega_2{}^2 - \omega_1{}^2)/\alpha \quad \text{(Multiplying terms in parenthesis)}$$

$$2\alpha\theta = \omega_2{}^2 - \omega_1{}^2 \quad \text{(Multiplying both sides by } 2\alpha\text{)}$$

$$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta \qquad (9)$$

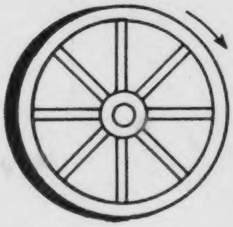
Equation 9 also has an analogue in rectilinear motion: $v_2^2 = v_1^2 + 2as$. Its use is illustrated in Example D.

EXAMPLE D: ROTATION OF A FLYWHEEL

Given: A wheel initially rotating at 4 rev/s decelerates at a rate of 2 rev/s^2.

Find: The number of revolutions it makes before stopping.

Solution: In this problem, the units can be left in rev/s without converting them to rad/s. The final angular velocity ω_2 is zero.



$$\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$$

$$0 = (4\text{ rev/s})^2 + 2(-2\text{ rev/s}^2)\theta$$
$$0 = 16\text{ rev}^2/\text{s}^2 - (4\text{ rev/s}^2)\theta$$
$$\theta = (16\text{ rev}^2/\text{s}^2)/(4\text{ rev/s}^2)$$
$$\theta = 4\text{ rev}$$

The parallels between translation and rotation are summarized in Table 1.

TABLE 1. ANALOGY BETWEEN TRANSLATION AND ROTATION

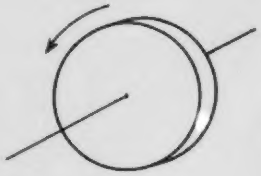
Concept	Translation	Rotation
Displacement	s	θ
Velocity	$v = \frac{s}{t}$	$\omega = \frac{\theta}{t}$
Acceleration	$a = \frac{(v_2 - v_1)}{t}$	$\alpha = \frac{(\omega_2 - \omega_1)}{t}$
Constant Acceleration	$v_2 = v_1 + at$ $s = v_1 t + \frac{1}{2} at^2$ $v_2^2 = v_1^2 + 2as$	$\omega_2 = \omega_1 + \alpha t$ $\theta = \omega_1 t + \frac{1}{2} \alpha t^2$ $\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$

Example E shows how two of the equations describing rotation are solved for the problem of a flywheel.

EXAMPLE E: ROTATION OF A FLYWHEEL

Given: The angular acceleration of a flywheel is 2 rad/s^2. It rotates through 50 radians in a 5-second time interval sometime after it started from rest.

Find: How long the flywheel had been turning just before the beginning of the 5-second interval.



Solution: The angular velocity at the beginning of the 5-second interval is found as follows:

$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$

$$50 \text{ rad} = \omega_1(5 \text{ s}) + \frac{1}{2}(2 \text{ rad/s}^2)(5 \text{ s})^2$$

$$\omega_1 = (50 \text{ rad} - 25 \text{ rad})/5 \text{ s}$$

$$\omega_1 = 5 \text{ rad/s}$$

This value is the angular velocity at the end of the time interval between the beginning of rotation and the beginning of the 5-second interval. The initial angular velocity ω_1 at time $t = 0$ is zero since the flywheel started from rest.

$$\omega_2 = \omega_1 + \alpha t$$

$$5 \text{ rad/s} = 0 + (2 \text{ rad/s}^2)t$$

$$t = (5 \text{ rad/s})/(2 \text{ rad/s}^2)$$

$$t = 2.5 \text{ s}$$

Thus, the flywheel had been spinning (from rest) for 2.5 s before it was found to turn through 50 radians in the next five seconds.

SUMMARY

"Rotary motion" is the motion of a body turning about an axis. Rotary motion occurs in a turning wheel, the rotating crankshaft of an automobile engine, and a pulley wheel attached to the spinning shaft of an electric motor. Notice the difference between circular and rotary motion. In circular motion, the object as a whole travels along a circular path. In rotary motion, the object simply spins. While a bicycle wheel spinning on a stationary axle undergoes rotary motion, a part of the wheel, say the valve stem of the tire, experiences circular motion.

For rotary motion to be uniform, the object must spin about a fixed axis at a definite rate. The movements of the hands of an electric clock are examples of uniform rotary motion. If either the direction of the axis or the rate of spin varies, the rotary motion is variable. The movements of automobile wheels as the car is driven at different speeds, and the movements of a spinning top as it falls down, are examples of variable rotary motion.

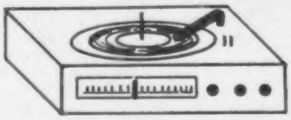
The rate of rotary motion is called "angular velocity." For an object in uniform rotary motion, angular velocity equals the angle through which the object turns divided by the time required for this rotation. Angular velocity may be measured in revolutions per minute. However, it includes not only the rate of rotation, but the direction of the axis of rotation. Thus, angular velocity is a vector quantity, represented by a vector parallel to the axis of rotation. The length of the vector indicates the magnitude of the angular velocity.

"Angular acceleration" is the change in angular velocity divided by the elapsed time.

ANGULAR VELOCITY AND ANGULAR ACCELERATION OF A TURNTABLE

MATERIALS

Phonograph turntable.
Stopwatch.
Weights.
Chalk.



PROCEDURE

1. Make a small chalk mark on the rim of the turntable. Set the speed at 45 rpm. Measure the time in seconds required for the chalk mark to pass a reference point of your choosing through ten revolutions. Record this time in the Data Table.

2. Convert the time to minutes by dividing by 60, and record in Data Table. Calculate angular velocity in rpm by dividing angular displacement $\theta = 10$ rev by time in minutes. Convert to rad/s, and record both values. How close is your value to 45 rpm?

3. Repeat Steps 1 and 2 for a speed of 33-1/3 rpm.

4. Reset speed to 45 rpm, and start turntable. Start stopwatch, and turn turntable off simultaneously. Stop watch when turntable comes to a complete stop. Record time in seconds in Data Table.

5. Convert time in seconds to minutes by dividing by 60. Record time in minutes in Data Table. Calculate angular deceleration α of turntable in r/min^2, using measured value of ω_1 in r/min from Step 2. (Remember that the final angular velocity ω_2 is zero.) Calculate α in rad/s^2, using ω_1 in rad/s and time in seconds. Is α positive or negative?

6. Repeat Steps 4 and 5, after placing a weight of 100g on turntable.

7. Explain how this experiment demonstrates the principles explained in the Discussion section.

DATA TABLE

Time required for 10 revolutions at 45 rpm = ______________ s

Time required for 10 revolutions at 33-1/3 rpm = ____________ s

Time required for turntable to decelerate = ___________ s

Time required for weighted turntable to decelerate = ________ s

RESULTS

Time required for 10 revolutions at 45 rpm = ___________ min

Angular velocity ω = ___________ rpm

Angular velocity ω = ___________ rad/s

Time required for 10 revolutions at 33-1/3 rpm = __________ min

Angular velocity ω = __________ rpm

Angular velocity ω = __________ rad/s

Time required for turntable to decelerate = _____________ min

Angular deceleration α = _______________ r/min^2

Angular deceleration α = _______________ rad/s^2

Time required for weighted turntable to decelerate = _______ min

Angular deceleration α = _______________ r/min^2

Angular deceleration α = _______________ rad/s^2

SELF CHECK

1. State and define the rotational analogue to each of the following:

   Initial velocity, v_1.
   Final velocity, v_2.
   Constant acceleration, a.
   Elapsed time, t.

Displacement, s.

$$v_2 = v_1 + at.$$

$$s = v_1 t + \frac{1}{2} at^2.$$

$$v_2^2 = v_1^2 + 2\,as.$$

2. Find the linear speed of a point on a record at the needle position (a) at the beginning of the recording, where the radius is 15 cm, and (b) at the end of the recording, where the radius is 7.4 cm, for a phonograph speed of 78 rpm.

3. An automobile has wheels of radius 38 cm and travels at 25 m/s. Find (a) the angular speed of the wheels about the axle; (b) the angular deceleration if the wheels are brought to a stop after 30 turns; (c) the distance traveled during braking. (Hint: the rim speed of the tire is equal to the translational speed of the automobile.)

4. The angular speed of the flywheel of a steam engine is 160 r/min. The wheel comes to rest in 25 hours when the steam is turned off. Find the number of revolutions made by the wheel before it comes to rest.

5. Using the equation (4) in the module and its rotational analogue, derive the relationship between rotational and translational acceleration.

6. If the angular acceleration of a flywheel is 5 rad/s^2 and the intial angular velocity of the flywheel is 10 rad/s, what is the time required for the flywheel to make 10 revolutions?

3M6

RADIAL ACCELERATION AND CENTRIPETAL FORCE

When rotational motion was studied previously, the concepts of angular displacement, angular velocity, and angular acceleration were used. In order to advance the study of angular motion, the concepts of radial (centripetal) acceleration, contripetal force, and centrifugal force are discussed here. These concepts will be used in studying the causes and effects of circular motion. The operation of governors and centrifugal pumps which depend upon centripetal acceleration and centripetal forces also will be examined. In the laboratory, the student will determine the centripetal acceleration and centripetal force related to a given rotating object.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate the resulting angular displacement and angular velocity for a rotating body at a specified time, given its initial angular velocity and angular acceleration.
- Define "tangential velocity" of an object in circular motion.
- Sketch on a diagram the radial (centripetal) acceleration and centripetal force acting on a mass in circular motion.
- Calculate the tangential velocity of an object in circular motion, given its lap time and the radius of its path.
- Calculate the radial (centripetal) acceleration of an object undergoing circular motion, given the tangential velocity and radius of the circle.
- Calculate the centripetal force on an object, given its centripetal (radial) acceleration and its mass.

- Describe, using a diagram, how the following devices work:

  A mechanical governor.
  A centrifuge.
  A centrifugal pump.

- Given the appropriate equipment, determine the centripetal acceleration and centripetal force of a rotating object.

DISCUSSION

The main variables introduced in studying angular motion are angular displacement θ, angular velocity ω, and angular acceleration α. Constant, or average, angular velocity was defined as "the change in angular displacement θ, divided by the time t, during which the change takes place." Constant, or average, angular acceleration was defined as "the change in angular velocity ω, divided by the time t, during which the change takes place."

Angular terms and equations are given in Table 1, along with the analogous linear terms and equations.

TABLE 1: ANGULAR TERMS & EQUATIONS RELATED TO LINEAR ONES

Angular Term or Equation		Linear Term or Equation
θ	←—— DISPLACEMENT ——→	x
ω	←—— SPEED ——→	v
α	←—— ACCELERATION ——→	a
$\omega = \omega_o + \alpha t$		$v = v_o + at$
$\omega^2 = \omega_o^2 + 2\alpha\theta$		$v^2 = v_o^2 + 2ax$
$\theta = \frac{w_o t + \alpha t^2}{2}$		$x = \frac{v_o t + at^2}{2}$

Some of the angular equations in Table 1 are employed to solve problems in Examples A and B.

EXAMPLE A: CONSTANT ANGULAR VELOCITY

Given: A flywheel rotates at a constant angular velocity of 5 rev/s for 5 s.

Find: The angular displacement in rev and degrees.

Solution: Since the angular velocity is constant, the angular acceleration is zero.

$$\theta = \frac{\omega_o t + \alpha t^2}{2}$$

$$= (5 \text{ rev/s})(5 \text{ s}) + (0)$$

$$\theta = 25 \text{ rev}$$

Since there are 360 degress in one rev —

$$\theta = (25 \text{ rev})(360 \text{ deg/rev})$$

$$\theta = 9000 \text{ deg}$$

EXAMPLE B: ANGULAR ACCELERATION

Given: A rotating shaft starts from rest with a constant angular acceleration of 20 rev/s^2.

Find: The angular velocity and angular displacement after 10 s.

Solution: The angular velocity is given by —

$$\omega = \omega_o + \alpha t$$

$$= (0) + (20 \text{ rev/s}^2)(10 \text{ s})$$

$$\omega = 200 \text{ rev/s}$$

The angular displacement is given by —

$$\theta = \frac{\omega_o t + \alpha t^2}{2}$$

$$= \frac{(0) + (20 \text{ rev/s}^2)(10 \text{ s})^2}{2}$$

$$\theta = 1000 \text{ rev}$$

Figure 1 depicts (1) a particle traveling in a circular path which has rotated through an angle θ, and (2) the corresponding tangential displacement s. A simple relation between θ, s, and the radius r is given by Equation 1.

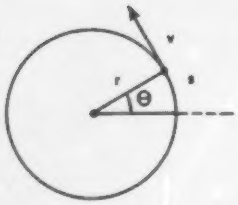


FIG. 1

$$s = r\theta \qquad (1)$$

where

s = tangential displacement.
r = radius.
θ = angle of circular path in radians.

Recall that the angle corresponding to a complete circle is defined as 2π radians. Then, according to Equation 1, the path length, or circumference, is indicated by Equation 2.

$$c = 2\pi r$$

where

c = circumference.

"Tangential velocity" is defined as "the tangential displacement divided by time," and is given by Equation 2.

$$v = \frac{s}{t} \tag{2}$$

where

v = tangential velocity.
s = tangential displacement.
t = time.

From Equation 1, the tangential velocity of Equation 2 takes the form in Equation 3.

$$\begin{aligned} v &= \frac{(r\theta)}{t} \\ &= r\left(\frac{\theta}{t}\right) \\ v &= r\omega, \text{ since } \omega = \frac{\theta}{t} \end{aligned} \tag{3}$$

where

ω = angular velocity.

This velocity is tangent to the circle, as shown in Figure 1.

If a particle is traveling in a circular path and has zero angular acceleration, then its tangential velocity will have the same magnitude at all points along its path; however, the direction of this tangential velocity will be changing continually.

Figure 2a shows a particle of mass m traveling in a circular path on the end of a string. If the string breaks, as in Figure 2b, the particle will follow a straight-line path, tangent to the circle and in the plane of the circle, with a speed given by Equation 2.

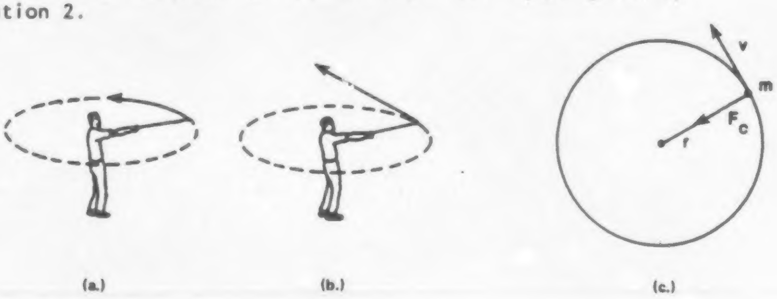


FIG. 2

Recall from Newton's first law that a particle will travel in a straight line, at constant speed, unless a force acts upon it. Because the mass in Figure 2a is not traveling in a straight line, it must have a force acting upon it in order for its motion to change. In the case of a circular path, the mass can be traveling at a constant tangential speed. Thus, the magnitude of the vector velocity does not change; rather, its direction changes.

Newton's second law states that the net force acting upon an object is equal to the product of its mass times its acceleration.

$$F = ma \tag{4}$$

where

F = force acting upon an object.
m = mass.
a = acceleration.

Figure 2c shows the force that must be acting upon the mass to produce a circular path. (The force must be constant in magnitude and always directed toward the center.) Since the acceleration must be in the same direction as the force, it also acts toward the center of the circle. This force, necessary to produce circular motion, is called the "centripetal force." ("Centripetal" means "center seeking.") The corresponding acceleration is called the "centripetal," or "radial" acceleration.

In Figure 2a, the force and acceleration act along the string in order to keep the mass traveling in a circular path. If the string breaks as in Figure 2b, the force suddenly becomes zero; and according to Newton's first law, the mass must move off tangentially in a straight line.

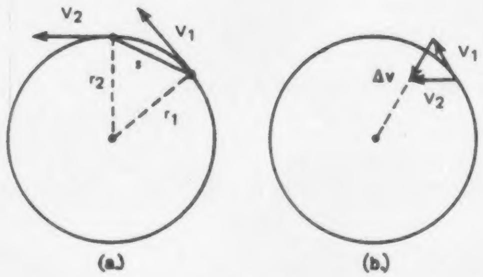
Newton's second law is now written for centripetal force as Equation 5.

$$F_c = ma_c \tag{5}$$

where

F_c = centripetal force.
a_c = centripetal acceleration.

An equation for centripetal acceleration in Equation 5 can be found by a consideration of Figure 3. Figure 3a illustrates a mass in circular motion at two different times, separated by a distance s and having corresponding tangential velocities



(a)

(b)

FIG. 3

v_1 and v_2. Figure 3b shows the two tangential velocities added together as vectors to result in the change in velocity Δv, which is directed toward the center. The triangles (r,s,r) and (v_1,Δv, v_2) are similar, as indicated by Equation 6.

$$\frac{s}{r} = \frac{\Delta v}{v_1} \tag{6}$$

The magnitudes of the velocities v_1 and v_2 are equal ($v_1 = v_2 = v$). Δv is solved for by Equation 7.

$$\Delta v = \frac{vs}{r} \tag{7}$$

The acceleration a_c is the change in velocity divided by the time, or $a_c = \Delta v/t$; therefore, Equation 7 can be used to obtain $a_c = vs/rt$ or $a_c = (v/r)(s/t)$.

By use of the relationship $v = s/t$, centripetal acceleration can be written in the form of Equation 8, which gives the simple relation between the tangential velocity and the centripetal acceleration.

$$a_c = \left(\frac{v}{r}\right) v$$

$$a_c = \left(\frac{v^2}{r}\right) \tag{8}$$

Centripetal force can now be written as in Equation 9.

$$F_c = ma_c = \frac{mv^2}{r} \tag{9}$$

Recall that v is the tangential velocity as expressed in Equation 3. The centripetal acceleration given by Equation 8 also is called the "radial acceleration."

EXAMPLE C: CENTRIPETAL ACCELERATION AND FORCE

Given: A 3000-lb car travels at constant speed on a circular track having a radius of 200 ft. The car requires 15 s to complete one lap.

Find: Find (a) the tangential velocity, the (b) centripetal acceleration and the (c) centripetal force.

Solution: (a) The tangential velocity can be found by use of these equations:

$$v = \frac{2\pi r}{t}$$

$$= \frac{2\pi\ (200\ \text{ft})}{15\ \text{s}}$$

$$v = 83.8\ \text{ft/s}$$

(b) Equation 8 gives the centripetal acceleration:

$$a_c = \frac{v^2}{r}$$

$$= \frac{83.8\ \text{ft/s}^2}{200\ \text{ft}}$$

$$a_c = 35.1\ \text{ft/s}^2,$$

directed toward the center of the circular path.

(c) The centripetal force is given by Equation 9:

$$F_c = ma_c = (w/g)\ a_c$$

$$= \left(\frac{3000\ \text{lb}}{32\ \text{ft/s}^2}\right)(35.1\ \text{ft/s}^2)$$

$$F_c = 3291\ \text{lb},$$

directed toward the center of the circular path.

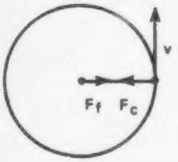


FIG. 4

Consider again the example of a mass traveling in a circular path on the end of a string, as illustrated in Figure 4. The centripetal force F_c acts inward on the mass toward the center of the circle. Newton's Third Law — "For every action there is an equal and opposite reaction" — indicates that there is an outward acting force F_f, called the "centrifugal force," which acts outward upon the center support. These two forces, centripetal (inward) and centrifugal (outward), are equal in magnitude, opposite in direction, and act upon different objects. If the string were being held by a hand, the centrifugal force would be acting outward on the hand toward the mass.

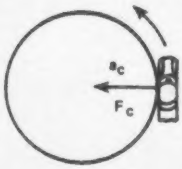


FIG. 5

Consider a car traveling in a circular path on a level road, as depicted in Figure 5. The centripetal acceleration and force act inward toward the center upon the tires of the car. In this case, it is a force caused by friction between the tires and road. The centrifugal force is the frictional force produced by the tires acting outward upon the road.

When a car travels in a circular path, a person in the car seems to experience an outward force; that is, a force tending to throw him or her outward from the center of the circle.

Actually, the seat exerts an inward force upon the person, causing him to travel in a circular path. The person tends to travel in a straight line, but the centripetal force acting inward (provided by the seat) causes him to travel in a circular arc.

Consider two particles m and M, which have the same size and which are suspended in a uniform fluid in a centrifuge (Figure 6). The density of M is greater than the density of m. As the centrifuge spins, both experience the same centripetal acceleration, $a_c = v^2/r$, since the radius and the tangential velocities are the same. The centripetal force upon M, necessary to hold M in place, is given by Equation 10.

$$F_{cM} = Ma_c \qquad (10)$$

The centripetal force for M must be greater than that to hold m in place, since M > m, as indicated by Equation 11.

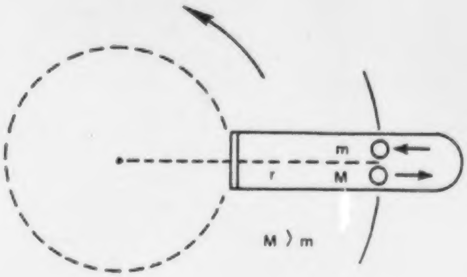


FIG. 6

$$F_{cm} = ma_c \qquad (11)$$

However, the fluid exerts the same fluid force on both masses, since they are of the same size. Because of this, the masses will separate, with the larger mass M moving further outward in the tube than the smaller mass m. The centrifuge uses this principle to separate particles of varying densities. The particles of greater density move toward the bottom (farther from the center) of the tube.

If the same particles are placed on ends of identical springs and swung in a circle, the one with the greater mass will stretch its spring more because it requires a greater restoring (centripetal) force. The larger mass will rotate with a greater radius, as in the centrifuge.

Another application of centripetal force in circular motion is illustrated in Figure 7. The fluid enters the pump at the center inlet and is given a spinning motion. Its natural tendency is to move in a straight line, tangent to its circular motion, unless an inward centripetal force keeps it in a circular path. Since there is no string or spring tied to each particle, there is no inward force. Thus, the fluid flows outward along lines tangent to its circular motion at each point.

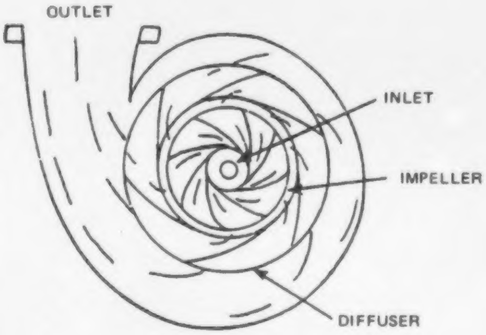


FIG. 7

Because the fluid moves outward, it is called a "centrifugal pump." Centrifugal means "center fleeing."

SUMMARY

Centripetal force is the inward force upon an object moving in a circular path. It is the radial force exerted by the constraining system required to keep the object moving in its curved path around a central point. Whenever rotational motion occurs, centripetal force is present because such a force is necessary to change the direction of object motion from the straight line that it normally would follow to a curved path. Before the magnitude of the force required to keep a given object moving in a circle at a certain speed can be determined, the acceleration of the object must be found. This radial, or centripetal, acceleration is the change in velocity divided by the time interval to express a rate.

LABORATORY

CENTRIPETAL ACCELERATION & CENTRIPETAL FORCE

MATERIALS

Centripetal force apparatus including weights and weight hanger
(Sargent-Welsh #0930 or equivalent).
Meter stick.
Stop watch or timer.

PROCEDURE

In this experiment, the student will determine the centripetal acceleration of a rotating object and find the corresponding centripetal force by two different methods. The percentage difference between these forces will be calculated.

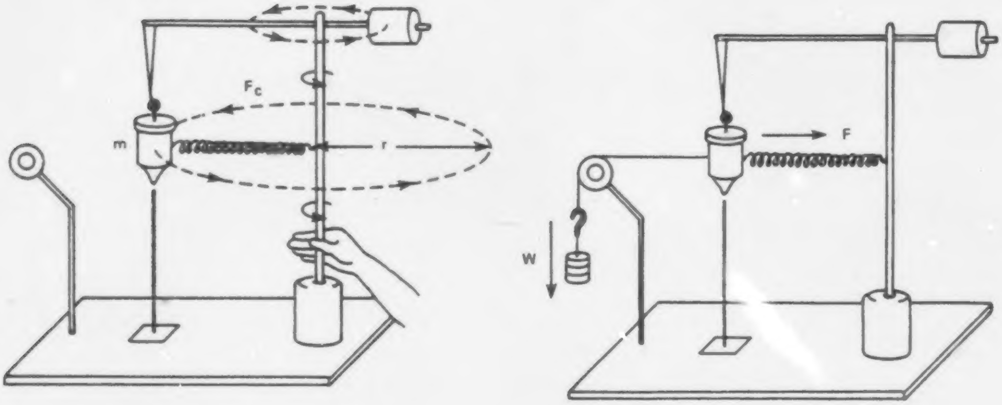
Figure 8a displays the centripetal force apparatus as it is rotated. The mass m swings outward due to the rotation, but the spring supplies the inwardly-directed centripetal force F_c necessary to keep the mass traveling in a circular path. The greater the rotational speed, the further out the mass swings. Figure 8b shows the stationary spring being stretched the same distance as in Figure 8a, but due to a force supplied by weight W. For equilibrium to exist in Figure 8b, F must equal W.

Since the spring is displaced the same amount in Figure 8a and 8b, the two forces F_c and F must be equal.

1. Rotate apparatus as shown in Figure 8a. Adjust speed until the mass m swings directly above radius indicator rod. Practice will allow rate to be held constant, so that the mass passes directly over the indicator rod on each revolution. Use the timer to determine the total time T_t for 30 rotations of mass, and record this in Data Table 1. Calculate the average period $T = T_t/30$ for each rotation. Record this value. Repeat this process three times, and find the average period for these three trials.

2. Measure and record radius r of rotational motion, and record this value.

3. Attach weight hanger as shown in Figure 8b. Place additional weights on hanger until rotating mass is held over radius indicator bar. Record this mass value M (weight hanger plus



(a.) (b.)

FIG. 8

added weights) in Data Table 2. Determine corresponding weight $W = Mg$ and record this value.

4. Calculate velocity of rotating mass illustrated in Figure 8a by dividing the total distance traveled ($2\pi r$) by the average period (T) from Data Table 1. Since the tangential velocity is the total distance traveled, divided by the time, then $v = 2\pi r/T$. Record the values of T, r, and v in Data Table 2.

5. The centripetal acceleration for the rotational motion now can be calculated from the equation $a_c = v^2/r$ and recorded in Data Table 2. Calculate the centripetal force acting upon the rotating mass m from the equation $F_c = ma_c$ and record in Data Table 2.

6. The force $F = W$ and the centripetal force F_c should be the same, since they produce an equal elongation of the spring. Determine the percentage difference in the measured values of these two forces. Record this value.

7. Repeat steps 1 through 6 for a different tension in the spring. This repetition can be achieved by changing the position of the radius indicator rod and by correspondingly changing the position of the rotating crossbar.

8. How would a radius of greater length affect the results?

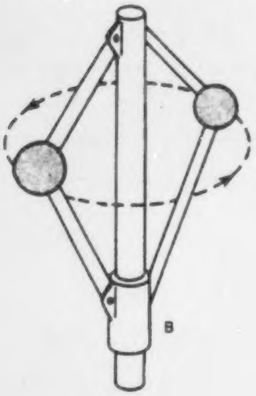
DATA TABLE 1				
Spring Tension Setting	Trial No.	Time for 30 rev T_t (s)	Period T (s)	Radius r (m)
Tension 1	1			
	2			
	3			
	Average			
Tension 2	1			
	2			
	3			
	Average			

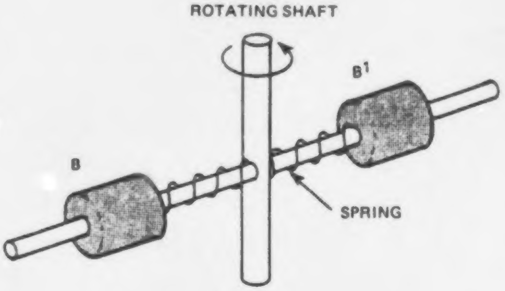
DATA TABLE 2		
Spring Tension Setting	1	2
Period	T = s	T = s
Radius of rotation	r = m	r = m
Mass to elongate spring	M = kg	M = kg
Weight to elongate spring	W = Mg = N	W = Mg = N
Speed of rotating body	v = m/s	v = m/s
Centripetal acceleration	$a_c = v^2/r =$ m/s^2	$a_c = v^2/r =$ m/s^2
Mass of rotating object	m = kg	m = kg
Centripetal Force	$F_c = ma_c$ N	$F_c = ma_c$ N
Percentage difference between F_c and W	%	%

SELF CHECK

1. A merry-go-round at a park is set into motion in such a manner that it completes one revolution each 3 seconds. A child weighing 50 lbs sits on the edge of the merry-go-round a distance of 6 ft from its center. Calculate (a) the tangential velocity of the child, (b) the centripetal acceleration, and (c) the centripetal force acting upon the child.

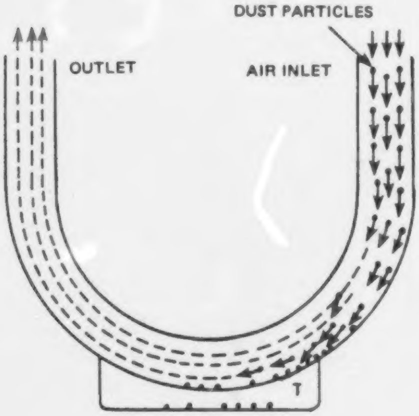
• 2. Two types of governors used for controlling the speed of devices are shown. The cylinders B and B' are free to slide as the rotational speed changes. Explain, using the ideas of centripetal acceleration and centripetal force, how the governors can be used to control the speed of a rotating object.



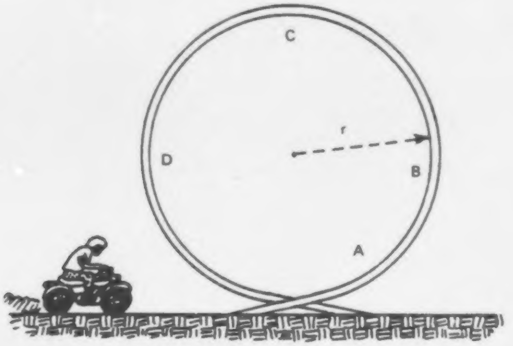

ROTATING SHAFT
B^1
B
SPRING

3. What is the difference between centripetal and centrifugal force?

4. A "cyclone-dust-remover" is shown in the adjoining figure. Explain how it can be used to collect dust particles in the collector T, which often contains water or a water spray.


DUST PARTICLES
OUTLET
AIR INLET
T

5. A motorcyclist loops-the-loop as shown in the adjoining figure. What is the minimum velocity he must have at C in order not to lose contact with the surface? Take $r = 16$ ft and $g = 32$ ft/s^2. (Hint: At point C, $F = mg = mv^2/r$.)



6. How long will it take a car to complete 3 laps around a circular track if the radius of the track is 100 meters and the car (mass 2000 kg) experiences a constant centripetal force of 72,000 N?

7. A boy of mass 40 kg sits 2 meters from the center of a merry-go-round. If the merry-go-round starts from rest and undergoes an angular acceleration of 1 rad/s^2 how many revolutions will the merry-go-round make before the boy experiences a centripetal force of 100 N?

8. What is the radius of a curve if a 2000 kg car experiences a centripetal force of 20,000 N when traveling around the corner with a speed of 20 m/s?

3E1

ELECTRIC CURRENT

The most fundamental "rate" or ratelike quantity that must be understood in relation to electrical systems is the concept of current, the rate at which the displacementlike quantity "charge" is moved from one point to another. Direct current in an electrical circuit is the rate at which charge moves past some point in the circuit when a constant electrical potential is applied. Alternating current is the rate of charge flow back and forth past a point in the circuit when an alternating voltage is applied.

In this module, the student will examine the concepts of direct and alternating current and in the laboratory will use a d.c. ammeter and an a.c. ammeter to measure electric currents.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the below terms:

    Current.
    Direct current.
    Alternating current.
    Ampere.
    Electron flow.
    Conventional current flow.

- Given any two of the following three factors, find the third:

    Current.
    Charge.
    Time.

- Draw and label a graph showing a.c. current as a function of time.

- Given a d.c. ammeter and a direct current source, measure the current within ±0.1 amps.
- Using a galvanometer and a direct current source, determine the direction of flow of the current.

DIRECT CURRENT

In an electrical conductor, some electrical charges — called "electrons" — are free to move in the material. When the poles of an electromotive force or battery are connected by a conductor, these free electrons respond by moving away from the point of negative potential toward the point of positive potential. The terminal of negative potential, called the "cathode," acts as a source of additional electrons; and the terminal of positive potential, called the "anode," acts as a "sink" that removes electrons from the conductor.

As long as an electrical potential difference exists, the electron motion through the conductor will continue. The conducting material itself has no net change in charge. For every electron that enters the conductor from the negative terminal, one electron leaves the conductor at the positive terminal.

To indicate the rate at which current flows through a conductor, a meter must "count" the electrons that pass a given point in some time interval. This process can be visualized by imagining a plane surface within the conductor perpendicular to the electron flow (perpendicular to the conducting wire) as shown in Figure 1.

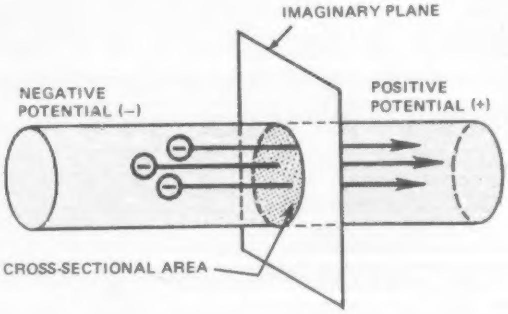


FIG. 1

When a charge Q passes through the surface in a time t, the current is defined by Equation 1 as follows:

$$I = \frac{Q}{t} \qquad (1)$$

where
I = current.
Q = charge.
t = time.

Direct current through a conductor is the charge per unit time that passes in one direction through a plane perpendicular to the applied electric field.

If the charge is measured in coulombs and the time is measured in seconds, resulting current is in "amperes." An ampere is defined as one coulomb per second. Examples A and B show the use of Equation 1 for solving problems.

EXAMPLE A: CURRENT CALCULATION

Given: 2.4 C (coulombs) passing through a copper wire in 0.6 seconds.

Find: Current.

Solution:

$$I = \frac{Q}{t}$$

$$= \frac{2.4\ C}{0.6\ s}$$

$$I = 4.0\ \frac{C}{s} = 4.0\ A$$

The current is 4.0 amperes.

EXAMPLE B: CHARGE CALCULATION

Given: A current of 62.5 amps flows for 16 ms.

Find: How much charge passes through the conductor.

Solution:

$$I = \frac{Q}{t}$$

$$Q = It$$

$$= (62.5\ \text{amps})(16\ \text{ms}) \ \ldots\ (\text{ms is equal to } 10^{-3}\ \text{s})$$

$$= (62.5\ \frac{C}{s})(16 \times 10^{-3}\ s)$$

$$Q = 1.0\ C$$

A charge of 1.0 coulomb passes through the conductor.

DIRECTION OF CURRENT FLOW

This module has described direct current in terms of electron flow. In metal conductors, the charge carriers are the electrons (negative carriers), which move from a point of negative potential toward a point of more positive potential.

In some semiconductor materials, in ionized gases, and in many solutions, charge is transferred by positive-charge carriers. Positive-carriers result from the removal of an electron at the positive terminal. Such positive-charge carriers move toward the negative terminal, where they receive an electron and become neutral. The result is the same as with electron flow: one unit of charge is transferred through the conductor.

The direction of current flow is somewhat confusing because, at the time electricity was discovered, the positive charge was thought always to be the moving particle. Some texts used in the study of electricity use "conventional current" direction, rather than electron flow direction. The direction of conventional current flow is from positive to negative, and the charge carriers are assumed to be positive. The direction of electron flow is from negative to positive, and the charge carriers are negative electrons.

Both conventional current and electron current are shown in Figure 2. The magnitudes in both directions are the same; only the directions are different.

ALTERNATING CURRENT

Direct current is the rate of charge flow that results when a constant voltage difference is maintained across an electrical conductor. If the voltage changes, the current also changes.

Alternating current (a.c.) flows first in one direction and then in the other, and results in a circuit when the polarity of the voltage source alternates as shown in Figure 3. At time t_0 in this figure, the applied voltage is zero; and no current flows, as is indicated by t_0 of Figure 4. At t_1 a maximum positive voltage is applied to one end of the conductor, while the other end remains at 0 volts. This voltage difference produces the electron flow shown by t_1 of Figure 4. At t_2 both voltage and current are again zero, and at t_3 an applied voltage of $-V_{max}$ causes a current flow in the opposite direction. At t_4 the voltage and current are both zero again. This waveform repeats at a frequency of 60 cycles/per second in ordinary household circuits.

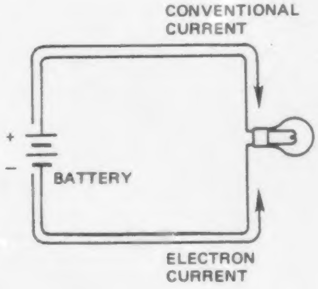


FIG. 2

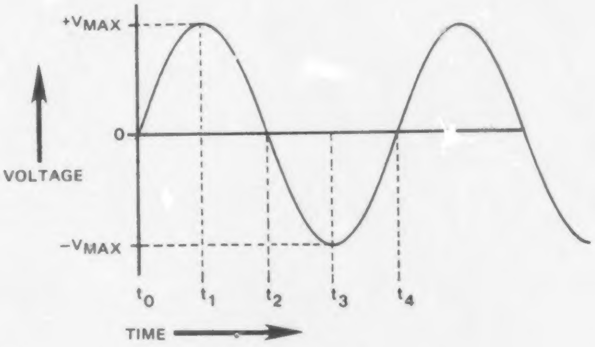


FIG. 3

SUMMARY

The rate at which charge flows through an electrical conductor is called the "current," which is measured in amperes. The "direction of current flow" is defined as the direction in which a positive charge would move for "conventional current" or the direction of motion of a negative charge (electron) for "electron current." "Direct current" (d.c.) is the current flow in a single direction that results when a constant voltage is applied to a circuit. "Alternating current" (a.c.) is the current flow in alternating directions that results when the polarity of the voltage source alternates.

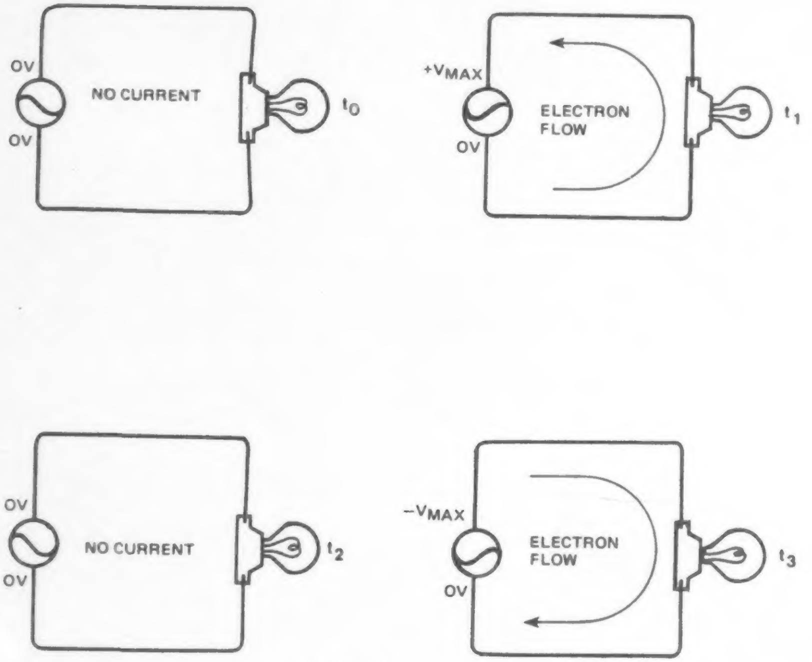

OV
OV
NO CURRENT
t_0
+V_{MAX}
OV
ELECTRON FLOW
t_1
OV
OV
NO CURRENT
t_2
−V_{MAX}
OV
ELECTRON FLOW
t_3

FIG. 4

CURRENT MEASUREMENT

MATERIALS

12-volt battery or d.c. power supply.
12-volt lamp and socket.
d.c. ammeter.
Clip leads.
Knife switch.

PROCEDURE

1. Connect circuit as shown in Figure 5. Notice that positive terminal of ammeter is connected to positive terminal of battery, and that negative terminal of ammeter is connected through lamp to negative terminal of battery. If polarity of meter is reversed, needle will be driven backward and can damage meter mechanism.
2. Close switch; observe both lamp and meter.
3. Record current in Data Table; open switch. Switch should be closed for short periods of time only to conserve battery's energy.
4. Disconnect circuit; reconnect it as shown in Figure 6.
5. Close switch; record current through this circuit.
6. Calculate average current for two situations.
7. Calculate average charge moved in one second.
8. Calculate number of electrons flowing through circuit in one second. (1 coulomb = 6.24×10^{18} electrons.)
9. Answer question in Data Table.

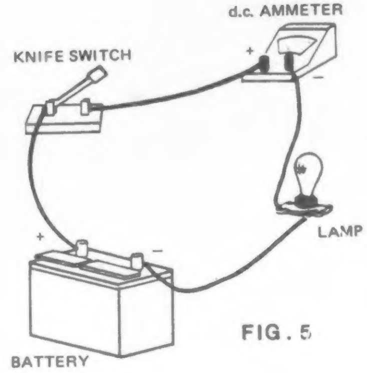


FIG. 5

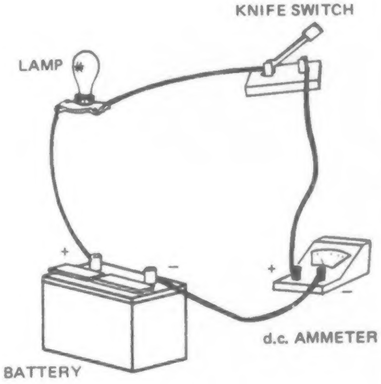
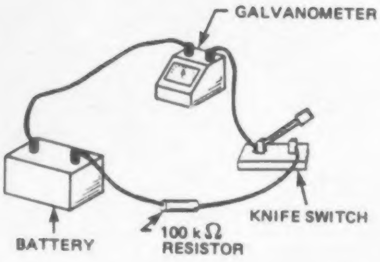


FIG. 6

DIRECTION OF CURRENT FLOW


GALVANOMETER
BATTERY
100 k Ω
RESISTOR
KNIFE SWITCH

MATERIALS

12-volt battery or d.c. power supply.
Clip leads.
Knife switch.
Galvanometer.
100-kΩ resistor.

PROCEDURE

1. Connect circuit as shown in Figure 6.
2. Close switch; note direction of deflection of galvanometer needle.
3. Indicate in Figure 7 direction of both conventional and electron current. Record direction and magnitude of needle deflection.
4. Reverse polarity of battery connections; repeat experiment.
5. Explain polarity and its importance.

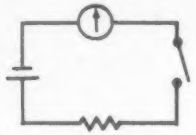


FIG. 7

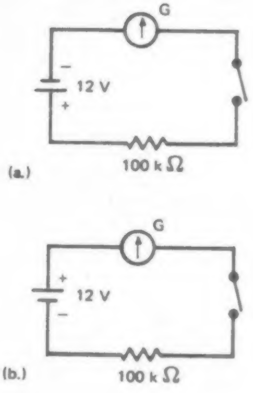

G
−
12 V
+
(a.)
100 k Ω
G
+
12 V
−
(b.)
100 k Ω

FIG. 8

DATA TABLE

Current Measurement

Current from first procedure, step 3 __________.

Current from first procedure, step 5 __________.

Average current __________.

What was shown by reversing circuit? ______________________

____

____

Average charge moved in one second ___________.

Number of electrons flowing in one second __________.

Direction of Current Flow

First Trial

Indicate in Figure 7a direction of conventional and electron current flow.

Direction of needle deflection _________.

Magnitude of needle deflection _________.

Second Trial

Indicate in Figure 7b direction of conventional and electron current flow.

Direction of needle deflection ________.

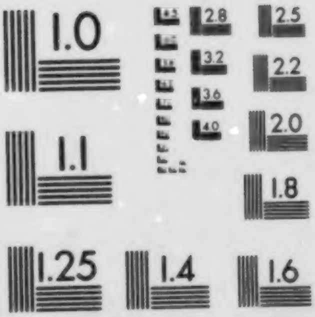
Magnitude of needle deflection ________.

SELF CHECK

1. Define:

   Current.
   Direct current.
   Alternating current.
   Ampere.
   Electron flow.
   Conventional current flow.

2. A current of 16 amps flows through a conductor. How much charge is transferred in 25 ms?

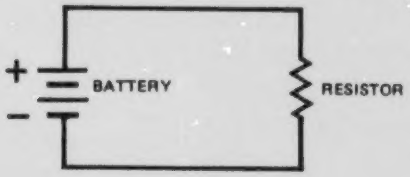

1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS
STANDARD REFERENCE MATERIAL 1010a
(ANSI and ISO TEST CHART No. 2)

3. If one minute is required for 10 C of charge to flow through a conductor, what is the current?

4. How much time is required for a current of 25 mA to transfer a charge of 0.5 C?

5. In the below circuit sketch, indicate (1) direction of electron flow with a solid line and (2) direction of conventional current flow with a dashed line.



6. Draw a graph showing alternating current as a function of time in an a.c. circuit.

7. Recalling that W = QV, find the current required to do 50 joules of work in 40 seconds if the operating voltage is 24 volts.

8. How many electrons flow through a cross-section of a wire in 3 minutes if the current through the wire is 3 amperes? (Hint: $1\ e^- = 1.602 \times 10^{-19}$ C.)

3E2

FREQUENCY

The membrane of a speaker vibrates 10,000 times in 20 seconds to produce sound of a certain pitch. The frequency of vibration of the speaker (and the resultant sound wave), therefore, is 500 cycles per second. Frequency is a quantity that provides a measure of the number of repetitive events that occur in a given time and, therefore, is a rate at which certain events occur.

In this module, the student will learn how the quantities frequency and time relate to each other in repetitive events. The importance of known frequencies for time measurement will be noted. In addition, the concept of spatial frequency will be introduced. In the laboratory, the student will measure the characteristic frequencies associated with some simple electrical, mechanical, and optical systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following concepts:

    Temporal frequency.
    Spatial frequency.
    Period.

- Given one of these two quantities, find the other:

    Frequency.
    Period.

- Given one of these two quantities, find the other:

    Spatial frequency.
    Spatial separation.

- Find the smallest interval of time that can be measured with different types of timekeeping devices.
- Given the appropriate equipment, measure the period of a pendulum and of an electrical wave.
- Given the appropriate equipment, determine the spatial frequency of the grooves in a diffraction grating, and calculate their spatial separation.

DISCUSSION

Frequency is a measure of the repetitive nature of events or of objects. Temporal frequency describes events that repeat after sepcific intervals of time have elapsed. Events that can be described in terms of temporal frequencies inlcude water dripping from a faucet, the ticking of a clock, or the motion of a planet around a star.

Spatial frequency describes objects that repeat over specific distances. Stripes on a highway, keys on a piano, and bottles in a carton are objects that can be well described by spatial frequencies.

Some objects and events can be characterized by both spatial and temporal frequencies. A train moving through a crossing is an example of such a system. The cars of the train are spaced evenly and, therefore, repeat after a certain distance. Because the train is moving, however, cars also pass through the crossing at definite, equal time intervals.

TEMPORAL FREQUENCY

Temporal frequency is defined as the rate at which a repetitive event occurs, as indicated by Equation 1.

$$f = \frac{n}{t} \tag{1}$$

where

f = temporal frequency.
n = number of repetitions.
t = time in seconds.

Example A illustrates how the equation for frequency is used to solve a problem. (Note: Hereinafter the term "frequency" will be used to refer to temporal frequencies.)

EXAMPLE A: FREQUENCY OF AN ELECTRICAL WAVE

Given: A commercial generator produces 240 electrical waves in four seconds and operates for one hour.

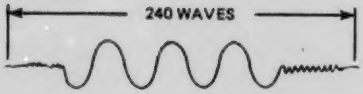
Find: (a) The frequency of the waves.
(b) The number of waves produced in this time.

Solution: (a) Use Equation 1.

$$f = \frac{n}{t}$$

$$f = \frac{240 \text{ waves}}{4 \text{ seconds}}$$

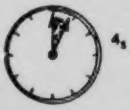
$$f = 60 \text{ waves/s}$$



(b)

$$f = \frac{n}{t}$$

$$\therefore n = f\,t \quad \text{(multiplying both sides by time)}$$



$$n = (60\ \frac{\text{waves}}{\text{s}})(1 \text{ hour})$$

$$= (60\ \frac{\text{waves}}{\text{s}})(1 \text{ hour})(\frac{60 \text{ min}}{1 \text{ hour}})(\frac{60 \text{ s}}{1 \text{ min}})$$

$$n = 216{,}000 \text{ waves}$$

Example A shows how to calculate the frequency of a repetitive event and demonstrates how a knowledge of the frequency allows the calculation of the number of events occurring in a given time.

The standard units of frequency are the same in the mks, cgs, and English measuring systems. Frequency is measured in Hertz (Hz), where —

$$1 \text{ Hz} = 1 \text{ cycle/second} = 1 \text{ event/second.}$$

"Cycles" or "events" cannot be expressed in terms of units of mass, displacement, or time; therefore, in a strict sense, frequency is measured in reciprocal time units:

$$1 \text{ Hz} = 1/\text{s} = 1 \text{ s}^{-1}$$

These units immediately lead to Equation 2, another form for the definition of frequency, completely equivalent to Equation 1:

$$f = \frac{1}{T} \qquad (2)$$

where
T = period (time) for one event.

Equation 2 is solved for T as shown in Equation 3.

$$fT = 1 \quad \text{(Multiplying both sides by T)}$$

$$T = \frac{1}{f} \quad \text{(Dividing both sides by f)} \tag{3}$$

Thus, the period of any repetitive event is simply the reciprocal of its frequency.

EXAMPLE B: PERIOD OF VIBRATION OF A TUNING FORK

Given: A tuning fork vibrates at a frequency of 800 Hz.

Find: The period of vibration of the tuning fork.

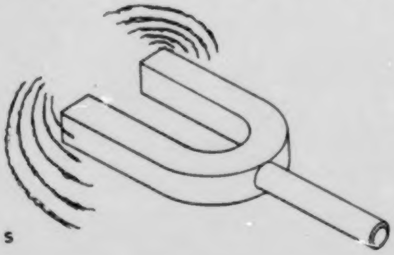
Solution: From Equation 3:

$$T = \frac{1}{f}$$

$$= \frac{1}{800\ \text{Hz}}$$

$$T = 0.00125\ \text{s}$$

Note: $\frac{1}{\text{Hz}} = \frac{1}{1/\text{s}} = \text{s}$



Many common energy systems exhibit repetitive motion and, hence, can be characterized by a specific period and frequency. Probably the most familiar such system is the simple pendulum, which consists of a massive object suspended by a light rod or string. If the pendulum is allowed to swing freely, its period will be constant. If the pendulum swings only through a small angle (approximately 5° or less), the period of the pendulum depends only upon its length and the acceleration of gravity as shown in Equation 4.

$$T = 2\pi\sqrt{\frac{\ell}{g}} \tag{4}$$

where

ℓ = length of pendulum.

g = acceleration of gravity.

For the past three centuries, the major application of the pendulum has been as a timer. Recently, however, pendula have been used to determine the acceleration of gravity to a high degree of precision. This determination is technologically important for oil exploration, since the acceleration of gravity over a large oil deposit often is somewhat less than the nominal value of 980 cm/s^2.

EXAMPLE C: FREQUENCY OF A SIMPLE PENDULUM

Given: A simple pendulum of length 20 cm swings in a region where the acceleration of gravity is 965 cm/s .

Find: Frequency of the pendulum.

Solution: First, use Equation 4 to find the period:

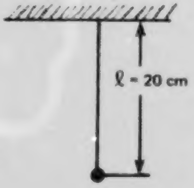
$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$= 2\pi\sqrt{\frac{20\ cm}{965\ cm/s^2}}$$

$$= 2\pi\ \ 0.0208\ s^2$$

$$= 2\pi\ (0.144\ s)$$

$$T = 0.904\ s$$



Use Equation 2 to find the frequency?

$$f = \frac{1}{T}$$

$$= \frac{1}{0.904\ s}$$

$$= 1.10/s$$

$$f = 1.10\ Hz$$

The frequency of the pendulum is 1.10 Hz.

By far, the most common (and oldest) application of the frequency concept is the measurement of time. In this application, time is measured by counting repetitive phenomena. If the frequency of the repetitive event is constant, then the period of the motion defines a basic unit of time. The smallest unit of time that can be measured by any repetitive event is equal to the period of the event; consequently, the higher the frequency of any repetitive phenomenon, the smaller the basic unit of time that it can measure — thus allowing more precise time measurements.

The first repetitive events used as time standards were the motions of the heavenly bodies. These motions were employed to define three basic units of time: the day (based upon the rotation of the earth on its axis), the year (revolution of the earth about the sun), and the month (motion of the moon with respect to the sun and the earth).

These standards of time were impractical for two reasons. First, the extremely small frequencies of the repetitive motions involved caused the time units to be too long for many applications. Second, the very nature of the motions made very difficult the precise measurements of their periods.

Many clever devices were developed to subdivide the smallest of these units (the day) into even smaller divisions (hours, minutes, and seconds). These devices included the sun dial, the hourglass and the water clock; but the basic unit of time remained the day.

In the early 16th century, Galileo studied the motion of simple pendula and deduced Equation 4. This simple discovery revolutionized time measurement. The smallest unit of time that could be measured with a pendulum (i.e., the period of the pendulum) is on the order of one second. Pendulum clocks were extremely reliable and remained the major timekeeping instruments for several hundred years. Their major drawbacks were their fragility and lack of portability.

Both of these problems were overcome with the introduction of the next timekeeping standard. A small balance wheel was mounted on an axle and caused to rotate back and forth by a fine spring. With suitable values for the mass and radius of the wheel and for the tension of the spring, the balance wheel could be made to oscillate many times each second. In addition, the balance-wheel mechanism proved far more rugged than pendulum clocks. The balance wheel also required much less room than the pendulum, making small clocks and watches possible.

Many modern clocks use the frequency of alternating voltage to determine their basic time unit. The frequency of this electrical votage is 60 Hz (50 Hz in most foreign countries) and divides the second even more finely than the balance wheel.

The natural vibrations of a quartz crystal provide even finer division of the second. Typical natural frequencies for these crystals are from one to ten megahertz (1 MHz = 10^6 Hz), allowing basic time units of $10^{-6} - 10^{-5}$ seconds, respectively.

The ultimate in timekeeping precision is presently the atomic clock. This device uses the frequency of electromagnetic radiation emitted from certain atoms or molecules (typically cesium or ammonia) for a time standard. The frequencies involved are generally in the range from 1 — 10 gigahertz (1 GHz — 10^9 Hz). One such clock, operated by the National Bureau of Standards, is accurate to within a few seconds in one million years.

SPATIAL FREQUENCY

Any object or event which repeats after a given distance can be characterized by a spatial frequency. The definition of spatial frequency can be stated in a form similar to Equation 1, as indicated by Equation 5.

$$k = \frac{n}{d} \qquad (5)$$

where

k = spatial frequency.
n = number of spatial events that occur in distance d.
d = distance.

The spatial frequency also can be expressed in terms of the spatial separation; that is, the distance between two consecutive objects or events. Spatial separation is defined by Equation 6.

$$k = \frac{1}{d} \qquad (6)$$

Notice the similarity between Equations 2 and 6. In Equation 6 the units of spatial frequency are reciprocal distance units; spatial frequency, therefore, has different units in each of the three systems of movement.

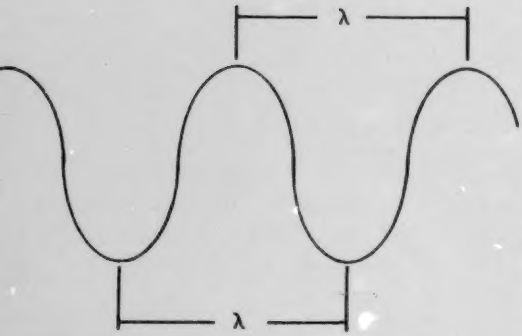


FIG. 1

One class of phenomena that is readily described by spatial frequencies is wave motion. The spatial separation of waves (for instance, the sound waves in Figure 1) is the distance between two consecutive peaks (or two consecutive troughs) and is called the "wavelength." Example D shows how the spatial frequency of waves is calculated.

EXAMPLE D: SPATIAL FREQUENCY OF WATER WAVES

Given: Ocean waves have peaks every 10 m.

Find: The spatial frequency of these waves.

Solution: Wavelength = spatial separation = 10 m.

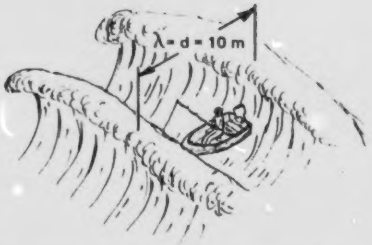
Then —

$$k = \frac{1}{d}$$

$$k = \frac{1}{10\ \text{m}}$$

$$= 0.1/\text{m}$$

$$k = 0.1\ \text{m}^{-1}$$



The concept of spatial frequency often is applied to optical instruments. Photographic film consists of fine grains of light-sensitive material (usually silver nitrate) on glass or cellulose. When exposed to light, these grains darken in proportion to the amount of light incident upon them to form the familiar photographic negative. Naturally, the clarity of detail in the negative will be better if the film contains many small, closely-packed grains, rather than large grains not so closely packed. The spatial frequency of the grains on the film is then a measure of the ultimate clarity (or resolution) of the negative. The greater the spatial frequency, the better the film resolution will be.

Another device called a "diffraction grating" can be used to bend, or deviate, a beam of light. The grating is a glass or plastic slide with many long, parallel scratches or grooves on its face. If the grating is illuminated with a beam of light of a single wavelength, it will break the beam into a diffraction pattern of alternating light and dark bands (Figure 2).

The brightest band is in the center of the pattern and is called the "central band." The distance from the central band to the nth outlying band (x in Figure 2) is related to the wavelength of the light and the distance ℓ from screen to grating by Equation 7.

$$nk\lambda = \frac{x}{\ell} \qquad (7)$$

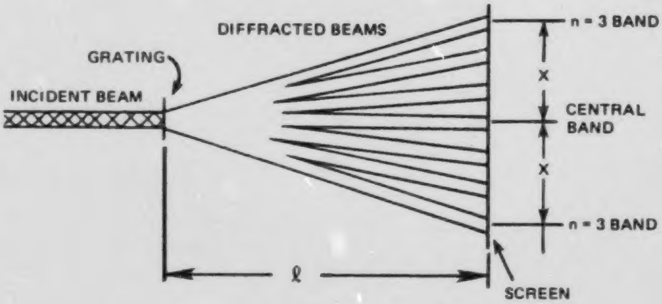


FIG. 2

where

k = spatial frequency of the grooves on the grating.

λ = wavelength of light.

x = distance from central band to outlying band.

ℓ = distance from screen to grating.

The diffraction grating is treated in more detail in Module 13L12.

EXAMPLE E: SPATIAL FREQUENCY OF DIFFRACTION GRATING GROOVES

Given: In Figure 2, $\ell = 5$ m, $\lambda = 5 \times 10^{-7}$ m, and the distance from the central band to the third outlying bright band is 3 m (0.03 m).

Find: The spatial frequency of the grating's grooves.

Solution: Note that n = 3 (third band) and that x = 0.03 m. From Equation 7:

$$nk\lambda = \frac{x}{\ell}$$

$$k = \frac{x}{n\lambda\ell} \text{ (dividing both sides by } n\lambda\text{)}$$

$$= \frac{(0.03 \text{ m})}{3\ (5 \times 10^{-7} \text{ m})(5 \text{ m})}$$

$$k = 4000/\text{m}$$

The grating has 4000 grooves per meter.

SUMMARY

Frequency is the rate at which events occur. "Temporal" frequency refers to events that repeat after specific time intervals have elapsed. Examples of temporal frequency are timekeeping devices, electrical waves, vibrations of a tuning fork, and repetition of a pendulum's motion. "Spatial" frequency describes objects that repeat over specific distances; for example, wavelength and the groove frequency of a diffraction grating.

LABORATORY

FREQUENCY OF ELECTRICAL OSCILLATIONS

MATERIALS

Signal generator.
Oscilloscope.
2 wire leads.

PROCEDURE

1. After instructor has adjusted oscilloscope in such a manner that electrical wave can be seen on the screen, set signal generator frequency to 200 Hz.
2. Use oscilloscope time base (s/div switch) to adjust the size of the wave until one wavelength is spread evenly over the entire screen. Record s/div switch setting in Data Table 1. (Note: "m/s" means milliseconds — 1 ms = 10^{-3} s.)
3. Measure number of divisions in one wavelength λ of electrical wave. Record this value in Data Table 1.
4. Calculate the period T and frequency f of the electrical wave.
5. Repeat this procedure for signal generator frequencies of 400 Hz, 500 Hz, and 1000 Hz.

DATA TABLE 1

	Signal Generator Frequency	200 Hz	400 Hz	500 Hz	1000 Hz
	s/div	_____ s	_____ s	_____ s	_____ s
	λ	_____	_____	_____	_____
	$T = s/div \times \lambda$	_____ s	_____ s	_____ s	_____ s
	$f = 1/T$	_____ Hz	_____ Hz	_____ Hz	_____ Hz

PERIOD OF A PENDULUM

MATERIALS

Pendulum (200 g hooked mass tied to approximately 150 cm of light string).
Stopwatch or electric timer.
Meter stick.

PROCEDURE

1. Suspend pendulum in a doorway. Measure and record length ℓ (distance from center of mass to suspension point at top of doorway).

2. Displace mass approximately 10 cm horizontally. Simultaneously release mass and start the timer. Stop timer after pendulum has completed 20 swings. (Mass must return to original release point to complete a swing.) Record elapsed time t in Data Table 2.

3. Calculate period T from t and number of swings.

4. Calcualte period T from ℓ, and acceleration of gravity ($g = 980\ cm/s^2$), using Equation 4.

DATA TABLE 2

Length of pendulum = ℓ = ________ cm

Number of swings = n = 20

Elapsed time = t = ______ s

$T = \frac{t}{20} =$ _________ s

$T = 2\pi\sqrt{\frac{\ell}{g}} =$ _________ s

SPATIAL FREQUENCY OF A DIFFRACTION GRATING

MATERIALS

He-Ne laser.
Student diffraction grating.

Metric ruler or tape measure.
White cardboard screen.

PROCEDURE

1. Set up laser, diffraction grating, and screen as shown in Figure 3. Distance should be at least 2 m. Turn on laser and align the beam so that a diffraction pattern appears on the screen.

   CAUTION: LASERS PRODUCE A HIGH INTENSITY BEAM OF LIGHT WHICH CAN DAMAGE YOUR RETINA IF DIRECTED INTO YOUR EYE. DO NOT LOOK DIRECTLY INTO THE LASER BEAM.

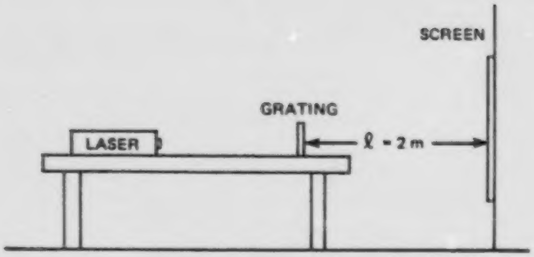


FIG. 3

2. Measure, and record distance ℓ in Data Table 3.

3. Measure and record the distance x from the center of the central band to the center of the fourth band to its left or right. (The value of n in Equation 7 for this band is then equal to 4.)

4. The He-Ne laser produces light of a single wavelengh, $\lambda = 6.328 \times 10^{-5}$ cm. Using this value for λ and the values of n, k, and ℓ already determined, calculate the spatial frequency k of the grooves on the grating.

5. Using this value of k, calculate the spatial separation of consecutive grooves d.

DATA TABLE 3

$\ell =$ ________ cm	$d = \frac{1}{k} =$ ________ cm
$\lambda = 6.328 \times 10^{-5}$ cm	$x =$ ________ cm
$k = \frac{x}{n\lambda\ell} =$ ______/cm	$n = 4$

SELF CHECK

1. Define the following:

    Temporal frequency.
    Period.
    Spatial frequency.

2. A train of identical boxcars travels at constant speed past an observer. The observer sees a car go by every four seconds. With what temporal frequency do the cars pass the observer? If each car is 20 m long, what is the spatial frequency of the boxcars?

3. A diffraction grating has a groove spacing of 0.002 mm. What is its spatial frequency?

4. Find the period of waves with the following frequencies:

    a. 50 Hz.
    b. 50 KHz.
    c. 50 MHz.

5. What is the smallest time measurable with each of the following devices:

    A pendulum of length 1.5 m (assume $g = 9.8\ m/s^2$).
    A 5.2 MHz quartz crystal.
    An atomic clock that uses electromagnetic radiation of frequency 5×10^{14} Hz as its reference.

6. What must the length of a simple pendulum on the moon be if it is to have the same period as a pendulum on the earth which has a length of 1 meter? Take g on the moon to be one sixth g on the earth.

3T1

HEATING AND COOLING RATE

Heat energy can be transferred into or out of an area so as to increase or decrease its temperature. Such heating or cooling is familiar. Consider heating a pot of food on a stove and then cooling the leftovers in the refrigerator. The heating or cooling rate is the change in temperature divided by the elapsed time. It is measured in units of degrees temperature per unit time.

In this module, heating and cooling rates under various conditions will be measured.

OBJECTIVES

To complete this module, the student should be able to:

- Define heating and cooling rate by means of an equation, stating the appropriate units, and how to distinguish between a heating rate and a cooling rate.
- Define three processes by which heat energy is transferred to produce heating or cooling.
- Calculate and plot the cooling rate versus time of several common systems given the required materials, temperatures and conditions.
- Calculate the final temperature of an object after cooling, given materials, temperatures and time span.
- Given proper equipment, measure heating and cooling rates of various materials.

DISCUSSION

In this module, the rate at which the temperature of a material changes, called the heating or cooling rate, will be discussed. The rate is defined by the equation:

$$\text{Rate of heating or cooling} = \frac{\text{temperature change}}{\text{elapsed time}}. \qquad (1)$$

In symbols this becomes:

$$R = \frac{T_2 - T_1}{t_2 - t_1} \qquad (2)$$

where

R = rate.
T_1 = temperature at time t_1.
T_2 = temperature at time t_2.

Assume that t_1 is the initial starting time. Then $t_2 - t_1$ is the elapsed time and will always be a positive number. If the temperature is increasing, T_2 is greater than T_1. Then Equation 2 defines a heating rate and the sign of the quotient will be positive. If T_2 is less than T_1, the sign in Equation 2 will be negative. Then Equation 2 defines a cooling rate.

The units of heating or cooling rate are units of temperature per unit time, for example °C per second or °F per hour.

Practical examples of heating and cooling are not difficult to find. Consider how the temperature in a room increases when the furnace turns on (Example A) or how the temperature of a pot of water increases when one turns on the stove beneath it. Simi-

EXAMPLE A: HEATING RATE OF A HOUSE

Given: On a chilly winter morning, the furnace has gone out and the temperature is 40° F. The furnace is started for 30 minutes.

Find: Heating rate if final temperature of the house is 70° F.

Solution: Using Equation 1, the elapsed time $t_2 - t_1$ is 30 minutes. The change in temperature $T_2 - T_1$ is 70° F - 40° F = 30° F. Then the heating rate is 30° F/30 minutes or 1° F per minute.

larly examples of cooling are common. Food placed in the refrigerator will cool from room temperature to the temperature of the refrigerator.

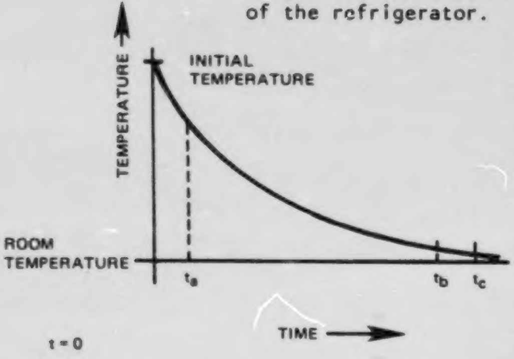


FIG. 1

In most cases the rate of heating or cooling will not be constant. For example, if a pan of hot water sits in a room to cool, the rate of cooling will be highest at first, when the water is still well above room temperature. When the water approaches room temperature, the cooling rate becomes slower. This is sketched schematically in Figure 1, which shows the cooling rate at various times. The rate of cooling is highest at $t = 0$. As the temperature comes closer to room temperature, the cooling rate decreases. In the elapsed time from $t = 0$ to $t = t_a$, the temperature drops considerably. In an equal elapsed time interval from t_b to t_c, when the sample temperature has come closer to room temperature, the change in temperature is less, and therefore, the cooling rate is lower (Example B).

EXAMPLE B: COOLING OF THE EARTH'S SURFACE

Given: Data included in the narrative solution.

Find: The cooling of the earth's surface after sunset. The decrease in temperature after sunset on a clear, windless night is due to removal of surface heat by radiation. This leads to a decrease in temperature which is proportional to the square root of the elapsed time after sunset. (This result is simply stated here without proof.) The temperature, T, is given by the equation:

$$T = T_s - K\sqrt{t}. \tag{3}$$

Solution: In this equation T_s is the temperature at sunset, K is a factor of proportionality which describes the cooling rate and t is the elapsed time after sunset. Consider a night on which the temperature at sunset is 70° F and the temperature at dawn, assumed to be 12 hours later, is 50° F. Equation 3 yields a curve of the form shown in Figure 2. This shows the temperature as a function of time during the night. The initial rate of decrease of temperature is fairly fast but becomes slower. The lowest temperature is reached just before dawn.

This, of course, is an approximation, and factors such as winds, clouds, motion of air masses and so on can change the shape of the curve from the idealized curve shown in Figure 2.

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If a substance undergoes a change of phase at a certain temperature, that is, if it melts or freezes, boils or condenses, the form of the heating/cooling curve giving temperature as a function of time will be different. This is illustrated in Figure 3, and would apply to a sample of ice initially at a temperature below 0° C, subjected to a constant heating rate starting at time t = 0. From time 0 to t_a (section A of the graph in Figure 3) the ice is warming at a constant rate until, at time t_a, it reaches 0° C. At that point, the temperature remains constant while heat energy is used to change the phase of the substance. The heat energy delivered to the ice is used to melt the ice. The temperature does not rise until all the ice is melted (section B of the curve in Figure 3). At time t_b, after all the ice is melted, the temperature will begin to rise again (section C of the curve). The heating rate is different than that in section A because the specific heat of liquid water is different from the specific heat of ice. This type of behavior is a common phenomenon observed whenever there is melting, freezing, boiling or condensation of a vapor and a break in the cooling curve and the temperature will remain constant until the change of phase is completed. Thus, the mixture of water and ice remains at the constant temperautre 0° C until all the ice is melted.

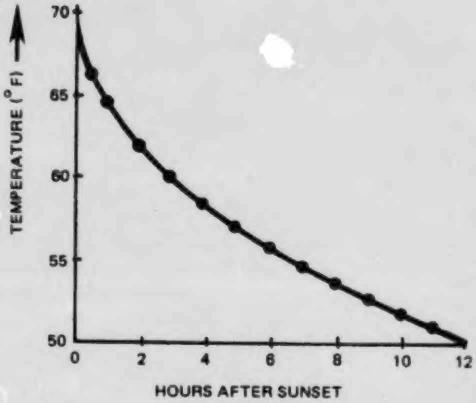

FIG. 2

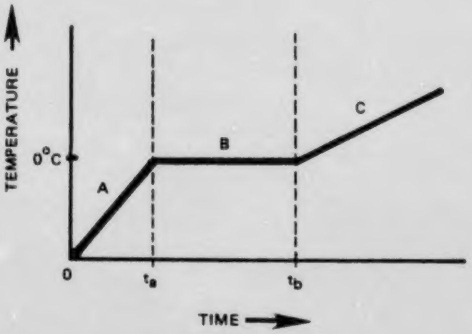

FIG. 3

Heat can be transferred from one place to another by three main processes:

Conduction.
Convection.
Radiation.

The first of these, conduction, involves transfer of heat energy through a material. The material can be solid, liquid or gaseous, and the heat is transferred through the material by atomic motions.

Convection occurs when heat is transferred through physical motion of a body of fluid, either a liquid or a gas. An example of convection by a liquid is provided by the operation of a hot water house furnace. Water is heated in the boiler and is transferred through pipes to a radiator where it gives up its heat, returning to the furnace to be reheated. This is an example of heat transmission by physical motion of the heated fluid, that is by convection.

The third way in which heat may be transmitted from one place to another involves no material. This method is called radiation. Heat transfer by radiation is familiar, as the energy in sunlight is transferred by radiation, and the heat delivered by the sun can be felt. Surfaces of all bodies emit energy by radiation and the amount of energy radiated from a surface increases sharply as the temperature of the surface increases. An example would be heat energy radiated from a fireplace.

LABORATORY

MATERIALS

Bunsen burner.
Beaker.
Thermometer 10° C to 110° C.
Stopwatch.
Double-walled beaker.
Antifreeze (ethylene glycol base).
Stand to support beaker over burner.
Graph paper.

PROCEDURE

1. Heat a beaker of water on burner until it reaches boiling temperature. Remove flame and record temperature and start the stopwatch. Keep the stopwatch running as the temperature is recorded every 20 seconds.
2. Measure temperature at recorded time intervals until the temperature of water is within ten degrees of room temperature. Plot temperature as a function of time on a piece of graph paper. Calculate the cooling rate for each pair of adjacent time intervals. Plot the cooling rate as a function of time on a piece of graph paper.
3. Next prepare a mixture of 50% antifreeze and 50% water, place this in the outer section of a double-walled beaker. Fill the inner section with plain water. Heat the entire apparatus over the burner. The experimental arrangement is sketched in Figure 4.
4. Gently stir with the thermometer continuously, and measure the temperature of the antifreeze-water mixture. Observe the temperature as a function of time, using the stopwatch, as the water in the inner beaker heats to its boiling point and then begins to boil away. Take temperature readings at appropriate time intervals.
5. Continue taking temperature readings while the water in the inner beaker is boiling. The temperature in the outer portion of the beaker should begin to rise again after all the water is gone. Stop heating the beaker before the temperature in the outer beaker reaches 227° F, the temperature at which a 50% mixture of antifreeze and water will begin to boil.
6. Plot the temperature of antifreeze-water mixture as a function of time on a piece of graph paper. From pairs of ad-

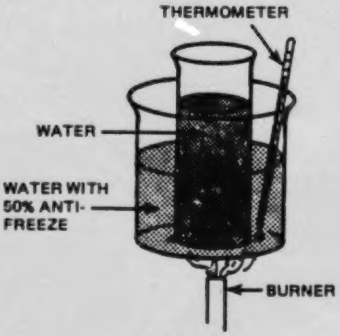

FIG. 4

jacent temperature measurements at different times, calculate the heating rate, using Equation 2. The elapsed time will be time between successive measurements of temperature. Then plot the heating rate versus time.

7. Interpret the form of the curve obtained. Discuss why the heating rate dropped to a lower value while the water in the internal part of the beaker was boiling.

SELF CHECK

1. State the mathematical formula for the heating or cooling rate. What are the appropriate units? How does one determine if the rate is to be interpreted as heating or cooling?
2. State three methods by which heat energy is transferred to produce heating or cooling.
3. Suppose that one places a bowl of jello in the refrigerator to cool. The initial temperature of the jello is 200° F. After 5 minutes its temperature is 150° F; after 10 minutes 110° F; after 15 minutes 80° F; after 20 minutes 60° F; after 25 minutes 50° F; after 30 minutes 44° F; after 35 minutes 40° F; and after 40 minutes 38° F. Plot the cooling rate versus time.
4. Suppose that one places a steel ingot in a high temperature furnace. In the early stages of the heating, the heating rate is nearly constant. If after the first 10 seconds the temperature of the ingot goes from 70° F (room temperature) to 200° F, what is the temperature of the ingot after 20 seconds?
5. What was the initial temperature of a cup of coffee if, after 30 minutes, the coffee is at a temperature of 27°C and the cooling rate (assumed to be linear) is 0.04 C°/s?
6. Assume that the rate at which a metal ingot of temperature 270°C cools when placed in a room (T = 20°C) is governed by the equation

$$R = -\ 0.1/\text{min}\ (T - 20°\text{C})$$

where T is the temperature of the ingot at a given time after it has started cooling. Find the rate of cooling when the temperature of the ingot is (a) 270°C, (b) 200°C, (c) 100°C, and (d) 25°C.

7. Plot a qualitative graph of temperature vs time for the situation described in No. 6. (Do not worry about the scale on the time axis.)

MOMENTUM

APPLICATION MODULES

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* Identifies concept support modules that introduce new physics.

4M1

ANGULAR MOMENTUM OF A FLYWHEEL

In any mechanical system involving rotational motion, angular momentum must be considered. One common, direct application of the concept of angular momentum is the flywheel. Flywheels are employed in most heat engines (internal combustion, steam) and in other types of mechanical systems where smooth, constant, rotary motion is desirable, such as tape players and stereo turntables.

In this module, the student will examine the concept of angular momentum. Angular momentum of several rotating systems will be calculated. In the laboratory, the angular momentum of a flywheel will be measured by two techniques and the results compared.

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Moment of inertia.
  - Angular momentum.
- Given mass, diameter, angular velocity and formulas, calculate the angular momentum and moment of inertia of a uniform disk and of a wheel with all its mass near its rim.
- Calculate angular momentum of a planetary body in circular orbit, given its mass, velocity and radius of its orbit.
- Determine angular momentum of a wheel by two methods and compare the results.

DISCUSSION

Before examining the concept of angular momentum, it is necessary to have an understanding of equations and quantities involved in describing circular motion. Table 1 lists some of the concepts already studied and introduces some new concepts. The first five entries should be familiar.

TABLE 1: ANALOG BETWEEN TRANSLATIONAL AND ROTATIONAL QUANTITIES

Concept	Translation	Rotation	Comments
1. Displacement	s	θ	$s = r\theta$
2. Velocity	$v = \frac{\Delta s}{\Delta t}$	$\omega = \frac{\Delta\theta}{\Delta t}$	$v = r\omega$
3. Acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta\omega}{\Delta t}$	$a = r\alpha$
4. Equations of motion	$v_2 = v_1 + at$ $s = v_1 t + 1/2\ at^2$ $v_2{}^2 = v_1{}^2 + 2as$	$\omega_2 = \omega_1 + \alpha t$ $\theta = \omega_1 t + 1/2\alpha t^2$ $\omega_2{}^2 = \omega_1{}^2 + 2\alpha\theta$	acceleration constant
5. Force, torque	$F = \frac{\Delta P}{\Delta t}$	$\tau = \frac{\Delta L}{\Delta t}$	$\tau = Fr$
6. Mass, moment of inertia	$m = \frac{F}{a}$	$I = \frac{\tau}{\alpha}$	
7. Momentum	$P = mv$	$L = I\omega$	$L = Pr$

In entry six, mass is defined as the force applied to an object divided by the acceleration caused by that force. The analogous concept in circular motion is called the moment of inertia. Moment of inertia is defined by Equation 1.

$$\text{Moment of inertia} = \frac{\text{torque}}{\text{angular acceleration}} \qquad (1)$$

In symbols: $I = \tau/\alpha$.

In the next entry, linear moment is defined as the product of mass and velocity. The corresponding concept of angular momentum is defined by Equation 2.

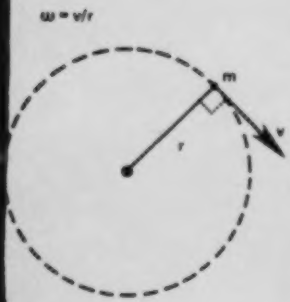
Angular moment = moment of inertia x angular velocity (2)

As $v = \omega r$.

In symbols: $L = I\omega$.

$\omega = \frac{v}{r}$.

Figure 1 shows a particle of mass m traveling with velocity v in an orbit of radius r. The linear momentum is mv. Angular momentum is the product of the linear momentum and the radius of the orbit.



$$L = mvr$$
$$= m(r\omega)r$$
$$L = m\omega r^2$$

From Equation 2:

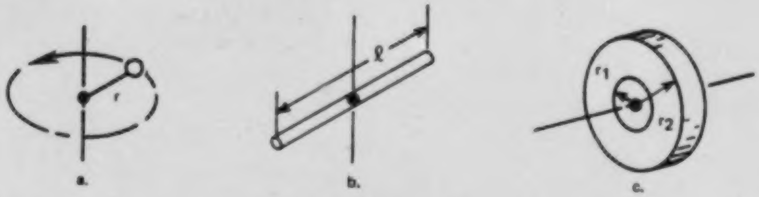
$$I = \frac{L}{\omega}$$
$$= \frac{m\omega r^2}{\omega}$$
$$I = mr^2$$

As $L = m\omega r^2$.

FIG. 1

Thus, the moment of inertia of a particle in a circular orbit is mr^2. Using this formula, it is possible to determine the moment of inertia of any uniform object. Figure 2 gives the moments of inertia of several common shapes.

FIG. 2



Particle in circular orbit
$I = mr^2$

Slender rod, axis through center
$I = \frac{1}{12} m\ell^2$

Annular cylinder
$I = \frac{1}{2} m \left(r_1^2 + r_2^2\right)$

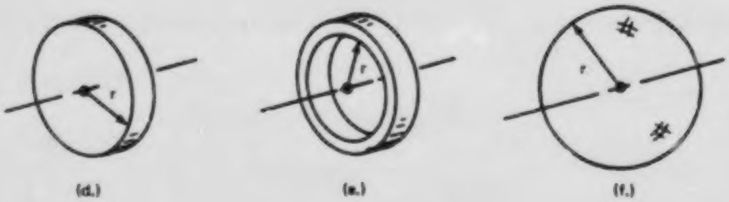


FIG. 2 (cont.)

(d.)

Solid cylinder
$I = \frac{1}{2} mr^2$

(e.)

Thin-walled hollow cylinder (wheel)
$I = mr^2$

(f.)

Solid sphere
$I = \frac{2}{5} mr^2$

Example A shows the use of Figure 2 and Equation 2 in solving a problem.

EXAMPLE A: ANGULAR MOMENTUM OF A DISK

Given: A uniform disk of radius 10 cm and mass 2 kg rotating about an axis through its center with an angular velocity of 0.5 rad/s.

Find: Its angular momentum.

Solution:

$$L = I\omega$$
$$= (\tfrac{1}{2}mr^2)\ \omega \quad \text{since } I = \tfrac{1}{2}mr^2$$
$$= (0.5)(2\ \text{kg})(0.1\ \text{m})^2(0.5/\text{s})$$
$$L = 0.005\ \frac{\text{kg} \times \text{m}^2}{\text{s}}$$

$$\text{Angular momentum} = 0.005\ \frac{\text{kg} \times \text{m}^2}{\text{s}}$$

Example B shows use of the equations in Table 1 in solving a more complex problem.

EXAMPLE B: FINAL ANGULAR MOMENTUM OF A DISK AND MASS

Given: The solid disk shown with a small pin on its circumference that holds a string in place at point A. The string is attached to a 0.1-kg mass. When the disk is released, it will begin to turn as the weight falls. The string will slip off the pin when the pin reaches point B. When this was done in the laboratory, 0.67 seconds elapsed between the release of the wheel and the pin reaching point B.

Find: Final angular momentum of the disk.

Solution A:

$\omega_1 = 0$ rad/s

$t = 0.67$ s

$G = 9.8$ m/s^2

As the weight falls, it falls through a distance ℓ.

$\ell = v_1t + 1/2\ at^2$

but $v_1 = 0$, therefore $v_1t = 0$

$\ell = 1/2\ at^2$

$a = \frac{2}{t^2}$

$= \frac{2\ (0.2\ \text{m})}{(0.67\ \text{s})^2}$

ause pin and mass are tied
ether, their acceleration
the same.

$a = 0.891$ m/s^2

ular acceleration
the disk

$\alpha = \frac{a}{r}$

$= \frac{0.891\ \text{m/s}^2}{0.15\ \text{m}}$

$\alpha = 5.94$ rad/s^2

$\omega_2 = \omega_1 + at$

but $\omega_1 = 0$

al angular velocity
disk

$\omega_2 = \alpha t$

$= (5.94\ \text{rad/s}^2)(0.67\ \text{s})$

$\omega_2 = 3.98$ rad/s

ember that "radian" is
a unit and can be
oped.

$I = 1/2\ mr^2$

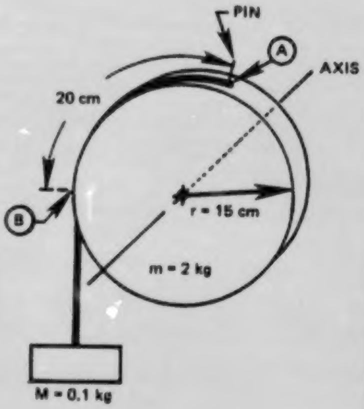
$= (0.5)(2\ \text{kg})(0.15\ \text{m}^2)$

$I = 0.0225\ \text{kg} \times \text{m}^2$

$L = I\omega$

$= (0.0225\ \text{kg} \times \text{m}^2)(3.98\ \text{rad/s})$

$L = 0.0895\ \frac{\text{kg} \times \text{m}^2}{\text{s}}$



Solution B:

The problem can be solved by an alternate method if $T = \frac{\Delta L}{\Delta t}$ from Table I is used. This gives:

$L = \tau t$

$\tau = Fr$, where F is force applied at point A

$F = MG - Ma$

$= M(G - a)$

$= (0.1\ kg)(9.8\ \frac{m}{s^2} - 0.891\ \frac{m}{s^2})$

$F = 0.891\ \frac{kg \times m}{s^2}$

$\tau = Fr$

$= (0.891\ \frac{kg \times m}{s^2})(0.15\ m)$

$\tau = 0.1337\ \frac{kg \times m^2}{s^2}$

$L = \tau t = (0.1337\ \frac{kg \times m^2}{s^2})(0.67\ s)$

$L = 0.0896\ \frac{kg \times m^2}{s}$

Thus, the problem can be solved by either method.

MG is gravitational force applied. Ma is the portion of that force used to accelerate that weight.

LABORATORY

MATERIALS

Bicycle wheel with tire.
100-g weight.
1-m length of string.
Meter stick.
Laboratory timer or stopwatch.
Short, headless pin.
Spring scale.

PROCEDURE

1. Using scale, determine mass of bicycle wheel. Assume that all mass is located at rim. Record mass in data table.
2. Measure and record radius of wheel.
3. Insert pin into tire as shown in Figure 3, perpendicular to tire surface and with only about 2 mm exposed.
4. Tie a loop in one end of string. Measure and mark a distance of 50 cm from loop. Tie 100-g weight to other end of string.
5. Arrange equipment as shown in Figure 4.
6. Simultaneously release wheel and start timer. Stop timer when string slips from pin. You may need to do this several times to get a good average value of time span.
7. Record all data; complete calculations portion of data table. (Refer to Example B.)

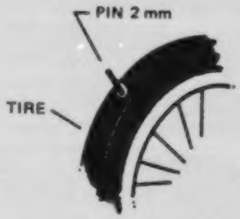


FIG. 3

DATA TABLE

DATA

Mass of bicycle wheel: m = __________ kg

Mass of weight: m = __________ kg

Radius of wheel: r = __________ m

Distance for uniform acceleration: s = __________ m

Time for uniform acceleration: t = __________ s

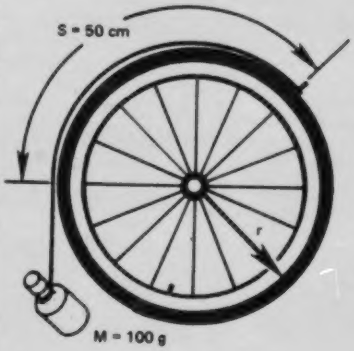


FIG. 4

CALCULATIONS

Moment of inertia: $I = mr^2 =$ _____ kg x (_____ m)^2

$I =$ _____ kg x m^2.

Acceleration of string: $a = \frac{2s}{t^2}$

$a = 2$ x _____ m ÷ (_____ s)^2

$a =$ _____ m/s^2.

Angular acceleration: $\alpha = \frac{a}{r}$

$\alpha =$ _____ m/s^2 ÷ _____ m

$\alpha =$ _____ rad/s^2.

Final angular velocity: $\omega = \alpha t$

$\omega =$ _____ rad/s^2 x _____ s

$\omega =$ _____ rad/s.

Angular momentum (first method): $L_1 = I\omega$

$L_1 =$ _____ kg x m^2 x _____/s

$L_1 =$ _____ kg x m^2/s.

Tangential force on wheel: $F = M\ (g - a)$

$F =$ _____ kg (9.8 m/s^2 - _____ m/s^2)

$F =$ _____ kg x m/s^2.

Torque: $\tau = Fr$

$\tau =$ _____ kg x m/s^2 x _____ m

$\tau =$ _____ kg x m^2/s^2.

Angular impulse: $L_2 = \tau t$

$L_2 =$ _____ kg x m^2/s^2 x _____ s

$L_2 =$ _____ kg x m^2/s.

Recall from Module 5-0:

Angular impulse = angular momentum

$$\%\ \text{variation} = \frac{\text{angular impulse - angular momentum}}{\text{average}} \times 100.$$

SELF CHECK

1. Define:
   - Moment of inertia.
   - Angular momentum.
2. A uniform disk has a mass of 3.6 kg and a diameter of 0.6 m. What is the moment of inertia? ($I = 1/2\ mr^2$.)
3. A wheel of radius 12 cm has a mass of 0.68 kg. If all mass is considered to be at the rim, what is its moment of inertia? ($I = mr^2$.)
4. A 100-g weight is swung at constant speed in a circle at the end of the 1-m string. It takes 1.2 seconds for one complete revolution. What is its angular momentum?
5. A dish 32 cm in diamter has a mass of 4.8 Kg. It is spinning at 1500r/min. Calculate its' angular momentum using $L = I\omega$ and $L = \tau t$ and compare the results.

4M2

IMPULSE AND MOMENTUM

Impulse is the product of force and the duration of application of the force. An impulse applied to a mass causes a change in momentum of the mass. Excessive applied forces may deform the object permanently. Application of smaller forces over a longer duration can achieve the desired change in momentum, but with reduced mechanical stress. This module discusses impulse and change in momentum as it applied to mechanical components and systems of modern technology. In the laboratory, the student will measure impulse and change in momentum.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define impulse and momentum.
- Derive the relationship between impulse and momentum from Newton's second law.
- Given three of the following quantities, calculate the fourth:

  Mass of object.
  Change in velocity.
  Applied force (constant).
  Duration of application of force.

- Estimate the impulse from a graph of force-versus-time by approximating the area under the force-versus-time curve.
- Explain the importance of considering impulse in the design of machine parts.

- Given the appropriate equipment, measure the impulse and momentum change when a force accelerates a mass.

Newton's second law (Equation 1) states the relationship of a constant force, mass, and acceleration as follows:

$$F = ma \qquad (1)$$

where
F = force.
m = mass.
a = acceleration.

An applied force causes an acceleration of a mass. Acceleration is the change in velocity divided by the time interval over which that change occurs. Thus, Newton's second law may be written in the form of Equation 2.

$$F = m\frac{\Delta v}{\Delta t} \qquad \text{(substituting } a = \frac{\Delta v}{\Delta t}\text{)}$$

or

$$F\Delta t = m\Delta v \quad \text{(multiplying each side by } \Delta t\text{)} \qquad (2)$$

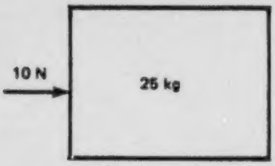
where
Δv = change in velocity.
Δt = time interval over which Δv occurs.

The quantity of the left side of Equation 2 is called the "impulse." It is the product of the applied force and the duration of action of that force. "Momentum" is the product of the mass of an object and its velocity. The quantity on the right side of Equation 2 is mass time change in velocity and represents change in momentum. Thus, Equation 2 may be stated as —

Impulse = Change in Momentum.

Notice that both impulse and momentum are vector quantities. Examples A and B show the use of Equation 2 in solving problems:

EXAMPLE A: CALCULATION OF CHANGE IN VELOCITY	
Given:	A force of 10 N is applied to a 25-kg mass for a time of 2 s.



Find: (a) Impulse
(b) Change in velocity.

Solution: (a) Impulse is defined as $F\Delta t$.

$$\begin{aligned} F\Delta t &= m\Delta v \\ &= (10\ \text{N})(2\ \text{s}) \\ F\Delta t &= 20\ \text{N}\cdot\text{s, to the right} \end{aligned}$$

(b)

$$\begin{aligned} F\Delta t &= m\Delta v \\ \Delta v &= \frac{F\Delta t}{m} \\ &= \frac{20\ \text{N}\cdot\text{s}}{25\ \text{kg}} \\ &= 0.8\ \frac{\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot\text{s}}{\text{kg}} \\ \Delta v &= 0.8\ \text{m/s, to the right} \end{aligned}$$

EXAMPLE B: CALCULATION OF DURATION OF FORCE

Given: A constant force of 5 N is applied to a 25 kg mass and changes its velocity by 10 m/s.

Find: The duration of the applied force.

Solution:

$$\begin{aligned} F\Delta t &= m\Delta v \\ \Delta t &= \frac{m\Delta v}{F} \\ &= \frac{(25\ \text{kg})(10\ \text{m/s})}{5\ \text{kg}\cdot\text{m/s}} \\ \Delta t &= 50\ \text{s} \end{aligned}$$

Figure 1 represents a constant force F_1 applied to an object and the velocity of that object as functions of time. The impulse of the force is the area $F_1\Delta t_1$. This impulse causes a momentum change of $m\Delta v$, where m is the mass of the object and Δv is the change in its velocity. The acceleration is the slope of the velocity of $\frac{\Delta v}{\Delta t_1}$.

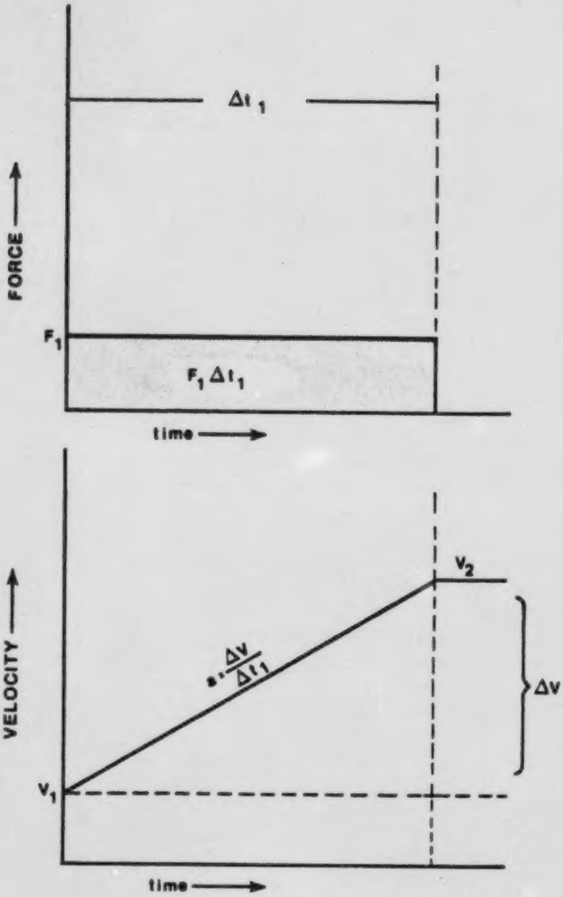

FORCE
Δt 1
F1
F1 Δt1
time
VELOCITY
v2
a = Δv/Δt1
Δv
v1
time

FIG. 1

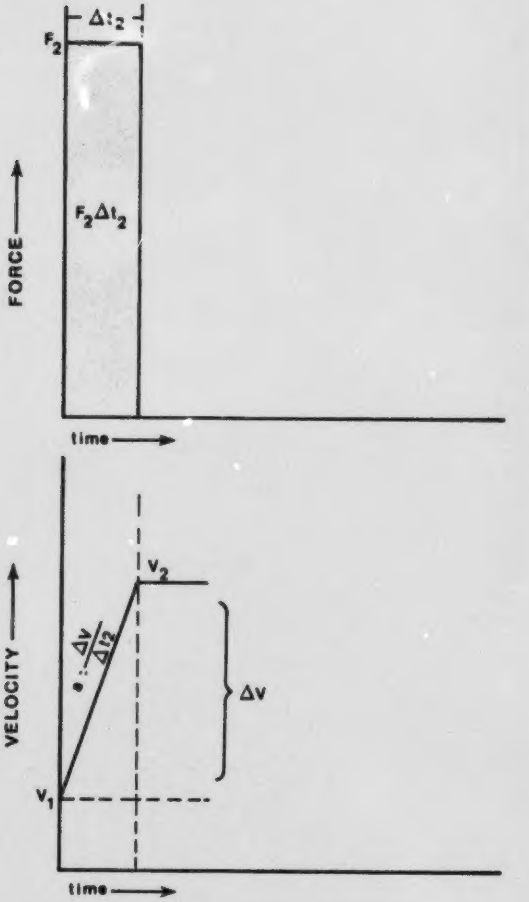

Δt2
F2
FORCE
F2Δt2
time
VELOCITY
v2
a = Δv/Δt2
Δv
v1
time

FIG. 2

Figure 2 shows a larger force F_2 applied for a shorter period Δt_2 to produce the same total impulse and the same net change in momentum ($m\Delta v$). Equal areas (shaded) under the force-versus-time curves represent the same impulse and cause the same change in momentum. If an applied force F is increased a hundred times (to 100F) and the time of application Δt is decreased to $\Delta t/100$, the impulse $F\Delta t$ equals $100F \frac{\Delta t}{100}$, so that both the impulse and the change in momentum remain the same.

The stress on the object being accelerated is proportional to the applied force. If the applied force is large enough, deformation may occur, although the change in velocity remains the same. A machine brought up to speed rapidly may be over-stressed. Reduction of the force and increase in the duration of application can achieve the desired change in momentum, but with reduced stress.

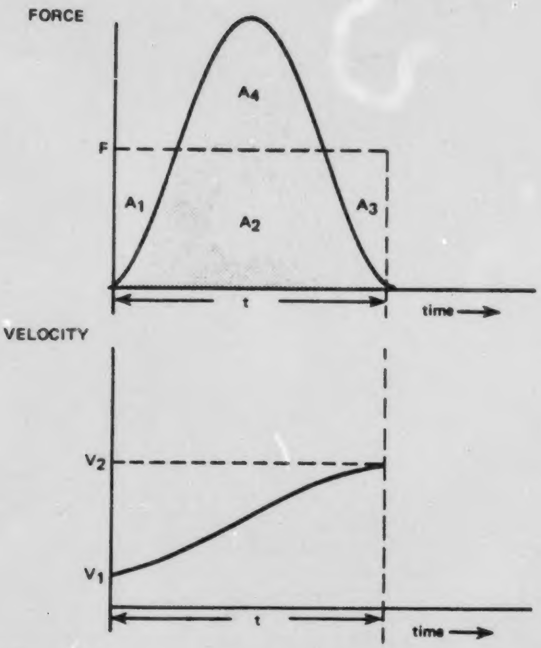


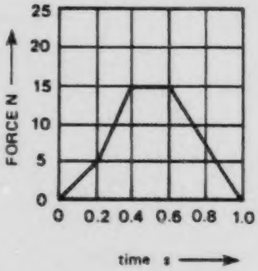
FIG. 3

A machine whose motion is described in Figure 1 has an initial velocity v_1. At the instant the force F_1 is applied, the acceleration changes, almost abrubtly, from zero to $a = F_1/m$. This effect is experienced as "jerk," which can be eliminated by gradually increasing the force — and, thus, the acceleration — from zero to the desired level.

The accelerating force applied to an elevator, for example, changes smoothly from zero to maximum and back to zero to reach the desired velocity, without the abrupt change in acceleration. This situation is shown in Figure 3. The impulse is the same as in Figures 1 and 2; but in the curve described in Figure 3, there is no abrupt jerk in the motion. The impulse depicted in Figure 3 is the sum of the areas $(A_2 + A_4)$. The impulse associated with a constant force F_1 (dotted line in Figure 3) is the sum of the areas $(A_1 + A_2 + A_3)$. These impulses are equal if $(A_1 + A_3 = A_4)$.

Example C demonstrates how to determine the change in the momentum of an object from an impulse graph.

EXAMPLE C: IMPULSE GRAPH



Given: A 10-kg object initially at rest is subjected to the force described in the graph shown.

Find: Final velocity of the mass.

Solution: The area of one square of the graph is 5N x 0.20 s = 1 N·s. The area under the impulse curve is 8.5 squares.

$$\text{Impulse} = 8.5 \text{ squares} \times 1 \text{ N·s/square}$$
$$F\Delta t = 8.5 \text{ N·s}$$
$$F\Delta t = m\Delta V$$
$$\Delta v = \frac{F\Delta t}{m}$$
$$= \frac{8.5 \text{ kg·m/s}}{10 \text{ kg}}$$
$$\Delta v = 0.85 \text{ m/s}$$

In an automotive engine, a cam working against a valve spring smoothly changes the acceleration of the valve. If the acceleration is too large or abrupt, the valve may "float" and not remain in contact with the cam, causing damage to the engine. Use of the proper cam shape and the maintaining of engine speed below the design limit decreases the applied force (and acceleration) and increases the duration of acceleration.

Newton's third law states that for every action, there is an equal and opposite reaction; therefore, if object A imparts an impulse to B, then B imparts an equal and opposite impulse to A. This law indicates that deformation of both objects may occur if the stress produced exceeds their elastic limits. Care must be taken to avoid this in all mechanical systems. Aircraft wings are subject to variable impulse loadings during flight, due to turbulent air or landings. Little can be done about the magnitude of the impulse during these situations; however, the force element of impulse can be reduced by allowing the wings to flex, thereby increasing the duration of the impulse. This controlled deformation serves as a shock absorber. Aircraft landing gears are designed to include shock-absorbing systems that for a given impulse increases the duration Δt of the applied force and, consequently, reduces the magnitude of the applied force. The elasticity of the shroud lines of a parachute serve this shock absorbing function and make the sudden opening of the chute and reduction of momentum more tolerable.

In crash situations, final momentum will be reduced to zero.

The impulse transmitted to the occupants, therefore, is equal to their initial momentum. The time must be lengthened to reduce the force applied to the occupants. This lengthening of time can be done, to some extent, by designing the vehicle to deform in a controlled manner. (Incidentally, inflatable air bags between the driver and the automobile frame are designed to do just that.)

Fluids in motion also possess momentum. The fluid in a pipeline may have a very large momentum because of its mass. If a valve is closed suddenly, the impulse applied to the valve by the change in momentum of the fluid may rupture the pipe at the valve head. For this reason, the flow in a large pipeline is shut off slowly. When water in the home is turned off quickly, the pipe may make a hammering noise because of the large forces resulting when the momentum of the flowing water is changed in a short time.

In rotational systems, the corresponding quantities are angular impulse and angular momentum. The angular impulse is the product of the torque and the duration of the application of that torque and is equal to the change in angular momentum. The same general principles apply to rotational systems for reducing wear and the probability of mechanical failure. The clutch or automatic transmission of an automobile slips upon the initial application of the engine torque to the drive train. This slippage reduces the torque in the drive train by allowing the momentum changes to occur over a longer period of time.

SUMMARY

When a force acts to accelerate an object, the velocity of the object and its momentum change. The impulse of the force is the product of the magnitude of the force and the duration of its application and is equal to the change in momentum of the object. Mechanical parts experience strain and wear proportional to the forces applied to them. When the momentum of a part is changed, minimum wear occurs if a minimum force is used to make that change. A common consideration in all areas of mechanical design is the reduction of strain through necessary momentum changes by applying smaller forces for longer times.

IMPULSE AND CHANGE IN MOMENTUM

MATERIALS

Airtrack.
Cart.
Photocell timer.
String.
Pulley.
Weight.
Balance.
Meterstick.

PROCEDURE

1. Measure and record length (ℓ) and mass (m_1) of cart and mass (m_2) of weight.

2. Set up equipment as illustrated in Figure 4. Photocell timer should be positioned in such a manner that weight hits floor before cart reaches timer. Be sure air track is level.

3. Hold cart in place. Measure and record height (h) of weight.

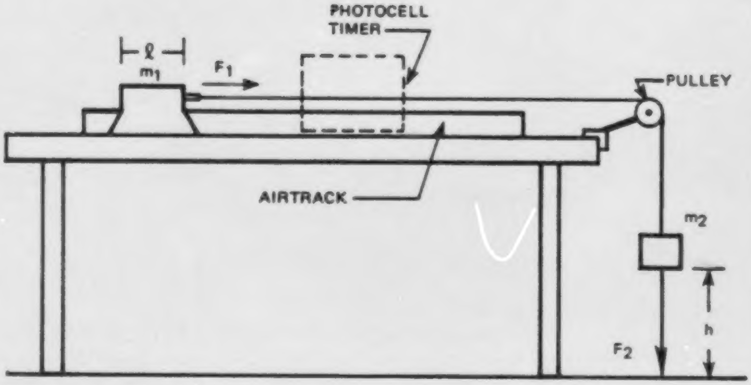


FIG. 4

4. Release cart. Record time (t) for cart to pass through photocell gate.

5. Complete calculations in Data Table to determine impulse and final momentum of cart.

6. Draw a graph of force versus time for this experiment, and show that the area under the curve represents both the impulse and the change in momentum.

7. If m_2 were three times as great, how would the results be affected?

DATA TABLE

Data:

Mass of cart: m_1 = ________

Mass of weight: m_2 = ________

Length of cart: ℓ = ________

Height of weight: h = ________

Time of cart through gate: t = ________

Calculation of Impulse:

Force causing motion of system: $F_2 = m_2 g$ = ________

Acceleration of system: $a = \frac{F}{m} = \frac{m_2 g}{(m_1 + m_2)}$ = ________

Force on cart: $F_1 = (m_1)\left(\frac{m_2 g}{m_1 + m_2}\right) = \left(\frac{m_1 m_2}{m_1 + m_2}\right) g$

F_1 = ________

Time for acceleration: $\Delta t = \sqrt{\frac{2h}{a}}$ = ________

Impulse: $F_1 \Delta t$ = ________

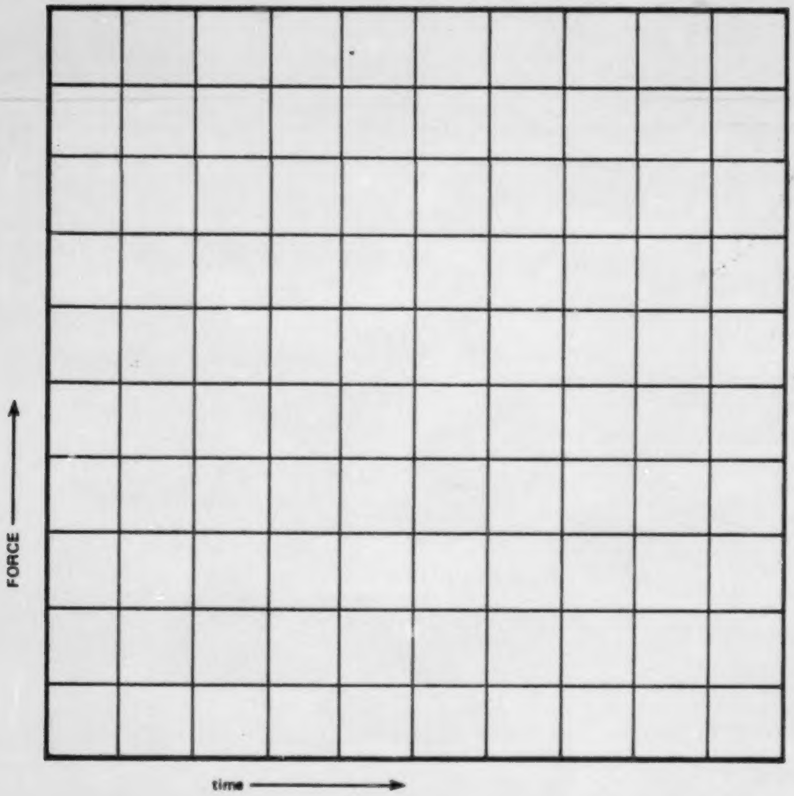
Calculation of Momentum Change:

Velocity of Cart: $v = \frac{\ell}{t}$ = ________

Final momentum of cart: $p = mv$ = ________

Momentum change = Δmv = ________

Impulse ____________________ Δ Momentum ______________________



SELF CHECK

1. Define:

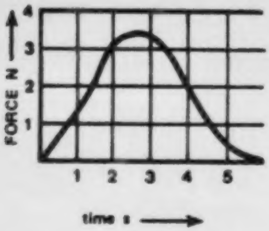
    a. Impulse.
    b. Momentum.

2. Derive the relationship of impulse to momentum change from the following:

    Force = Mass x Acceleration

3. A force of 16N acts on a 2-kg mass for 3.5 s. Determine the change in the velocity of the mass by —

   a. Finding its momentum change.
   b. Finding the acceleration of the mass.



4. The force described by the adjoining graph was applied to an object initially at rest. The area under the curve is measured to be roughly 10.5 Ns. Its final velocity was 16 m/s. Find its mass.

5. How long must a 50 N force be applied to a 25-kg mass to change its momentum by 1000 kg•m/s?

6. Describe the importance of considering impulse when designing parts for high-speed mechanical systems.

4M4

GYROSCOPES

A gyroscope is a mechanical device that consists of a spinning wheel, called a "rotor," mounted on an axle. The ends of the axle are supported in a mounting stand. The unique properties of this device are based upon the "gyroscopic effect." The gyroscopic effect can be observed in such simple motions as riding a bicycle or spinning a top, but also provides the operating principles for complex navigational aids.

In this module, the student will learn how the properties of the gyroscope depend upon torque and angular momentum. The student will solve problems involving these parameters and gain an understanding of the vector nature of the gyroscopic effect.

In the laboratory, the student will observe the effects of external forces on a spinning gyroscope. The student also will use the gyroscopic effect to determine the angular momentum of a spinning gyroscope. Finally, the student will construct a simple gyrocompass.

OBJECTIVES

Upon satisfactory completion of this module, the student should be able to:

- State the right-hand rule for determining the direction of a rotational vector.
- Label a diagram of a spinning gyroscope with the following vectors:

  Angular velocity, $\vec{\omega}$.
  Angular momentum, $\vec{L}$.
  Force, $\vec{F}$, acting on the gyroscope.

Torque, $\vec{\tau}$, acting on the gyroscope.
Angular momentum change, $\Delta\vec{L}$.
Angular velocity of precession, $\vec{\Omega}$.

- Given the magnitudes of two of the following vectors, find the magnitude of the third:

  Angular momentum, $\vec{L}$.
  Torque, $\vec{\tau}$.
  Angular velocity of precession, $\vec{\Omega}$.

- List the major parts of a gyrocompass, and explain the function of each.

- List the two methods for stabilizing a gyrocompass. Explain the effect of each on the magnitude of the torque $\vec{\tau}$, or the angular momentum $\vec{L}$.

- Find the direction of the gyroscopic effect on a moving vehicle.

- Given the appropriate equipment, demonstrate that the spin axis of a gyroscope tends to retain a fixed direction in space (in the absence of any torque on the gyroscope).

- Given the appropriate equipment, demonstrate that the direction of precession of a gyroscope is always perpendicular to the applied force.

- Given the necessary equipment, measure the precession rate of a spinning gyroscope and the torque acting upon it. Use these values to calculate the angular momentum of the gyroscope.

DISCUSSION

Before the gyroscopic effect can be discussed, a framework within which the pertinent rotational quantities can be treated as vectors must be developed. This requires a system for determining both the magnitude and the direction of such vector quantities as torque, angular velocity, and angular momentum.

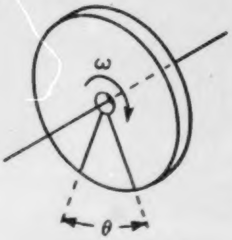


FIG. 1

Equations for determining the magnitudes of these quantities have been presented in previous modules. The magnitude of the angular velocity of the flywheel of Figure 1, for instance, can be calculated from Equation 1.

$$\omega = \frac{\theta}{t} \qquad (1)$$

where

ω = average angular velocity in rad/s.
θ = angle through which rotating body turns in time t; measured in rad/s.
t = time during which rotating body advances through angle θ.

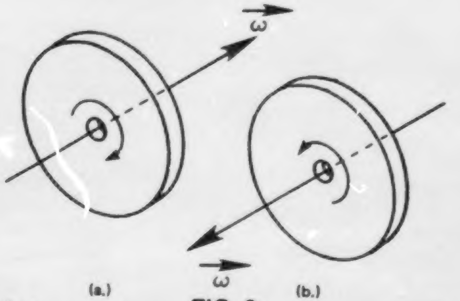


FIG. 2

The vector nature of angular velocity is evident from the fact that the flywheel can rotate in either of two directions — clockwise or counterclockwise. The vector representing $\vec{\omega}$ must then point in one direction to signify clockwise rotation and a different direction to signify counterclockwise rotation. The standard means of representing this vector is illustrated in Figure 2. The angular velocity vector of the clockwise-rotating flywheel lies along the axis of rotation pointing away from the viewer (Figure 2a). The angular-velocity vector of the counter-clockwise-rotating flywheel also lies along the axis of rotation, but points toward the viewer (Figure 2b).

In general, the vector associated with any rotational quantity lies along the axis of rotation in a direction determined by the right-hand rule. Wrap the fingers of the right hand around the axis of rotation in the direction of rotation. The extended thumb points in the direction of the rotational vector. Figure 3 indicates how to find the direction of $\vec{\omega}$ for the rotating flywheel using the right-hand rule.

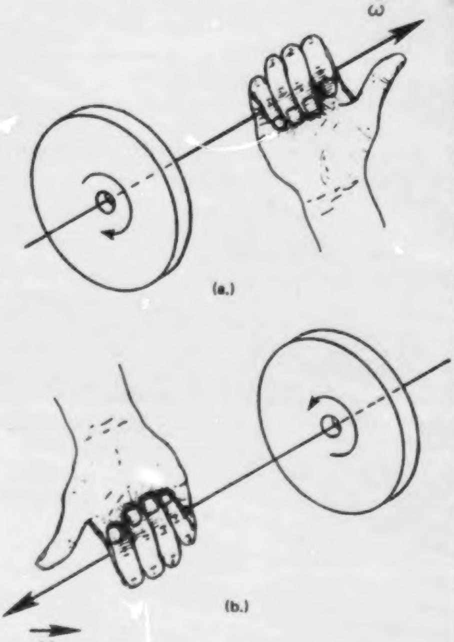


FIG. 3

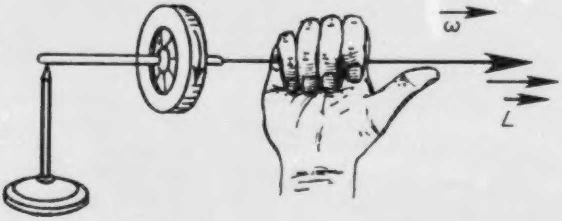
This is a convenient point at which to introduce two new symbols to represent vectors aligned perpendicular to the plane of the page. The symbol ⊙ represents a vector pointing straight up out of the page toward the reader. The symbol ⊗ represents a vector pointing straight down into the page, away from the reader.

Example A shows how the basic vector $\vec{\omega}$, $\vec{L}$, and $\vec{\tau}$ can be found for a spinning gyroscope supported at one end.

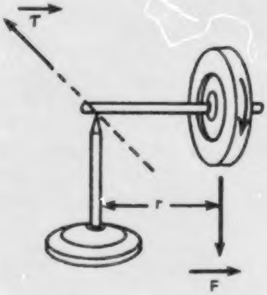
EXAMPLE A: ANGULAR VELOCITY, ANGULAR MOMENTUM, AND TORQUE ON A SPINNING GYROSCOPE

Given: A gyroscope of mass 100g, moment of inertia 900g cm^2, and angular speed 3000 r/min (rpm) is supported as shown. The distance from the end of the gyroscope to its center of gravity is 4.50 cm.

Find:
(a) Its angular velocity $\vec{\omega}$ in rad/s.
(b) Its angular momentum $\vec{L}$.



(c) The torque $\vec{\tau}$ acting upon it.



Solution: (a) To find the magnitude of $\vec{\omega}$ convert the units from rpm to rad/s.

$$\omega = 3000 \text{ rpm}$$

$$= (3000 \frac{r}{min})(2\pi \frac{rad}{r})(\frac{1 \text{ min}}{60 \text{ s}})$$

$$= \frac{6000\pi \text{ rad}}{60 \text{ s}}$$

$$\omega = 314 \text{ rad/s}$$

The direction of $\vec{\omega}$ is found by wrapping the fingers of the right hand around the spin axis in the direction of rotation, as shown. The angular velocity vector now can be represented as follows:

$$\xrightarrow{\quad \vec{\omega} \quad} \text{, where } |\omega| = 314 \text{ rad/s}$$

(b) Magnitude of $\vec{L}$:

angular momentum = moment of inertia x angular velocity,

$$L = I\omega \qquad \text{(Equation 4)}$$

$$= (900 \text{ g cm}^2)(314 \text{ rad/s})$$

$$L = 2.83 \times 10^5 \ \frac{\text{g cm}^2}{\text{s}}$$

Direction of $\vec{L}$:

The equation $L = I\omega$ is a vector equation, and indicates that the vector $\vec{L}$ equals the scalar I times the vector $\vec{\omega}$. Thus, L and ω have the same direction. The vectors $\vec{\omega}$ and $\vec{L}$ are shown below:

$$\xrightarrow{\quad \vec{\omega} \quad} \quad \xrightarrow{\quad \vec{L} \quad} \qquad |L| = (2.83)(10^5 \ \frac{\text{g cm}^2}{\text{s}})$$

(c) Magnitude of τ:

torque = force x lever arm,

$$\tau = Fr$$

The force on the gyroscope is its weight, so

force = mass x acceleration of gravity,

$$F = mg$$

$$= (100 \text{ g})(980 \ \frac{\text{cm}}{\text{s}^2})$$

$$F = 9.80 \times 10^4 \text{ dynes}$$

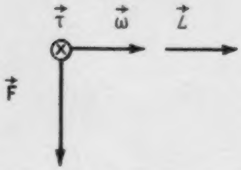
Then:

$$\tau = (9.80 \times 10^4 \text{ dynes})(4.50 \text{ cm})$$

$$\tau = 4.41 \times 10^5 \text{ dyne cm}$$

Direction of $\vec{\tau}$:

The force $\vec{F}$ acting upon the moment arm r, as shown in the figure above, tends to cause a rotation of the gyroscope assembly in a clockwise direction — as viewed by the reader. The product of F and r is equal to the torque. Thus, when the right-hand rule is applied to the figure shown above, the torque vector τ has the direction along the dashed line, as shown, away from the viewer. The vectors $\vec{\omega}$, $\vec{\tau}$, $\vec{L}$, and F are shown below:



PRECESSION

One of the most fascinating aspects of the gyroscope effect is that the gyroscope does not fall off the support. Instead, the free, unsupported end of the gyroscope moves in a horizontal circle with the support at its center. This motion is called "precession." The rate at which the spin axis precesses around the support is called the "angular velocity of precession," $\vec{\Omega}$.

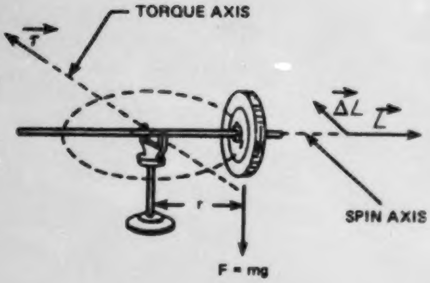


Figure 4 shows a gyroscope precessing around its supported end.

To understand precession, one may begin with the relation between the angular impulse $\tau\Delta t$ and the angular momentum change $\Delta\vec{L}$. This already has been presented as Equation 7 in Concept Module 4-0:

$$\Delta L = \tau\Delta t \qquad (2)$$

where

ΔL = change in angular momentum (angular impulse).
τ = torque.
Δt = elapsed time.

Since $\vec{\tau}$ and $\Delta\vec{L}$ are vectors, the corresponding vector relationship is given by Equation 3.

$$\Delta\vec{L} = \vec{\tau}\Delta t \qquad (3)$$

Equation 3 states that the effect of a torque $\vec{\tau}$ acting upon the gyroscope for a length of time equal to Δt is to change the angular momentum of the gyroscope by an amount $\Delta\vec{L}$. Furthermore, the angular momentum changes in the direction of the applied torque. Example B shows how $\Delta\vec{L}$ is calculated for the gyroscope of Figure 4.

EXAMPLE B: $\overrightarrow{\Delta L}$ FOR A GYROSCOPE

Given: The gyroscope of Figure 4 has a mass m of 0.2 kg and a lever arm r of 0.05 m.

Find: Its angular momentum change $\Delta\vec{L}$ after an elapsed time of 0.03 seconds.

Solution: (a) Calculate the magnitude of the torque acting:

$$\begin{aligned}\tau &= Fr\\ &= mgr\\ &= (0.2\ \text{kg})(9.8\ \text{m/s}^2)(0.05\ \text{m})\\ \tau &= 0.098\ \frac{\text{kg m}^2}{\text{s}^2}\end{aligned}$$

The direction of $\vec{\tau}$ is found by applying the right-hand rule around the torque axis.

(b) Find the magnitude of $\Delta\vec{L}$:

$$\begin{aligned}\Delta L &= \tau\Delta t\\ &= \left(0.098\ \frac{\text{kg m}^2}{\text{s}^2}\right)(0.03\ \text{s})\\ \Delta L &= 2.94 \times 10^{-3}\ \frac{\text{kg m}^2}{\text{s}}\end{aligned}$$

From Equation 2, the directions of $\vec{\tau}$ and $\Delta\vec{L}$ are the same.

$\vec{\tau}$
L
$\overrightarrow{\Delta L}$

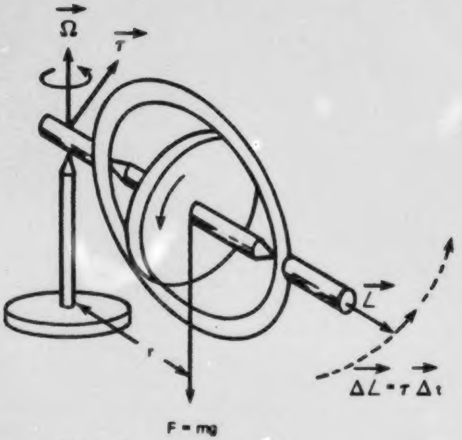


FIG. 5

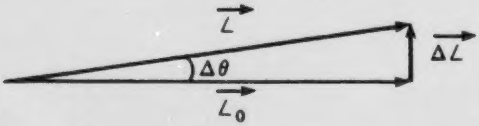


FIG. 6

Figure 5 shows a spinning gyroscope with its weight, torque, angular momentum, and angular momentum change vectors shown. The angular momentum change $\Delta\vec{L}$ after a short time Δt is given by Equation 3. The angular momentum $\vec{L}$ at the end of this time period must be the vector sum of the initial angular momentum $\vec{L}_0$ and $\Delta\vec{L}$:

$$\vec{L} = \vec{L}_0 + \Delta\vec{L}$$

Figure 6 shows these vectors added graphically. The final angular momentum $\vec{L}$ has approximately the same magnitude as $\vec{L}_0$, but is displaced through angle $\Delta\theta$ from $\vec{L}_0$. Since ΔL is perpendicular to L_0, the vector L_0 rotates but does not change in magnitude. Since $\vec{L}$ always lies along the spin axis, the spin axis rotates, or precesses, in the direction of $\Delta\vec{L}$. Notice that the torque vector rotates with $\Delta\vec{L}$, so that they remain parallel.

Figure 6 can be used to estimate the magnitude of the angular velocity of precession $\vec{\Omega}$, which is just the rate of change of angle $\Delta\theta$. From Figure 6 —

$$\tan(\Delta\theta) = \frac{\Delta L}{L}$$

Since $\Delta\theta$ is a very small angle,

$$\tan(\Delta\theta) \cong \Delta\theta \text{ (in radians)}$$

Then —

$$\Delta\theta = \frac{\Delta L}{L}$$

$$\Delta\theta = \frac{\tau\Delta t}{L} \quad (\text{since } \Delta L = \tau\Delta t)$$

$$\frac{\Delta\theta}{\Delta} = \frac{\tau}{L} \quad (\text{Both sides are divided by } \Delta t.)$$

$$\Omega = \frac{\tau}{L} \quad (\text{since } \Omega = \frac{\Delta\theta}{\Delta t})$$

Example C shows how to find the magnitude and direction of $\vec{\Omega}$ for a precessing gyroscope.

EXAMPLE C: ANGULAR VELOCITY OF PRECESSION FOR A GYROSCOPE

Given: The gyroscope of Example A ($m = 100$ g, $I = 900$ g cm^2, $r = 4.50$ cm) spins clockwise at the rate of 3000 rpm.

Find: Its angular velocity of precession $\vec{\Omega}$.

Solution: (a) Find $\vec{\tau}$:

$$\tau = Fr$$

$$= mgr \quad \text{(since } F = mg\text{, the gyroscope's weight)}$$

$$= (100\text{ g})(980\text{ cm/s}^2)(4.50\text{ cm})$$

$$\tau = 4.41 \times 10^5\ \frac{\text{g cm}^2}{\text{s}^2}$$

The direction of τ is found by applying the right-hand rule around the torque axis.

(b) Find $\vec{L}$:

$$L = I\omega \quad \text{(Equation 4)}$$

$$= (900\text{ g cm}^2)(3000\text{ rpm})$$

$$= (900\text{ g cm}^2)\left(3000\ \frac{\text{rev}}{\text{min}}\right)\left(2\pi\ \frac{\text{rad}}{\text{r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$L = 2.83 \times 10^5\ \frac{\text{g cm}^2}{\text{s}}$$

The direction of $\vec{L}$ is found by applying the right-hand rule around the spin axis.

(c) Calculate Ω:

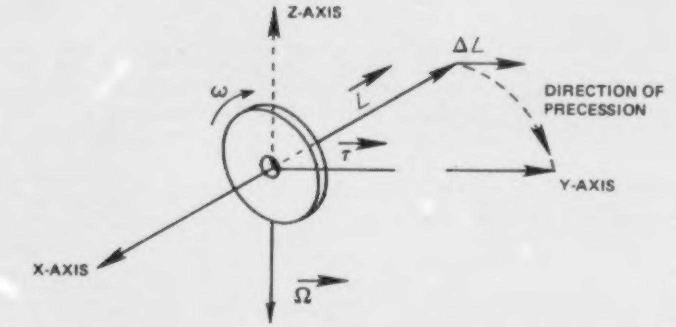
$$\Omega = \frac{\tau}{L}$$

$$= \frac{4.41 \times 10^5\ \frac{\text{g cm}^2}{\text{s}^2}}{2.83 \times 10^5\ \frac{\text{g cm}^2}{\text{s}}}$$

$$= \left(1.56\ \frac{\text{g cm}^2}{\text{s}^2}\right)\left(\frac{\text{s}}{\text{g cm}^2}\right)$$

$$= 1.56\ \frac{1}{\text{s}}$$

$$\Omega = 1.56\ \frac{\text{rad}}{\text{s}}$$


Z-AXIS
ΔL
ω
$\vec{L}$
DIRECTION OF PRECESSION
$\vec{\tau}$
Y-AXIS
X-AXIS
$\vec{\Omega}$

The direction of precession at each instant is the same as the direction of $\Delta\vec{L}$. Equation 3 indicates that $\Delta\vec{L}$ and $\vec{\tau}$ point in the same direction; so the gyroscope must precess in the direction of $\vec{\tau}$. Application of the right-hand rule around the precession axis indicates that the vector $\vec{\Omega}$ points down. (The gyroscope precesses along the dotted arc shown in the previous figure.)

GYROSCOPIC STABILITY

Examination of Equation 4 shows that as the torque on the gyroscope becomes smaller, Ω also decreases. If the torque is reduced to zero in a model such as shown in Figure 8, the gyroscope stops precessing; and the spin axis remains rigidly fixed in space. This aspect of the gyroscopic effect is called "gyroscopic stability."

A bicycle or motorcycle being ridden demonstrates gyroscopic stability and precession. The spinning wheels of the cycle are gyroscopes. With the rider's weight positioned in the same vertical plane as the wheels (as in Figure 7a), there is no torque exerted on the wheels. The precessional velocity is zero, and gyroscopic stability keeps the bicycle upright. This is why the act of riding a bicycle is a very simple task, but balancing on a stationary cycle is very difficult. When the rider leans to either side, his or her weight exerts a torque that acts upon the gyroscopes (wheels). The wheels precess, and the bicycle turns right or left (Figure 7b). When we learn to ride a bicycle, we do this without thinking.

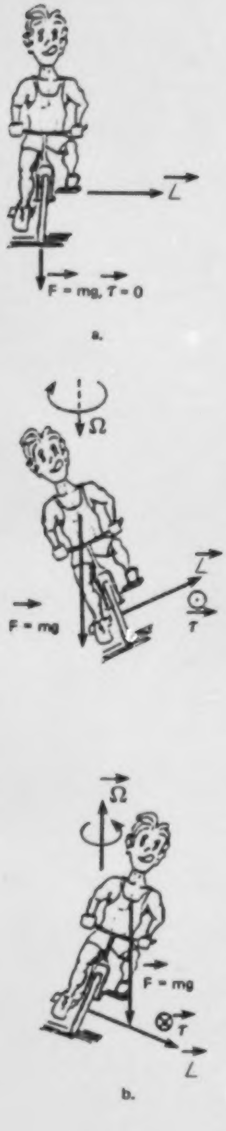

$\vec{L}$
$\vec{F} = mg, \vec{\tau} = 0$
a.
Ω
$\vec{F} = mg$
$\vec{L}$
$\odot$
$\vec{\tau}$
$\vec{\Omega}$
$\vec{F} = mg$
$\otimes\vec{\tau}$
$\vec{L}$
b.

FIG. 7

Gyroscopic stability allows the use of the gyroscope as a direction reference. A gyroscope mounted in such a manner that there is no torque acting upon it maintains its spin axis in the same direction indefinitely. This phenomenon is the basic operating principle of the gyrocompass diagrammed in Figure 8.

The gyroscope is mounted inside pivoted rings called "gimbals," which allow the base to be moved in any direction without exerting a torque upon the gyroscope. The inner gimbal allows the base to be turned left or right without exerting a torque upon the gyroscope. The outer gimbal allows the base to be tilted forward or backward without exerting a torque on the gyroscope. The gyroscope and gimbals are pivoted on jeweled bearings to reduce friction.

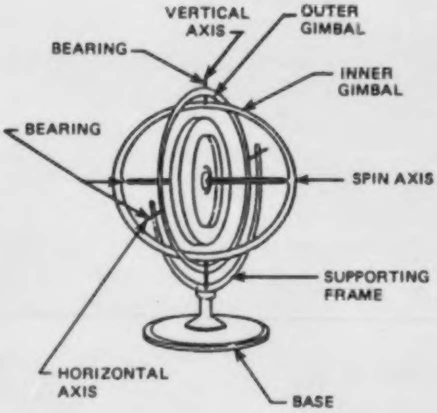


FIG. 8

In practice, no matter how finely the bearings are ground, they cannot be completely frictionless. Friction causes a small torque to be exerted on the gyroscope whenever the base is moved, resulting in precession, which rotates the spin axis out of alignment. This effect is minimized by making the amount of inertia of the rotor as large as possible and by spinning it at a very high angular velocity. Each of these steps serves to increase the angular momentum of the gyroscope. According to Equation 4, an increase in L causes Ω to decrease, making the precession less severe.

The gyrocompass is thus stabilized in two ways:

1. Gimbal mounting makes τ in Equation 4 very small.
2. Large values of I and ω make L in Equation 4 very large.

The gyrocompass used on most ships has a rotor 11 inches in diameter and 55 lbs in weight. It is spun by an electric motor at about 6000 rpm.

GYROSCOPIC EFFECT

In addition to the navigational uses of the gyrocompass, the gyroscopic direction finder has many applications. It is used in road mapping to measure accurately the curves in highways. It can measure curves and grades of sections of tract in railroad surveying. Gyroscopes also are attached to oil-drilling equipment to determine and to control the direction of drilling.

The gyroscopic effect often intrudes detrimentally on the operation of commonplace devices. The rotating flywheel of an engine is a gyroscope with a very large moment of inertia. The effects of this large gyroscope are seen in the flight characteristics of single-engine aircraft. Figure 9a shows an airplane in level flight. The propeller and engine flywheel rotate clockwise (as viewed from the pilot's seat). The angular momentum $\vec{L}$ is found by application of the right-hand rule around the spin axis. Figure 9b shows the vectors associated with the aircraft as it goes into a dive. The force $\vec{F}$ pulls the nose down. The torque $\vec{\tau}$ is found by applying the right-hand rule around the torque axis, which causes precession around the precession axis in the direction of $\Delta\vec{L}$. Thus, the aircraft veers left when the nose is dropped.

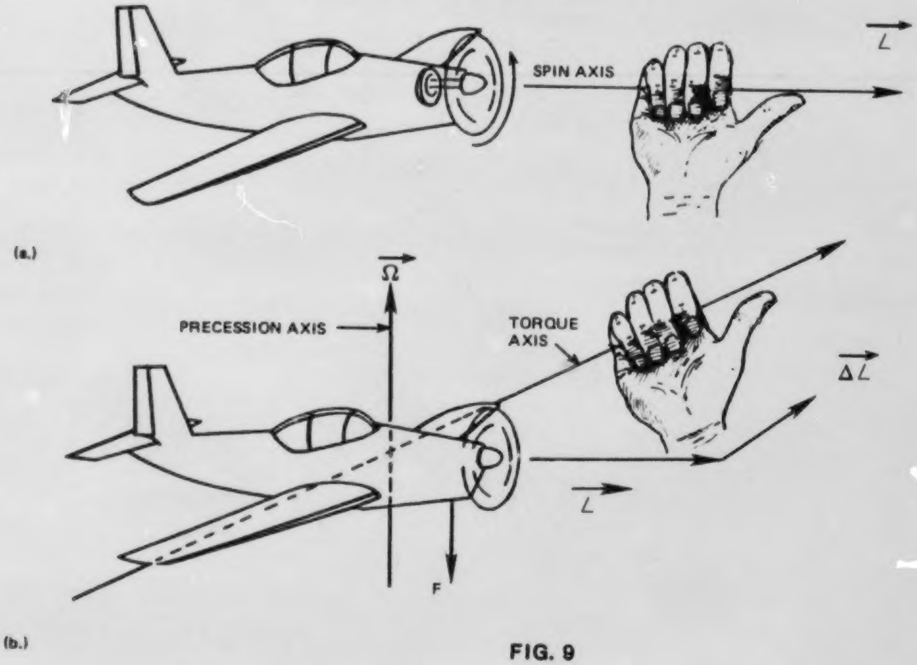


FIG. 9

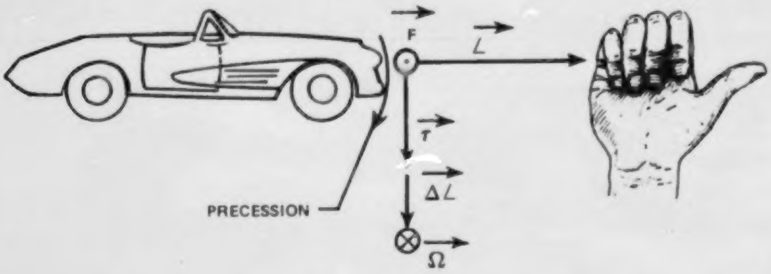
This effect is most noticeable when the angular momentum of the engine is small, as readily seen from Equation 4, since L appears in the denominator. The moment of inertia of the flywheel is constant, so the angular momentum can be changed only by changing the angular velocity of the engine. Therefore, the effect is most pronounced at low engine speeds (small angular momentum).

In twin-engine aircraft, the gyroscopic effect is nullified if each engine is designed to rotate in opposite directions. If the left engine rotates clockwise, the right engine rotates counterclockwise. This difference in direction causes the precessional motion of the engines to be equal in magnitude and opposite in direction, so that there is no net precession.

The gyroscopic effect of an engine in operation is illustrated in Example D.

EXAMPLE D: THE CORNERING AUTOMOBILE

Given: An automobile engine (and flywheel) rotates clockwise (viewed from the driver's seat). The auto makes a sharp right turn.



Find: The gyroscopic effect acting on the automobile.

Solution: Use the right-hand rule to find the direction of the angular momentum $\vec{L}$. The force on the auto is to the right, causing downward torque. The angular momentum change $\Delta\vec{L}$ is also downward. Since the precession is in the same direction as $\Delta\vec{L}$, the front end of the auto drops as the car goes around the corner.

SUMMARY

The gyroscope is a wheel and axle mounted in gimbals in such a manner that it is free to turn in any direction. When the wheel is caused to turn rapidly, it remains in a fixed position (by conservation of angular momentum), even if the base is moved. If a force is applied, however, at right angles to the axis of rotation, "precession" occurs. Precession is the movement of the axis at right angles to the rotation vector and to the torque applied to the axis. The gyroscopic effect is exhibited by bodies rotating at sufficient speed; these bodies include engine crankshafts. This effect is employed in such instruments as gyrocompasses (which indicate true north), direction indicators, automatic pilots, horizon indicators, and turn-and-bank indicators.

LABORATORY

THE GYROSCOPIC EFFECT

MATERIALS

Demonstration gyroscope.
Unsharpened pencil.
String.
Stand or mount from which gyroscope can be hung.

PROCEDURE

1. Set the gyroscope spinning, and hang it with its spin axis horizontal. (See Figure 10.) Record in Data Table 1 the direction of spin (clockwise or counterclockwise as viewed along the spin axis from the front end of the spin axis).
2. Push near or front end of spin axis to left with pencil. Record direction in which gyroscope tends to precess. Repeat with pushes to the right, up and down.
3. Verify observed precessions for each of the above cases by drawing vector diagrams. Each diagram should include $\vec{L}$, $\vec{F}$, $\vec{\tau}$, $\vec{\Delta}$, and $\vec{\Omega}$.
4. Explain "precession."

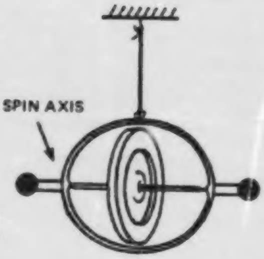


FIG. 10

PRECESSION RATE OF A GYROSCOPE

MATERIALS

Demonstration gyroscope and support.
Needle marker.
Meter stick.
Timer with accuracy of 0.1 seconds or better.
Balance with accuracy of 1 g or better.

PROCEDURE

1. Measure and record in Data Table 2 the distances r_1 (from one end of gyroscope to edge of rotor) and d (the thickness of rotor). (See Figure 11.)

2. Use balance to measure mass of gyroscope. Record this value.

3. Start gyroscope spinning. Record direction of spin.

4. Place one end of gyroscope on support, and allow it to begin precessing. In order to start gyroscope off with pure precession, push outer end gently in direction of precession.

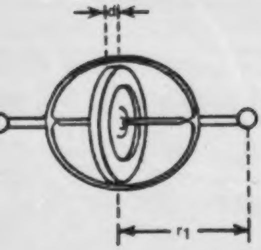


FIG. 11

5. Allow gyroscope to precess until spin axis is horizontal. Place a needle marker next to stand. (See Figure 12.) When axis of gyroscope precesses past marker, start timer. Stop timer when gyroscope next passes over needle marker. Record elapsed time.

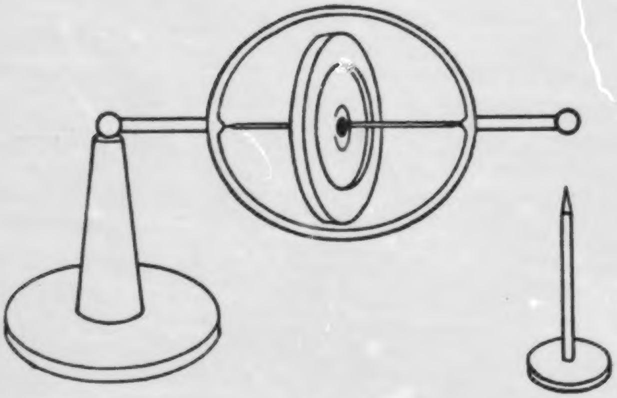


FIG. 12

6. Draw a vector diagram to verify the direction of precession.

7. Calculate weight F of gyroscope.

8. Calculate lever arm r of gyroscope.

9. Calculate magnitude of torque τ.

10. Calculate angular velocity of precession Ω.

11. Calculate angular momentum L of gyroscope.

GYROSCOPIC DIRECTION REFERENCE

MATERIALS

Demonstration gyroscope.
Two needle markers.
String.
Stand or mount on which gyroscope can be hung.

PROCEDURE

1. Hang spinning gyroscope with its spin axis horizontal, as illustrated in Figure 13. Mark the position of each end of axis with a needle marker.
2. Unhook string from mounting stand, and walk around room holding string from which gyroscope is suspended.
3. Return to mounting stand, and rehang gyroscope. The ends of spin axis still should be aligned with needle markers.
4. Why do the ends of the spin axis remain aligned?

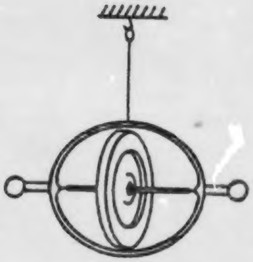


FIG. 13

DATA TABLE 1: THE GYROSCOPIC EFFECT

Direction of spin: ________________.

Direction of push	Direction of precession
Left	____________________
Right	____________________
Up	____________________
Down	____________________

DATA TABLE 2: PRECESSION RATE OF A GYROSCOPE

Data:

r_1 = ______________ cm $\qquad$ d = ______________ cm

Mass = m = ______________ g

Direction of spin: ______________

Elapsed time = t = ______________ s

Angular displacement, $\theta = 2\pi$ rad

Calculations:

$F = (m)(980\ \frac{cm}{s^2}) =$ ______________ $\frac{g\ cm}{s^2}$

$r = r_1 + (1/2)d =$ ______________ cm

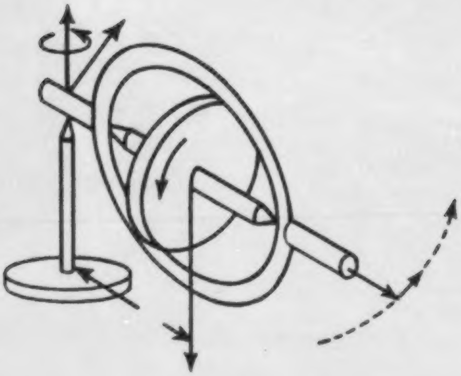
$\tau = Fr =$ ______________

$\Omega = \theta/t =$ ______________ rad/s

$L = \tau/\Omega =$ ______________ $\frac{g\ cm^2}{s}$

SELF CHECK

1. State the right-hand rule for determining the direction of a rotational vector.



2. The rotor of a gyroscope spins as shown at left. Label the diagram with the following vectors:

   Angular velocity, $\vec{\omega}$.
   Angular momentum, $\vec{L}$.
   Force (weight), $\vec{F}$.
   Torque, $\vec{\tau}$.
   Angular momentum change, $\Delta\vec{L}$.
   Angular velocity of precession, $\vec{\Omega}$.

3. A spinning gyroscope precesses at the rate of 0.3 rad/s and has angular momentum 5000 rpm. Find the torque acting upon the gyroscope. Moment of inertia, $I - 900\ \text{g}\cdot\text{cm}^2$.

4. List the major parts of a gyrocompass, and explain the function of each.

5. List the two methods of stabilizing a gyrocompass. Explain the effect of each upon the magnitude of $\vec{\tau}$ or $\vec{L}$.

6. The aircraft of Figure 9a turns to the left. Find the direction of the gyroscopic effect on the aircraft.

RESISTANCE

APPLICATION MODULES

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* Identifies concept support modules that introduce new physics.

5M2

EFFECTS OF LUBRICANTS

When any two surfaces in contact move against each other, friction is present. The term "friction," as pointed out in Concept Module 5-0, refers to any resistance to this movement. In some applications, friction can be useful; in others, extremely harmful. Nails and screws depend upon friction to hold them in place. Fan belts and brakes are other examples of devices that depend upon friction to function properly. Friction in the moving parts of engines, on the other hand, can be very harmful. As friction in an engine increases, useful output power decreases. Increased friction causes heat, and heat causes expansion. Ultimately, if this heat is not removed, pistons or bearings may "freeze," or "lock up." In some cases, heat buildup may be sufficient to cause melting of bearings or other moving parts.

This module explains how lubricants overcome or reduce friction. It also lists some of the basic properties of lubricants. In the laboratory, the student will investigate the effects of various lubricants upon static and kinetic friction.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the coefficient of friction.
- List and explain three factors that contribute to friction.
- Define cohesion and adhesion, and describe how each is reduced by lubricants.
- List three common types of friction, and state which ones are reduced effectively by lubricants.
- State three reasons for lubricating moving parts.
- List three essential characteristics of lubricants.
- Given the appropriate equipment, demonstrate how the coefficient of friction is changed with the use of lubricants.

CAUSES OF FRICTION

There are three principal causes of friction in solids. They are:

Surface finish.
Cohesion and adhesion of molescules.
Weight or force holding the surfaces together.

SURFACE FINISH

Microscopic inspection of surfaces that appear smooth to the eye reveals surface irregularities. The interlocking of these irregularities prevents, because of friction, two surfaces from sliding over each other (with ease). Figure 1 shows what two relatively smooth surfaces look like if greatly magnified.

Surface finish of a material is a major factor in causing friction. These surface irregularities cause a grinding action as the surfaces move over each other. Motor oil in an automobile is changed periodically to remove the material ground off in this process. This grinding action also can cause considerable heating.

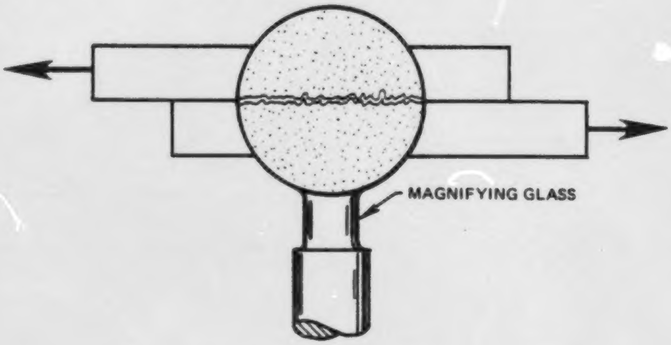


FIG. 1

MOLECULAR ATTRACTION

When two surfaces of the same type of material are in contact, molecules of one surface have an attraction for the molecules of the other surface. This attraction between like molecules is called "cohesion," which can be demonstrated by placing two pieces of flat glass against each other and pressing to remove the air between them. Cohesion will provide enough friction to make difficult the sliding of the top piece of glass. Cohesion causes friction in similar materials. Lubricants reduce cohesion by providing a film between the two surfaces.

Some surfaces of different materials will also have a molecular attraction when placed in contact. Molecular attraction between unlike molecules is called "adhesion." Glue, adhesive tape, tar and paint are well known for their adhesive properties.

Soaps and detergents are specially formulated so that they will break down adhesive forces and allow dirt and impurities to be washed away. Detergents are added to some lubricants in order that impurities can be dissolved and carried away from moving parts by the lubricants.

WEIGHT OR FORCE

Everyday experience shows that to drag a 200 lb bag of sand across a floor is more difficult than to drag a 100 lb bag. The force of friction that causes such objects to be difficult to slide is expressed by Equation 1.

$$f = \mu N \tag{1}$$

where

f = force of friction.
μ = coefficient of friction.
N = normal force pressing the level surfaces together (weight).

Equation 1 shows that as normal force is increased, the force required to move the object is increased. Equation 1 is used in solving a problem in Example A.

EXAMPLE A: FORCE OF DRY SLIDING FRICTION	
Given:	A 150-pound metal tool box is pushed on a wood floor. There is a coefficient of friction value of 0.5 between the box and the floor.
Find:	The force required to keep the box sliding at a constant speed.
Solution:	$f = \mu N$ $= (0.5)\ (150)$ $f = 75$ pounds

KINDS OF FRICTION

There are three kinds of friction:

- Starting, or static, friction.
- Sliding, or kinetic, friction.

- Rolling friction.

Static friction is generally greater than sliding friction. Lubricants are effective in reducing static and sliding friction, but have little effect on rolling friction.

PURPOSE OF LUBRICANTS

Moving parts are lubricated to reduce friction, wear and heat. Lubricants are produced in many forms, the most common ones being oils, greases and graphite compounds. Each lubricant is used for a different purpose, but must meet three conditions:

- It must be able to withstand the heat generated during operation.
- It must flow freely and therefore keep a thin film between all moving parts.
- And it should prevent corrosion.

If a lubricant cannot withstand the heat of operation, it will soon vaporize or become so thin that its lubricating qualities are lost. Similarly, if a lubricant flows too slowly, it will not reach all the moving parts. Finally, if a lubricant causes a chemical reaction which corrodes the moving parts, it can cause permanent damage to the parts.

A lubricant separates moving parts and allows them to move on a thin film, as shown in Figure 2.

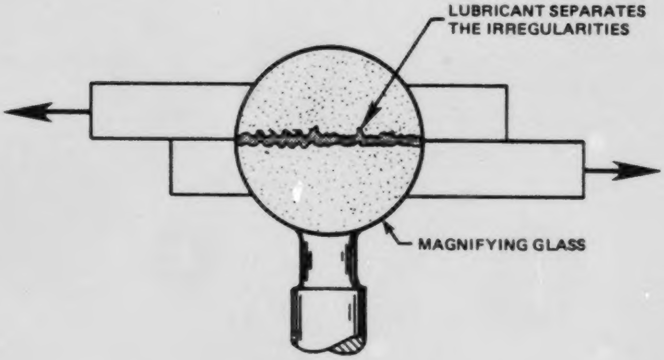


FIG. 2

SUMMARY

Lubricants help to overcome the resistance caused by friction between surfaces in contact. The most common lubricants are oils, greases, and graphite compounds - all of which must be able to withstand heat of operation, to flow freely and to prevent corrosion.

LABORATORY

THE EFFECT OF LUBRICANTS UPON STATIC AND KINETIC FRICTION

MATERIALS

Three rectangular blocks, approximately 4" x 4" x 1", with one large smooth surface (aluminum, cast iron, steel).
One large aluminum plate (6" x 18" minimum).
Three lubricants: oil (light-grade), oil (heavy-grade) and grease.
Spring balance (scaled in dynes).

PROCEDURE

In the following experiment, the student will investigate the effect of various lubricants on static and kinetic friction, using several surfaces.

1. Place aluminum block on aluminum plate, as shown in Figure 3.

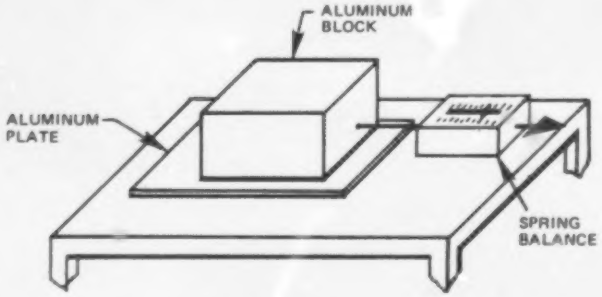


FIG. 3

2. Gently pull spring balance, and record in Data Table the static frictional force (f_s, in dynes) required to start block in motion.

3. Divide this force by weight (weight = mass x gravity) of the aluminum block. (In this case , normal force is just

weight of block.) The resulting number is the coefficient of static friction $\mu_s = \frac{f_s}{N}$.

4. Continue to pull aluminum block at a constant rate and record the required kinetic frictional force (f_k, in dynes). It should be less than f_s.
5. Divide this force by the weight of the aluminum block. This gives the coefficient of kinetic friction. $\mu_k = \frac{f_k}{N}$.
6. Replace aluminum block with cast iron, and repeat steps 1 through 5.
7. Replace cast iron with steel block, and repeat steps 1 through 5.
8. Record all values in appropriate boxes in Data Table.
9. Repeat previous steps in friction experiment, using grease as a lubricant on aluminum plate.
10. Calculate coefficient of static and kinetic friction, and record results in Data Table.
11. Remove grease from all parts, and clean surfaces. Dry all surfaces.
12. Repeat entire experiment using a thin film of heavy-grade oil as a lubricant.
13. Calculate coefficients of kinetic and static friction for this case, and record results in Data Table.
14. Again thoroughly clean and dry all parts.
15. Repeat the experiment, using a thin film of light-grade oil as a lubricant.
16. Calculate coefficients of static and kinetic friction as before, and record results in Data Table.
17. How do the three lubricants used in this procedure compare with reference to their effect upon friction?

DATA TABLE

	No Lubricant (Dry)			
Block	Static	μ_s	Kinetic	μ_k
Aluminum	F_s = ________		f_k = ________	
	N = ________		N = ________	
Cast iron	f_s = ________		f_k = ________	
	N = ________		N = ________	
Steel	f_s = ________		f_k = ________	
	N = ________		N = ________	

	Lubricant (Grease)			
Block	Static	μ_s	Kinetic	μ_k
Aluminum	f_s = ________		f_k = ________	
	N = ________		N = ________	
Cast iron	f_s = ________		f_k = ________	
	N = ________		N = ________	
Steel	f_s = ________		f_k = ________	
	N = ________		N = ________	

	Lubricant (Heavy-Grade Oil)			
Block	Static	μ_s	Kinetic	μ_k
Aluminum	f_s = ________		f_k = ________	
	N = ________		N = ________	
Cast iron	f_s = ________		f_k = ________	
	N = ________		N = ________	
Steel	f_s = ________		f_k = ________	
	N = ________		N = ________	

	Lubricant (Light-Grade Oil)			
Block	Static	μ_s	Kinetic	μ_k
Aluminum	f_s = ________		f_k = ________	
	N = ________		N = ________	
Cast iron	f_s = ________		f_k = ________	
	N = ________		N = ________	
Steel	f_s = ________		f_k = ________	
	N = ________		N = ________	

SELF CHECK

1. Write the equation defining the coefficient of friction, and identify each term.

2. Explain how each of the following contributes to frictional forces:

   Surface finish.
   Molecular attraction.
   Weight or pressure.

3. Define cohesion and adhesion, and explain how modern lubricants reduce each.

4. State the effect of lubricants on the three types of friction.

5. State three quantities that can be reduced by the use of lubricants.

6. List three essential characteristics of lubricants.

7. What is the mass of a wooden crate if the coefficient of sliding friction is 0.3 and a horizontal force of 40N is required to move the block across a horizontal surface with a constant speed?

8. A force of 20N is required to slide a 20 kg object at a constant speed along a track when the track is unlubricated. If lubricating the track decreases the coefficient of sliding friction by a factor of 10, what force is required to move the object along the lubricated track at constant speed?

9. The coefficient of sliding friction between a metal box and an unlubricated track is 0.2. When the track is lubricated the coefficient of sliding friction is 0.01. Find the ratio of the work required to move the box through a distance ℓ before lubrication to that required after lubrication.

10. What is the coefficient of static friction between a 35 kg wooden crate and a lubricated floor if a force of 30N applied at an angle of 45° above the horizontal just starts the crate in motion?

11. What is the coefficient of sliding friction between an object and a lubricated track if a force of 40N is required to move the box at a constant velocity along the track when it is not lubricated and a force of only 5N is required when the track is lubricated. The coefficient of sliding friction between the box and the unlubricated track is 0.3.

5M3

ROLLING FRICTION

Friction is a form of resistance present in all moving mechanical systems. Although it may be reduced by several means, it cannot be eliminated completely. Friction in a system can be reduced effectively if sliding-friction components are replaced with rolling-friction ones.

The physical mechanisms that produce rolling friction and the relation of these mechanisms to sliding friction will be examined in this module. Methods of reducing rolling friction will be discussed. In the laboratory, the student will determine experimentally the coefficient of rolling friction for several mechanical systems.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Discuss the influence of surface irregularities on sliding and rolling friction.
- Describe surface deformation and its effect upon rolling friction.
- Define coefficients of static and rolling friction.
- Given a body and the horizontal forces applied through its center to start and maintain a constant rolling velocity, calculate its coefficients of static and kinetic rolling friction.
- Given the appropriate equipment, design and carry out an experiment to determine coefficients of static and rolling friction.

DISCUSSION

Figure 1 shows an enlarged view of the surfaces of two objects in contact. As described in Concept Module 5-0, "Resistance," even the smoothest surfaces have small irregularities. These tiny hills and valleys lock together to produce a physical barrier to motion. The resulting resistance is called "sliding friction."

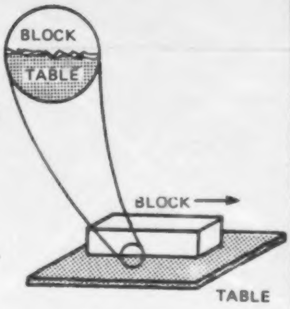


FIG. 1

Figure 2 shows how the effect of this type of friction is reduced when a round object rolls across a flat surface. The surface irregularities and friction still are present; but their effect is minimized, because the two surfaces do not slide. At the point of contact, they are motionless with respect to one another. The forward surface of the round object moves downward, causing the point of contact to move forward. At the same time, the rear surface is lifted away from the irregular surface of the flat object. Some friction still is present as the two surfaces make and break contact, but its effect is small compared with what is properly called "rolling friction."

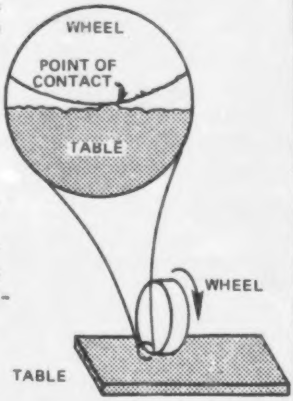


FIG. 2

Figure 3 illustrates the origin of rolling friction. In a real system, surfaces of the objects are not perfectly rigid, and some surface deformation occurs around the point of contact. At points A and B, the round object bulges slightly. From point C to point D, it is slightly flattened; and the plane surface on which it rests is indented. When the object rolls, both surfaces change shape as the point of contact moves forward. Internal resistance of objects to this change in shape produces rolling friction. Magnitude of the frictional force produced depends upon normal force between the objects and rigidity of their surfaces. Rigid surfaces (such as steel on steel) undergo less change in shape, producing smaller frictional forces than flexible surfaces, such as rubber on wood.

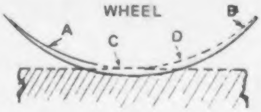


FIG. 3

Figure 4 shows the forces at work when a round object rolls across a plane surface at a constant velocity. The applied force F_a is horizontal and acts through the center of mass. The frictional force F_f is of equal magnitude, but opposite in direction. It acts tangent to the circular object at the point of contact. N is the normal force, equal and opposite to the weight of the round object. The torque produced by the force F_a about the point of contact causes the object to rotate. As with sliding friction, greater force is required to start the motion than to maintain it. The coefficients of static and kinetic rolling friction are defined by Equations 1 and 2.

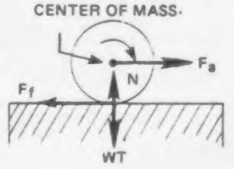


FIG. 4

$$\mu_s = \frac{F_s}{N} \qquad (1)$$

where

μ = coefficient of static rolling friction.

F_s = applied force to start rolling.
N = normal force.

$$\mu_k = \frac{F_k}{N} \qquad (2)$$

where

μ_k = coefficient of kinetic rolling friction.
F_k = applied force to maintain rolling friction.

Example A shows how to use Equations 1 and 2 in solving a practical problem.

In most practical cases, these coefficients are much smaller than those of sliding friction. In addition, the difference between static and kinetic coefficients of rolling friction is less than the corresponding difference for sliding friction.

In many mechanical systems, sliding friction exerted on shafts rolling in plain bearings can be reduced drastially when roller or ball bearings are installed, thereby replacing the sliding friction by a lesser rolling friction. In sliding friction, resistance to motion is reduced if the sliding parts are separated with a thin film of lubricating oil. Use of rolling friction bearings, however, simplifies both the lubrication and the wear problems. In Figure 5, two common types of rolling friction bearings are shown, one with ball bearings and the other with roller (cylindrical) bearings.

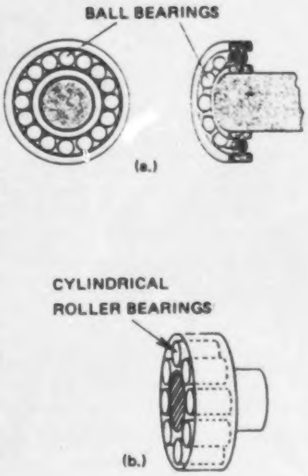
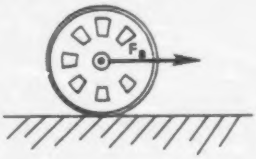


FIG. 5

EXAMPLE A: CALCULATION OF STATIC AND KINETIC FRICTION

Given: A 12-lb wheel is rolled across a wooden floor. A 9-ounce force is required to start it rolling, and a 7.5-ounce force is required to keep it rolling at a constant velocity.

Find: Coefficients of static and kinetic rolling friction.



Solution: Static friction:

$$\mu_s = \frac{F_s}{N}$$

$$\mu_s = \frac{(9\text{ oz})\left(\frac{1\text{ lb}}{16\text{ oz}}\right)}{12\text{ lb}}$$

$$= \frac{0.5625\text{ lb}}{12\text{ lb}}$$

$$\mu_s = 0.047$$

Kinetic friction:

$$\mu_k = \frac{F_k}{N}$$

$$= \frac{(7.50\text{ oz})\left(\frac{1\text{ lb}}{16\text{ oz}}\right)}{12\text{ lb}}$$

$$= \frac{0.469\text{ lb}}{12\text{ lb}}$$

$$\mu_k = 0.039$$

SUMMARY

Rolling friction (the resistance to rolling movement in a mechanical system) is not as great as other friction (the resistance of sliding friction). Ball and roller bearings — both of which exhibit rolling friction in movement — have little rotational resistance. The efficiency of a mechanical system is increased when components that exhibit high sliding friction are replaced with those that exhibit rolling friction.

LABORATORY

DETERMINATION OF ROLLING FRICTION

MATERIALS

1-ft x 3-ft piece of pile carpet and 4 tacks.
Level wooden surface at least 1 ft x 3 ft.
Bicycle wheel.
Spring balance with 10-kg capacity.
Spring balance with 250-g capacity.
Set of metric weights with hangers.
Bicycle pump.
String with metal rings.
Bowling ball with axle as shown in Figure 6.

Note: Axle is formed by drilling two small opposing holes in bowling ball and inserting a small finishing nail in each hole.

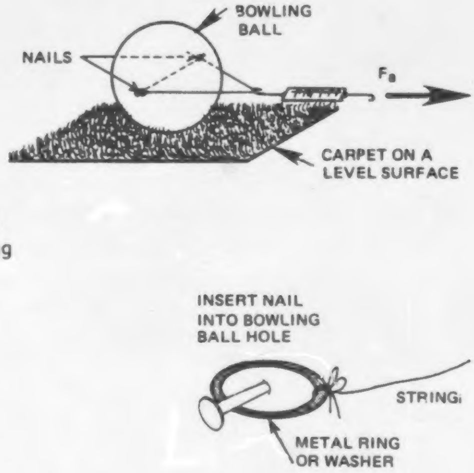

BOWLING BALL
NAILS
F_a
CARPET ON A LEVEL SURFACE
INSERT NAIL INTO BOWLING BALL HOLE
STRING
METAL RING OR WASHER

FIG. 6

PROCEDURE

1. Use large spring balance to weigh and determine masses of bowling ball and bicycle wheel. Record these values in Data Table.

2. Secure carpet on level surface with tacks. Place bowling ball on surface; attach small spring scale with string and metal rings as shown in Figure 6.

3. Apply a horizontal force F_a sufficient to cause ball to roll. Measure force necessary to start ball rolling and force necessary to keep ball rolling at a constant velocity. Record in Data Table. Note: Extreme care must be taken in measuring these small forces. Several attempts may be necessary to obtain reasonably consistent and accurate data.

4. Repeat step 3 on a wood surface without carpet; enter results in Data Table.

5. Set up bicycle wheel on wooden table as shown in Figure 7. Hang equal weights on opposite sides of wheel to bring total mass of wheel and weights to approximately 10 kg. Adjust air pressure in tire until tire is flattened noticeably at point of contact (trial 1, Figure 7).

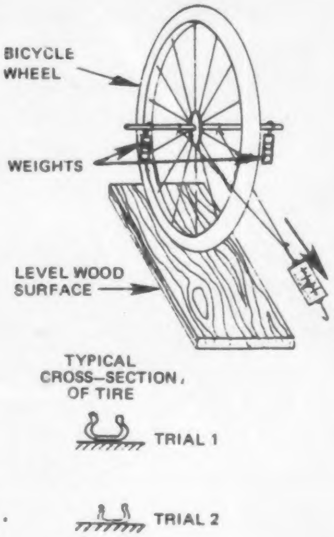

BICYCLE WHEEL
WEIGHTS
LEVEL WOOD SURFACE
TYPICAL CROSS-SECTION OF TIRE
TRIAL 1
TRIAL 2

FIG. 7

6. Hold top of wheel lightly to balance it; apply a horizontal force F_a to cause wheel to roll. In Data Table, record forces necessary to start and maintain rolling.

7. Repeat step 6 with tire fully inflated; enter into Data Table.

8. Calculate coefficients of static and kinetic rolling friction, using Equations 1 and 2.

9. Are other sources of friction contributing to the rolling friction determined in each experiment? If so, identify their source, and determine whether sliding or rolling friction is involved.

DATA TABLE

Mass of bowling ball = ____________ kg.
Mass of bicycle wheel = ____________ kg.

Rolling Object	Mass (kg)	Rolling Surface	Type of Friction	Applied F_a Force (g)	Coefficient of Rolling Friction (Calculated)
Bowling ball		Carpet	Static		μ_s =
Bowling ball		Carpet	Kinetic		μ_k =
Bowling ball		Wood	Static		μ_s =
Bowling ball		Wood	Kinetic		μ_k =
Flat tire		Wood	Static		μ_s =
Flat tire		Wood	Kinetic		μ_k =
Inflated tire		Wood	Static		μ_s =
Inflated tire		Wood	Kinetic		μ_k =

SELF CHECK

1. Describe how small surface irregularities produce sliding friction.

2. Explain why surface irregularities do not contribute greatly to rolling friction.

3. Write a one- or two-sentence definition for the coefficient of static and rolling friction.

4. In Figure 6, the mass of the ball is 7.25 kg, and the force necessary to start it rolling gives a scale reading of 180 g. Calculate the coefficient of static rolling friction.

5. In Figure 7, mass of the wheel and weights is 9.68 kg and force necessary to keep it rolling is 95 g. Calculate the coefficient of kinetic rolling friction.

6. What force is required to keep a 40 kg wheel rolling with a constant speed across a horizontal floor if the coefficient of kinetic rolling friction is 0.04?

7. What is the coefficient of kinetic rolling friction between a road and a bicycle if the bicycle coasts (i.e., no pedaling) down a 5° incline at a constant speed?

8. What is the minimum force required to keep a 100 kg wagon from rolling down a 15° incline? The coefficient of static rolling friction between the wagon and the incline is 0.03 and the force is applied parallel to the incline.

5F2

VISCOSITY

The viscosity of a liquid can be described as internal resistance to motion of the liquid. It is the primary source of resistance when a liquid flows through a pipe, when an object moves through a liquid or when two surfaces separated by a lubricant move relative to each other. In this module the effects of viscosity in lubrication systems are presented. Methods of measuring viscosity and the effect of temperature on viscosity of lubricants also will be discussed. In the laboratory, the student will measure the viscosity of several lubricating oils at varying temperatures.

OBJECTIVES

Upon completion of this module, the student should be able to:

- State a two or three sentence definition of viscosity.
- Describe the flow of a lubricant between a moving and a stationary surface.
- List two advantages and three disadvantages of high viscosity lubricants.
- Describe in a short paragraph the effect of temperature on the viscosity of lubricants.
- Describe in two or three sentences, each of the following types of viscometers:

  Cup.
  Kinematic.
  Rolling sphere.

- Describe in a short paragraph the nomenclature used to indicate the viscosity of motor oils.
- Given the appropriate equipment, measure in the laboratory the relative viscosity of three motor oils at two temperatures.

DISCUSSION

The viscosity of a fluid is defined as the internal resistance to fluid flow. While the concept can be applied to gases, it is used more often as a description of resistance in liquid-flow systems. Higher viscosity means a greater resistance to flow. As an example, common motor oils have a higher viscosity than water, since they flow less easily than water. Concept Module 5-0, "Resistance," contains a discussion of how lubricants are used to reduce sliding friction between two surfaces. The resistance to motion present when a lubricant is used is due primarily to the viscosity of the lubricant. Figure 1 shows the flow of lubricant between a moving and a stationary surface. The lubricant forms thin layers that move relative to one another. The layer in contact with the moving surface moves at the same velocity as the moving surface. The next layer moves at a smaller velocity and the next at a still smaller velocity. The lubricant layer in contact with the stationary surface is itself stationary. The resistance to motion is the total resistance present as each layer of lubricant slides over the next.

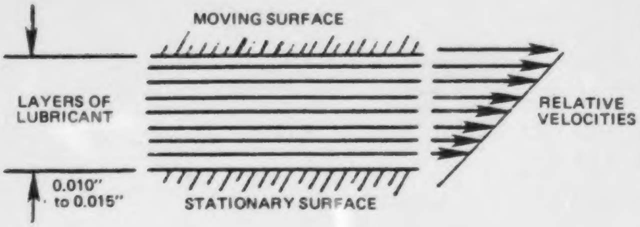


FIG. 1

The same phenomenon is present when a liquid flows through a pipe. In this case, the lubricant layers are concentric cylinders, with fluid velocity being greatest at the center of the pipe and decreasing to zero at the walls.

In mechanical systems, lubricants are chosen to provide minimum resistance for moving parts during normal operation. The details of lubricant choice are beyond the scope of this module, but the following characteristics will provide some insight into lubricant choice:

Higher viscosity lubricants flow less easily and will remain between moving parts longer than those of lower viscosity, which allows a lower rate of lubricant replacement.

High-viscosity lubricants provide a thicker oil film, which results in less friction for rougher surfaces. High-viscosity lubricants have greater internal resistance, which reduces their effectiveness at high speed.

The greater thickness of lubricant resulting from higher viscosity is a barrier to heat flow.

Higher viscosity lubricants result in greater resistance in pipes and are more difficult to deliver and pump.

Another important consideration is the operating temperature of the lubricant. The viscosity of a liquid decreases as its temperature increases. The rate of decrease in viscosity with increasing temperature varies greatly from one liquid to another. In general, higher viscosity lubricants exhibit a greater change in viscosity over a given temperature range than lower viscosity lubricants.

VISCOSITY MEASUREMENT

Three types of instruments are commonly used for viscosity measurements. Each type of instrument is immersed in a temperature bath to stabilize the lubricant temperature at some standard value. The basic characteristics of each device, and the units commonly used to express viscosity for each, are discussed below. Figure 2 shows a cup viscometer. This device consists of a disk immersed in a cup containing the test liquid. The disk is rotated by a motor or spring, and the viscosity of the test liquid produces a resistance to the rotary motion. Similar instruments employ rotating cylinders, cones or paddles, depending upon the viscosity range of the fluid to be tested. Cup viscometers measure either the torque necessary to achieve a constant, rotational velocity or the velocity produced by a constant, applied torque. In either case, the unit of viscosity measure is usually the centistoke (1 centistoke = 1/100 stoke). The stoke is a complicated unit, which accounts for the force necessary to move a given surface area through a liquid at a given velocity. The density of the liquid also is included in the calculation. As a standard, the viscosity of distilled water at 20°C is 1.007 centistoke (cSt).

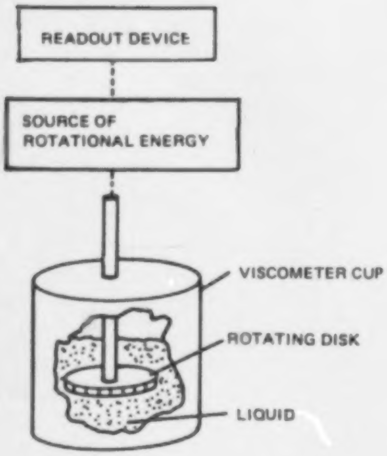


FIG. 2

Figure 3 shows one type of kinematic viscometer. In these devices, a known volume of liquid is placed in an open container with a calibrated outlet tube. The plug is removed from the tube, and the time required for the container to empty is measured. Several models of kinematic viscometers are in use; but the most common is the Saybolt Universal viscometer, in which 60 cm^3 of liquid flows through a capillary tube 1.225 cm long. The viscosity measured with this device is the time measured in Saybolt Universal Seconds SUS), necessary for the container to empty.

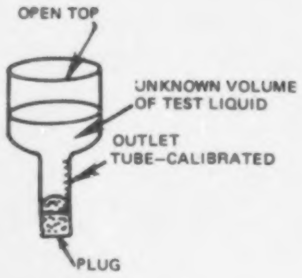

OPEN TOP
UNKNOWN VOLUME OF TEST LIQUID
OUTLET TUBE—CALIBRATED
PLUG

FIG. 3

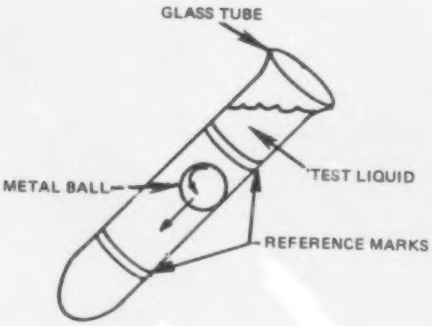

GLASS TUBE
TEST LIQUID
METAL BALL
REFERENCE MARKS

FIG. 4

A rolling sphere viscometer is shown in Figure 4. This device consists of a glass tube closed at one end and filled with the test liquid. A metal sphere with a diameter much less than the inner diameter of the tube is dropped in the upper end of the tube and rolls through the test liquid to the bottom. The time necessary for the ball to travel between two reference marks is measured. This time is divided by the time necessary for the ball to roll the same distance through some standard liquid to obtain the relative viscosity.

Since all these methods of specifying viscosity are difficult to apply in actual situations, the Society of Automotive Engineers adopted the SAE specification for automotive engine lubricants. The SAE number indicates the viscosity of the lubricant at a specific temperature. These values are given in Saybolt Universal Seconds in Table 1.

TABLE 1. SAE NUMBER/VICOSITY

Viscosity at - 18°C (0°F) SUS*			Viscosity at 99°C (210°F) SUS*	
SAE No.	Min.	Max.	Min.	Max.
5W	---	4,000	--	--
10W	6,000[a]	<12,000	--	--
20W	12,000[b]	48,000	--	--
20	---	---	45	<58
30	---	---	58	<70
40	---	---	70	<85
50	---	---	85	110

[a] Minimum viscosity at - 18°C (0°F) may be waived if viscosity at 99°C (210°F) is not less than 40 SUS.

[b] Minimum viscosity at - 18°C (0°F) may be waived provided viscosity at 99°C (210°F) is not less than 45 SUS.

*SUS - Saybolt Universal Seconds.

In the SAE nomenclature system, numbers followed by the letter W indicate low temperature lubricants and those without the W indicate high temperature lubricants. The larger the SAE number, the higher the viscosity. Multi-grade engine oils are blended to have a lower change in viscosity with increasing temperature than single grade oils. Thus, SAE 10W-40 has the viscosity of SAE 10W oil at -18°C and the viscosity of SAE 40 oil at 99°C, which provides proper lubrication during normal operation, but reduces lubricant resistance for starting cold engines.

SUMMARY

Viscosity is the internal friction, or resistance to motion, of a fluid. Higher-viscosity lubricants flow less easily than those of lower viscosity, but have a thicker oil film, greater internal resistance, and greater barrier to heat flow. Various pieces of equipment require lubricants having a specific viscosity.

DETERMINING VISCOSITY OF LUBRICANTS AT VARYING TEMPERATURES

MATERIALS

Water.
SAE 10W oil.
SAE 40 oil.
SAE 10W-40 oil.
Standard wall glass tubing, 25 mm outer diameter, 50 cm long, one end closed, calibration marks 40 cm apart.
Test tube holder.
1/4 inch diamter steel ball bearing.
1 liter beaker.
Hot plate.
Thermometer.
Lab timer.

PROCEDURES

1. Place all oil in refrigerator overnight to lower temperature.
2. Mount glass tube in test tube holder, as shown in Figure 5. (Angle of inclination of tube with horizontal is 10°.)
3. Fill tube with water to within 1 cm of top.
4. Insert thermometer; measure and record water temperature in Data Table.
5. Drop ball bearing into open end of tube. Measure and record time for ball to roll between two reference marks.
6. Remove tube from test tube holder; empty tube, and dry thoroughly. Do not change orientation of test tube holder; that is, do not change inclination from 10° standard arranged in step 2.
7. Pour about half a quart of cooled SAE 10W oil into beaker; measure and record temperature.
8. Replace tube; fill with SAE 10W oil.
9. Drop ball into open end of tube. Measure and record time for ball to roll between two reference marks.

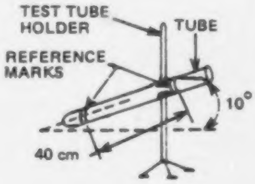


FIG. 5

10. Return SAE 10W oil to beaker. Heat over hot plate to about 100°C. Measure and record oil temperature. (Caution: Hot oil can cause severe burns. Handle with care.)

11. Return heated SAE 10W oil to tube, and repeat measurement. Record time in Data Table.

12. Remove oil from tube, and clean tube thoroughly.

13. Repeat steps 7 through 12 with SAE 40 and SAE 10W-40 oil.

14. Divide each time recorded by the time measured for water to determine viscosity of oil relative to that of water.

15. Explain the relationship between viscosity and temperature of each lubricant tested.

DATA TABLE

Liquid	Temperature (°C)	Time (s)	Relative Viscosity
Water			1.00
SAE 10W	(Cold)		
SAE 10W	(Hot)		
SAE 40	(Cold)		
SAE 40	(Hot)		
SAE 10W-40	(Cold)		
SAE 10W-40	(Hot)		

SELF CHECK

1. Viscosity is a measure of what characteristic of a liquid?

2. Draw and label a diagram illustrating the flow of a lubricant between a moving and a stationary surface.

3. Describe two problems that would result from using a more viscous motor oil than that for which an engine is designed.

4. What happens to the viscosity of the motor oil as an automobile engine warms up?

5. Name each of the following viscometers:
   a. A disk or cylinder is rotated in a cup of liquid and torque is measured.
   b. A liquid flows from a reservoir and the time required is measured.
   c. A metal sphere rolls down a tube filled with a liquid and the time for a specified distance is measured.

6. Describe the difference between each of the following pairs of motor oil:
   a. 20 and 20W.
   b. 30 and 10W-30.

7. A metal sphere rolls from one reference mark in a glass tube filled with a certain liquid and inclined at a small angle to a second mark in 7.0 seconds. The same sphere rolls between the same two marks in 1.2 seconds when the glass tube is filled with water and inclined at the same angle. What is the negative viscosity of the liquid?

8. Would one expect a measurement of relative viscosities using a rolling sphere viscometer to be more accurate if the glass tube is inclined at 10° or inclined at 70°?

9. It has been found experimentally that the viscosity of a particular liquid decreases linearly with increasing temperature in the range from 20°C to 80°C.
   a. If the viscosity at 20°C is 40 SUS and at 80°C is 25 SUS, write a relationship which gives the viscosity V of this liquid as a function of temperature T.

   b. What is the viscosity of the liquid at 27°C?

10. A rolling sphere viscometer is used to measure the viscosity of a certain oil relative to water. It is known that the relative viscosity of the oil is 25. If it takes the metal sphere 1.7 seconds to traverse the distance between the two marks when the glass tube is filled with water how long will it take the metal sphere to roll from one mark to the next if the tube is inclined at the same angle and filled with the given oil?

11. If it takes 6 seconds for a metal sphere to traverse the distance between the two reference marks of the glass tube of a rolling sphere viscometer when the tube is filled with the liquid of Problem 9 and the temperature of the liquid is 20°C, how long will it take when the temperature of the liquid is 43°C?

5F3

FLUID RESISTANCE IN PIPES

There are many causes for energy losses in any system that uses fluid power. The major cause of energy loss in a fluid-power system is friction. Concept Module 5-0 explained that this friction is due to the fact that the fluid molecules bump against one another and against the pipe walls. Friction produced in this manner is manifested as heat given off to the surrounding air or to some cooling media. Heat always represents a loss of energy in a fluid-power system. This loss always shows up as a decrease in head, or pressure, across the system and is labeled as a frictional loss, or frictional drop. In the laboratory, the student will investigate how the friction losses in a pipe are affected when the pipe's length or diameter is changed and when the viscosity of the fluid that flows through it is changed.

OBJECTIVES

Upon completion of this module, the student should be able to:

- List the four factors that affect fluid flow rate through a pipe, and state how changes in each of these affect fluid flow rate.
- Draw and label diagrams, showing fluid flow rate as a function of each of the above quantities.
- Draw and label a diagram of pressure differential as a function of fluid flow rate, and identify the streamline and turbulent regions.
- Explain why streamline flow usually is employed in fluid power systems.
- Given the absolute viscosity of a fluid, the flow rate through a pipe, and the pipe length and diameter, calculate the pressure differential of the pipe.

- Given the absolute viscosity of a fluid, the pressure differential across a pipe and the pipe length and diameter, calculate the fluid flow rate for laminar flow.
- Given the appropriate equipment, determine (1) the head loss of a fluid flowing through various lengths and diameters of straight pipe, and (2) the effect produced upon the head loss when two fluids of differing viscosity are used.

DISCUSSION

Fluid flow in a pipe always is accompanied by the friction of fluid molecules colliding with one another and with the walls of the pipe; and consequently, by the loss of energy available to do work. Another way of stating this is that there must always be a pressure drop in the direction of flow, as illustrated in Figure 1.

If the valve in Figure 1 is closed, there is no fluid flowing in the pipe. Thus, the pressure p_1 is the same as p_2, and both are equal to the head of liquid in the cylinder, as shown by Equation 1.

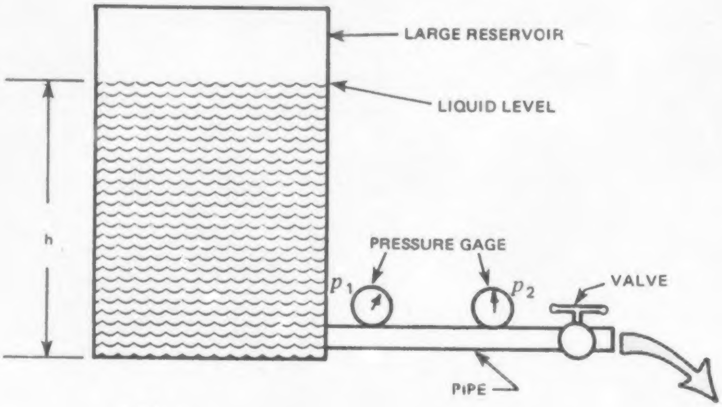


FIG. 1

$$p_1 = p_2 = \rho g h \qquad (1)$$

where

$p_1 = p_2$ = pressure in lb/ft^2.

ρg = weight density of liquid in lb/ft^3.

h = depth of liquid in ft.

Example A shows the use of this equation in solving a problem.

EXAMPLE A: CALCULATING LIQUID HEAD

Given: The depth of water in a barrel is 2 ft.

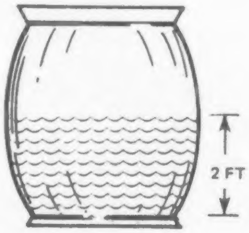
Find: Pressure at the bottom of the barrel.

Solution: Water has a weight density of $\rho g = 62.4\ \text{lb}/\text{ft}^3$.

$$p = \rho g h$$

$$= (62.4\ \frac{\text{lb}}{\text{ft}^3{}_2})(2\ \text{ft})$$

$$p = 124.8\ \frac{\text{lb}}{\text{ft}^2}$$



Or, if the pressure is to be in lb/in^2 —

$$p = 124.8 \frac{lb}{ft^2} \left(\frac{1\ ft^2}{144\ in^2}\right)$$

$$p = 0.866 \frac{lb}{in^2}$$

If the valve in Figure 1 is opened, allowing the liquid to flow, pressure p_2 is less than pressure p_1. This pressure drop is caused by the fluid resistance in the pipe.

As stated in Concept Module 5-0, "Resistance," if the outflow of water is collected and measured for a measured time interval, the flow rate of the liquid at a constant pressure can be determined. This constant pressure is simply the liquid head shown in Figure 1 and is assumed to be constant if the reservoir level does not drop too much during the collecting time. This flow rate and pressure differential Δp is shown in Figure 2 as point 1.

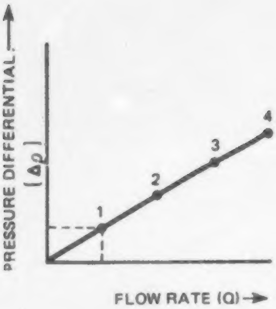


FIG. 2

If this experiment is repeated for progressively greater depths of liquid, the flow rate increases linearly for greater pressure differential Δp; thus, points 2, 3, and 4 of Figure 2 are obtained. The resulting graph shows how pressure differential (forcelike quantity) across a pipe changes with fluid flow rate through the pipe. Concept Module 5-0, "Resistance," stated that the plotting of a forcelike quantity against a rate produces a graph in which the slope of the line at any point is a measure of the resistance offered by the system for that flow rate.

As long as the flow rate increases linearly with the pressure differential Δp, the liquid flow in the pipe is laminar (streamlined). A point soon is reached, however, when the pressure that pushes the fluid through the pipe increases so much that the flow becomes turbulent. When turbulent flow sets in, the resistance increases, and greater pressure is required to increase the flow rate from this point onward.

Figure 3 shows the effect of turbulence upon the pressure required to move liquid through a pipe at a given rate. Since the slope represents resistance, in the turbulent region the resistance obviously is increasing continuously; and ever-larger pressure differentials are required to increase the flow rate.

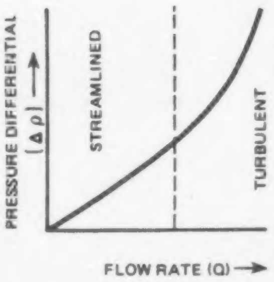


FIG. 3

The point at which fluid flow becomes turbulent can be determined from the Reynolds number, given by Equation 2.

$$N_R = \frac{6.32\ Q}{d\eta} \qquad (2)$$

where
Q = flow rate (lb/h).
d = pipe diameter (in).
η = absolute viscosity (centipoise).

If the Reynolds number is less than 2000, the liquid flow probably will be laminar; if greater than 10,000, the flow will be turbulent. If the Reynolds number is between 2000 and 10,000, the flow may be laminar, turbulent, or a combination of [illegible] two. Examples B and C illustrate the use of Reynolds number to determine whether fluid flow is laminar or turbulent.

EXAMPLE B: CALCULATION OF REYNOLDS NUMBER

Given: Water is flowing in a 1-inch pipe at a rate of 40 gal/min. The absolute viscosity of water is $\eta = 1$ centipoise.

Find: Whether the fluid flow is laminar or turbulent.

Solution: Weight density of water = 8.33 lb/gal.

$$Q = (40\ \text{gal/min})\left(\frac{60\ \text{min}}{\text{h}}\right)(8.33\ \text{lb/gal})$$

$$Q = 19992\ \text{lb/h}$$

$$N_R = \frac{6.32Q}{d\eta}$$

$$= \frac{(6.32)(19992\ \text{lb/h})}{(1\ \text{in})(1.0\ \text{cp})}$$

$N_R = 126349$ (Notice that all units are dropped in calculating N_R.)

Since $N_R \gg 10{,}000$, the flow is turbulent.

EXAMPLE C: CALCULATION OF REYNOLDS NUMBER

Given: Motor oil is flowing in a 1-inch pipe at a rate of 40 gal/min. The viscosity of the motor oil is 2000 cp, and the specific gravity is 0.916.

Find: Whether the fluid flow is laminar or turbulent.

Solution:

$$Q = (40\ \text{gal/min})(60\ \text{min/h})\left(\frac{8.33\ \text{lb}}{\text{gal}}\right)(0.916)$$

$$Q = 18313\ \text{lb/h}$$

$$N_R = \frac{6.32\ Q}{d\eta}$$

$$= \frac{(6.32)(18313\ \text{lb/h})}{(1\ \text{in})(2000\ \text{cp})}$$

$$N_R = 57.87$$

Since $N_R << 2000$, the flow will be laminar.

Since virtually all practical fluid flow systems operate in the streamline region of Figure 3, the remainder of this module will discuss only streamline flow. For streamline flow, the flow rate Q and the pressure differential Δp are related by Equation 3.

$$Q = \frac{1176\ d^4}{\eta \ell}\ \Delta p \qquad (3)$$

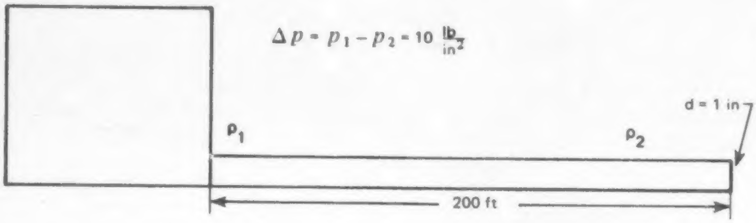
where

Q = flow rate (gal/min).
d = pipe diameter (in).
η = absolute viscosity (centipoise).
ℓ = pipe length (ft).
Δp = pressure (lb/in^2).

Example D shows the use of this equation to calculate flow rate through a pipe.

EXAMPLE D: CALCULATION OF FLOW RATE

Given: Water is flowing through a pipe 1 inch in diameter and 200 ft in length. The pressure differential along the pipe is 10 lb/in^2.



Find: The flow rate.

Solution: $Q = \frac{1176\ d^4}{\eta \ell} \Delta p = \frac{(1176)(1)^4(10)}{(1)(200)}$

$Q = 58.8$ gal/min.

Notice that the units are not used in this calculation. The value 1176 is chosen to give the flow rate in gal/min if all other quantities are expressed in the units specified for the equation.

Equation 3 states that the flow rate of a viscous fluid through a tube is directly proportional to the pressure difference between the ends of the tube; directly proportional to the 4th power of the diameter of the tube; and inversely proportional to both the length of the tube and the viscosity of the fluid. Now the effect on flow rate when each of these quantities is varied independently of the others shall be presented.

Figure 4 shows flow rate as a function of pressure differential for a fixed pipe length and diameter and for a fixed fluid viscosity in the laminar flow region. The flow rate increases linearly with the pressure differential A change in the value of any of the fixed quantities will produce another straight line with a different slope. The dotted line, for example, shows flow rate as a function of pressure differential for a pipe of smaller diameter. The same effect could have been achieved by use of a greater pipe length or a fluid of higher viscosity.

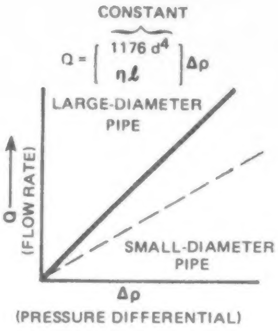


FIG. 4

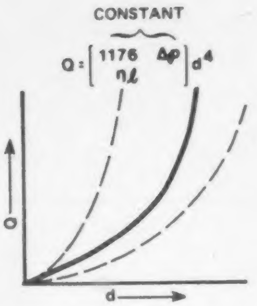


FIG. 5

In Figure 5, all quantities in Equation 3 are held constant, except the pipe diameter. In this case, flow rate increases as the fourth power of diameter. A change in the value of any of the fixed quantities results in a curve of the same general shape as indicated by the dotted lines in the figure.

Variation of only the length of the pipe results in the graph shown in Figure 6. In this case, doubling of the pipe length cuts the flow rate in half and the curve approaches zero asymptotically.

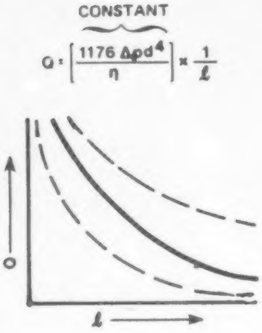


FIG. 6

Figure 7 shows similar curves that result when fluids of differing viscosity flow through the same pipe. These curves have the same shape as those in the graph of flow rate versus pipe length in Figure 6.

If Equation 3 is rearranged to express pressure differential in terms of the other quantities, the result is Equation 4.

$$\Delta p = \left[\frac{\eta \ell}{1176\ d^4}\right] Q \qquad (4)$$

This equation states that the forcelike quantity Δp is the product of some constant (that depends upon the exact pipe and fluid) and the fluid flow rate. The form of this equation is the same as for Ohm's Law in electrical systems. The quantity $\left[\frac{\eta \ell}{1176\ d^4}\right]$ can be thought of as the resistance-like term; in fact, the electric resistance of a wire is given by:

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{(\pi/4) d^2}$$

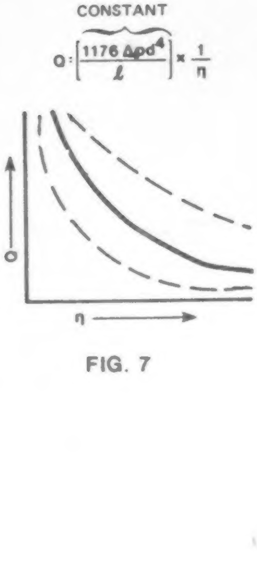


FIG. 7

where

ρ = resistivity.
ℓ = length of resistor.
A = area of resistor $\frac{\pi}{4} d^2$.

The "resistance" of fluid flowing through a pipe is then

$$\text{"R"} = \frac{\eta \ell}{1176\ d^4}$$

This similarity is shown in Figure 8.

Unifying Equation:

FORCELIKE QUANTITY = RESISTANCE TERM X RATELIKE QUANTITY

Electrical Systems:

Voltage = Resistance x Current

$$V = RI$$

$$V = \left[\frac{4\rho\ell}{\pi d^2}\right] I$$

Fluid Systems:

Pressure differential = Resistance x Flow Rate

$$\Delta p = RQ$$

$$\Delta p = \left[\frac{\eta\ell}{1176\ d^4}\right] Q$$

FIG. 8

SUMMARY

Fluid flow in pipes may be either streamlined (laminar) or turbulent. In the laminar flow region, the resistance to flow is a constant; thus, a constant fraction of system power is needed to overcome this resistance. For turbulent flow, resistance always is higher than for laminar flow; and the resistance increases further as flow rate increases. Thus, a greater fraction of total system power therefore is used to overcome resistance. Fluid power systems usually are designed to operate with laminar flow, because of the higher efficiency afforded by lower resistance.

Factors affecting the fluid flow rate in a pipe are pressure differential, pipe diameter, pipe length, and fluid viscosity. Connectors, restrictions, valves, and bends in pipes also affect fluid flow in practical systems.

FLUID RESISTANCE IN PIPES

MATERIALS

Large tank with outlet near base.
Three pieces of glass tubing with diameters and length ratios shown in Figure 9.
Matching-bore rubber tubing.
A scale for weighing the fluid.
One quart container.
Clamp for rubber tubing.
2 types of fluid, preferably water and light oil.

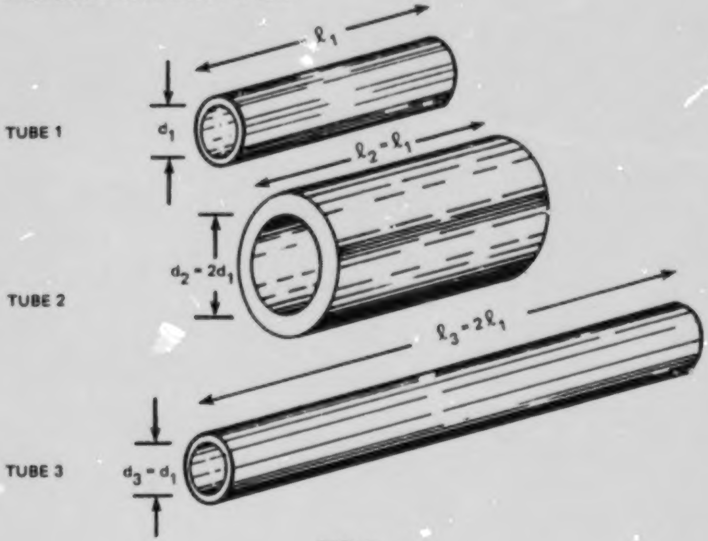

TUBE 1
d_1
ℓ_1
$\ell_2 = \ell_1$
TUBE 2
$d_2 = 2d_1$
$\ell_3 = 2\ell_1$
TUBE 3
$d_3 = d_1$

FIG. 9

PROCEDURE

1. Connect glass tube 1 to bottom outlet of large container, using short piece of rubber tubing as a connector, as shown in Figure 10. Place clamp on rubber tubing to prevent fluid flow.

2. Fill large reservoir, and measure liquid head (Δh); record this value in Data Table. Calculate pressure differential across pipe, and record in Data Table.

3. With tubing in horizontal position, open clamp, and allow the fluid to flow until quart container is filled. Record time required. Calculate and record fluid flow rate.

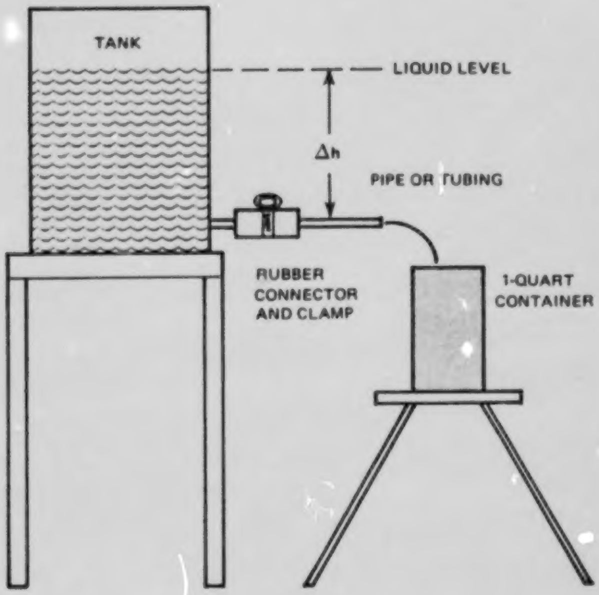

TANK
LIQUID LEVEL
Δh
PIPE OR TUBING
RUBBER CONNECTOR AND CLAMP
1-QUART CONTAINER

FIG. 10

4. Repeat steps 1-3 for other two pieces of tubing. Record all data in Data Table.

5. Repeat above procedure, using tube 3 and oil instead of water.

6. Draw the following graphs*:

   - Flow rate vs. tube diameter, using trials 1 and 2. Note that (d = 0, Q = 0) must be a point on the curve. (See Figure 5.)

   - Flow rate vs. tube length, using trials 1 and 3. Note that curve must approach axes asymptotically. (See Figure 6.)

   - Flow rate vs. viscosity, using trials 3 and 4. Note that curve must approach axes asymptotically. (See Figure 7.)

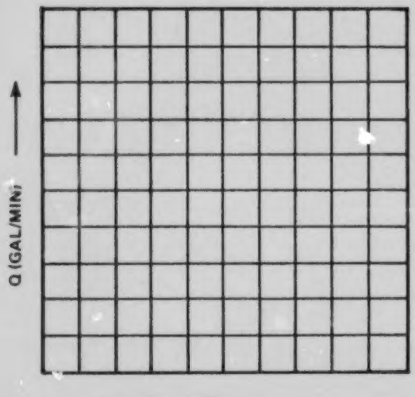
7. State briefly the relationship of pipe diameter, pipe length, and viscosity to fluid flow rate; and support your statements with your experimental data.

*Note: The graphs drawn in step 6 are based upon only two points. In general, one should plot graphs based upon more than two points; however, the trend predicted by only two points here will be seen to be in agreement with the plots shown in Figures 5, 6 and 7. Thus, in the interest of brevity, only two points will be "required" for the measurements plotted in step 6.

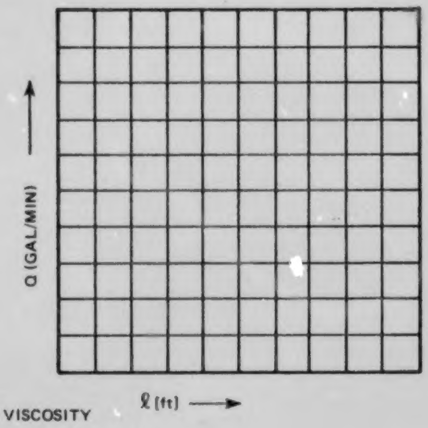
DATA TABLE

Trial	Tube	Δh(ft)	Δp(lb/in^2)	ℓ(ft)	d(in)	t(s)	Q gal/min	η(cp)
1	1(ℓ,d)							H_2O
2	2(ℓ,2d)							H_2O
3	3(2ℓ,d)							H_2O
4	3(2ℓ,d)							oil

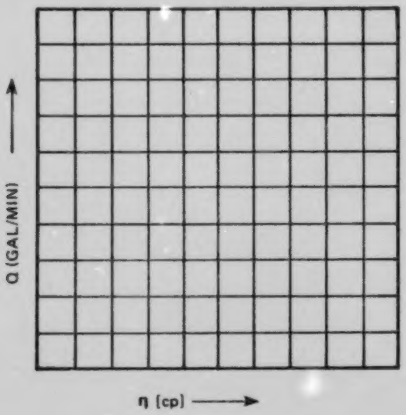
FLOW RATE VERSUS TUBE DIAMETER



FLOW RATE VERSUS TUBE LENGTH



FLOW RATE VERSUS VISCOSITY



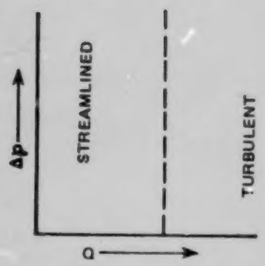
SELF CHECK

1. State how laminar fluid flow rate through a pipe changes as each of the following quantities is changed:

   (a) Pressure differential.
   (b) Pipe diameter.
   (c) Pipe length.
   (d) Fluid viscosity.

2. Draw graphs showing laminar fluid flow rate as a function of the following quantities:

   (a) Pressure differential.
   (b) Pipe diameter.
   (c) Pipe length.
   (d) Fluid viscosity.



3. Draw a typical curve of pressure differential versus fluid flow rate on the graph at the left.

4. Water (η = 1.0 cp) flows through a pipe 10 feet in length and 1/2 inch in diameter at a rate of 2 gal/min. Determine the pressure differential.

5. Oil (η = 1000 cp) flows through a pipe 8 feet long and 2 inches in diameter with a pressure differential of 18 lb/in^2. Determine the flow rate.

6. A fluid flows through a pipe at a rate of 32 gal/min. Determine the flow rate for each of the following conditions:

   (a) Double pipe length.
   (b) Double pipe diameter.
   (c) Decrease pressure differential by one-half.
   (d) Increase fluid viscosity to four times the original value.

7. Why is laminar flow preferred to turbulent flow in fluid power systems?

8. The Reynolds number of a particular oil is 100 when the oil is flowing at a rate of 45 gal/min through a 1.5 inch diameter pipe. What is the pressure differential between the ends of the pipe under these conditions if the pipe is 6 ft long? The specific gravity of the oil is 0.87.

9. What is the "fluid resistance" in a length of pipe if the flow rate is 100 gal/min and the pressure differential is 26 lb/in^2?

VALVES AND REGULATORS

Valves are devices used to control the flow of fluids and gases. The term "valve" is a general term and other words commonly used to mean the same thing are "cocks," "taps," and "faucets." There are many types of valves and an almost infinite number of applications in our industrial society. Regulators, more correctly identified as "pressure regulators," are devices that maintain a constant output pressure at different flow rates. This module discusses valves and regulators as variable fluid resistors. In the laboratory, the student will measure the resistance of valves and regulators.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define "fluid resistance," and discuss valves and regulators as variable fluid resistors.
- Explain the difference between valves and regulators in terms of function and design.
- Draw and label diagrams of the following types of valves:
  - a. Screw-down stop valve.
  - b. Plug cock.
  - c. Gate valve.
- Draw and label a diagram of a gas-pressure regulator, and explain its operation.
- Given the appropriate equipment, take the necessary data and draw resistance curves for a valve and for a pressure regulator.

DISCUSSION

RESISTANCE IN FLUIDAL SYSTEMS

In Concept Module 5-0, "Resistance," the term "resistance" was defined as the ratio of a forcelike quantity to a rate. In fluidal systems, the fluidal resistance is the ratio of pressure to volume flow rate, which can be represented as the slope of a line, as shown in Figure 1. This figure depicts the resistance of a fixed-fluid resistance in which the pressure drop across the resistor increases as the flow rate increases.

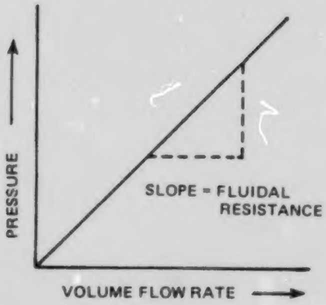


FIG. 1

In the fluid system, illustrated in Figure 2, resistance of the glass tubing is determined by the ratio of the pressure differential across the tubing to the volumetric flow rate through the tubing. If several pieces of tubing having equal lengths but different diameters are used, each will have its own characteristic curve, as shown in Figure 3. The tube with the greatest slope (pipe No. 1) has the greatest resistance. The vertical dashed line in Figure 3 indicates that for a given flow rate, a smaller-diameter pipe will have a greater pressure drop than a larger-diameter one. The horizontal line shows that at a constant pressure drop, a larger-diameter pipe (which has lower fluidal resistance) will allow a greater flow rate than a smaller one. Thus, both pressure drop and flow rate are related to the diameter of the pipe carrying the fluid.

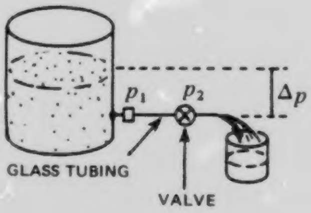


FIG. 2

In fluidal systems, control is accomplished by varying the size of an opening through which a fluid (either gas or liquid) passes. A valve is a device used to control flow rate; whereas, a regulator is a device used to control differential pressure. Both are variable fluid resistors in which resistance is changed by varying the area of an opening.

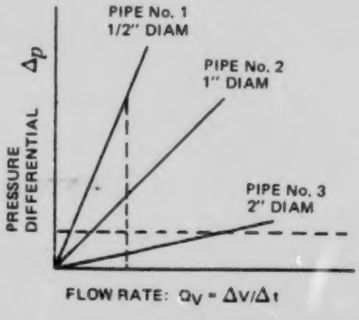


FIG. 3

VALVES

Ideally, a valve is a flow-control device that operates with a constant pressure drop. Enlargement of the opening in a valve reduces the resistance and increases the flow rate. When the valve is closed, no liquid flows (large resistance, equivalent to an open electrical circuit); and when the valve is completely open, the liquid flows at its maximum rate (small resistance, equivalent to a shorted electrical circuit).

The pressure drop across a valve actually is constant only if the valve is the only significant resistance in the system. In practical fluidal systems, other resistances have an effect upon the pressure drop across the valve.

Figure 4 illustrates a fluidal system containing a closed valve compared with a graph of pressure in the system as a function of distance along the pipes. When the valve is closed, the entire pressure differential between points A and B appears across the valve.

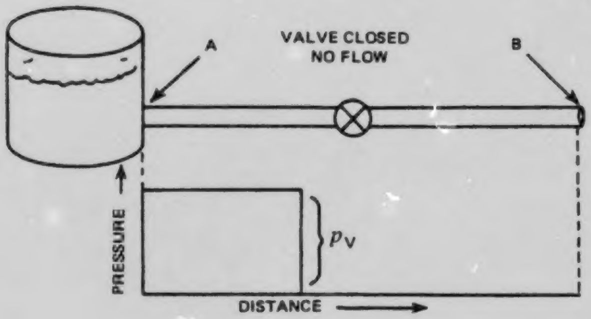


FIG. 4

Figure 5 depicts the same system with the valve open to allow fluid flow. In this case, there is a pressure drop along each of the pipes, and the differential pressure across the valve is much less than the total system pressure differential.

A valve has a fixed value of resistance for each of its settings, which means that both flow rate through the valve and differential pressure across the valve vary with the pressure applied to the system.

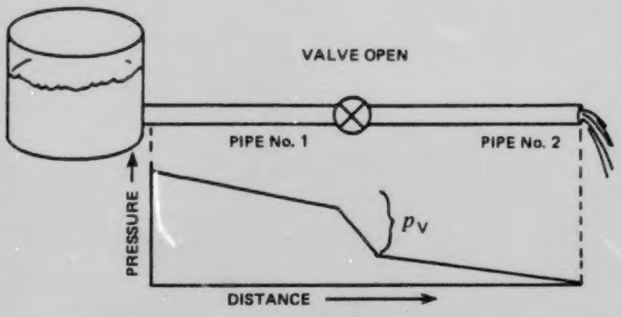


FIG. 5

TYPES OF VALVES

Perhaps the most familiar valve is the water tap, or faucet. This type of valve is technically closed as a screw-down stop valve. Figure 6 consists of a schematic of this type of valve. Another type of valve commonly found in the laboratory and industrial setting is the plug cock (Figure 7). This valve is a simpler one, in which the fluid passage is a hole in a rotatable plug that has a slightly-tapered shape and is fitted into the body of the cock. The cock is opened or closed by rotation of the plug through an angle of 90°. Some degree of flow control is possible if the plug of the cock is set in an intermediate position. This type of valve is not used if accurate control of flow rate is desired.

A feature common to most manually-operated flow control valves is the stem, or screw spindle, surmounted by a hand-wheel. The fluid (gas or liquid) flows through an opening whose edge forms the valve seat. The ordinary domestic water tap illustrates the principle (Figure 6). The stem is connected to a disk usually provided with a replaceable sealing washer to make the actual contact with the seat and, thus, to stop the flow of water. To open the tap, the disk is raised by rotating the handwheel in the counterclockwise direction until the stem is screwed out of the valve body. Clockwise rotation brings the valve disk into contact with the seat and, thus, closes the tap. In some valves, the stem, instead of being provided with a disk, ends in a conical point inserted into a conical hole of the seat. The point closes into the hole when the stem is screwed home. This type of valve is known as a "needle valve" and is found in carburetors. The angle valve (Figure 8) is another variation of this concept.

Larger valves used for controlling the flow of liquids in pipelines, water mains, etc., often are of the kind illustrated in Figure 9.

This device is known as a "gate valve," or "sluice valve." In this valve, a wedge-shaped gate is actuated by a threaded stem and a handwheel. The gate bears against two seat faces to shut off the flow. The fluid flows through the gate valve in a straight

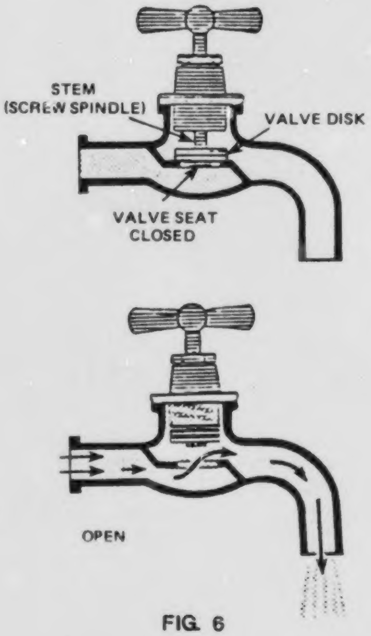


FIG. 6

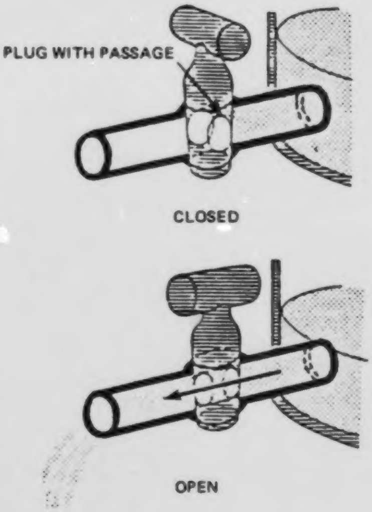


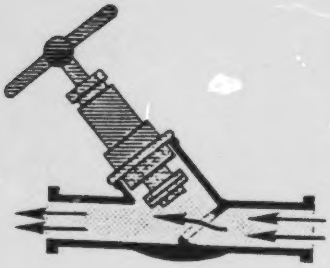
FIG. 7

line, causing the flow resistance to be minimized. The valve stem must be sealed at its point of entry into the valve body in order to prevent leakage of the fluid. The seal usually is formed by a "stuffing box" (Figure 10), which is a cylindrical recess filled with packing compressed by a sleeve, known as a "gland," to make a tight joint.

The material inserted in the stuffing box is called "packing." It may be a compressible material, such as hemp or asbestos. The pressure exerted by the gland keeps the packing tightly pressed against the valve stem.

REGULATORS

Valves have variable resistances, but the resistance of any particular valve is fixed for any setting of that valve. Unless the valve is adjusted, its resistance remains constant.



ANGLE VALVE

FIG. 8

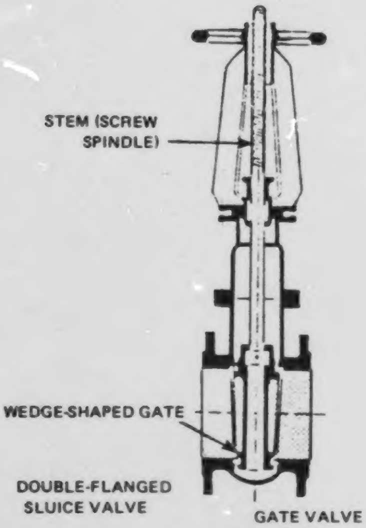
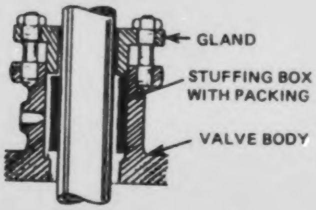

STEM (SCREW SPINDLE)
WEDGE-SHAPED GATE
DOUBLE-FLANGED SLUICE VALVE
GATE VALVE

FIG. 9


GLAND
STUFFING BOX WITH PACKING
VALVE BODY

STUFFING BOX

FIG. 10

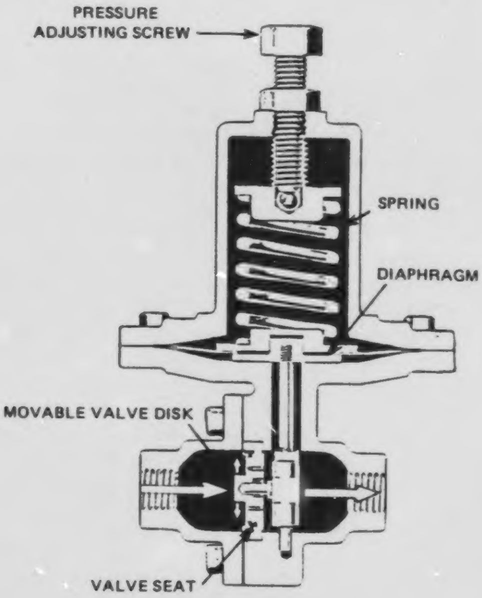

PRESSURE ADJUSTING SCREW
SPRING
DIAPHRAGM
MOVABLE VALVE DISK
VALVE SEAT

FIG. 11

A regulator is a variable fluidal resistor that responds to system changes to produce a constant pressure differential at varying flow rates. Figure 11 is a schematic of a typical gas-pressure regulator. The valve seat is a fixed disk containing a series of holes. A movable disk with a similar hole pattern slides vertically across the valve seat and is positioned by the motion of a diaphragm. The diaphragm is forced downward by spring tension from above and upward by the gas pressure down stream from the regulator. If down-stream pressure drops, the spring forces the diaphragm downward, opening the regulator to decrease resistance. This process allows a higher flow rate and brings steam pressure back to the desired level. If the down-stream pressure rises, it forces the diaphragm upward, increasing the resistance and reducing the flow. The operating pressure of the regulator is changed by adjustment of the spring tension on the diaphragm. Regulators of this type are used whenever a constant pressure is required in fluidal systems.

SUMMARY

Valves and regulators are variable fluidal resistors used to control flow rates and pressures in fluidal systems.

The resistance of a valve is adjustable, but is fixed for any valve setting. Thus, valves can be used to control flow rates, but not pressures; and flow rate through a valve changes as applied pressure changes.

Regulators are active devices whose fluidal resistance changes in response to changes in the system. A regulator senses the down-stream pressure and adjusts its resistance to maintain that pressure at a constant level.

DIFFERENTIAL PRESSURE OF VALVES AND REGULATORS

MATERIALS

Cylinder of nitrogen gas.
Pressure regulator for nitrogen cylinder.
Flow meter.
Pressure gage 0-20 psi.
Two valves.
Connecting tubing.

PROCEDURE

1. Connect equipment as in Figure 12. All gages indicate pressure above atmospheric. p_1 is input pressure to regulator. p_2 is output pressure of regulator. p_3 is input pressure of valve V_2.

   Output pressure of valve 2 is atmospheric pressure (0 psig).

2. Close V_1 and V_2. Turn regulator control counterclockwise until no spring tension can be felt. If in doubt, unscrew regulator control until it can be removed.

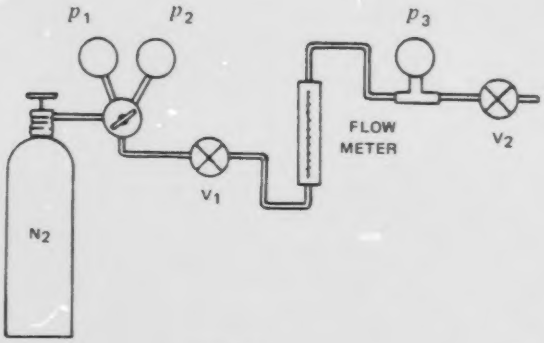


FIG. 12

3. Open nitrogen tank valve.

4. Set regulator for an output pressure of 10 psig on p_2.

5. Open V_1.

6. Adjust V_2 for several flow rates. Record p_1, p_2, and flow rate for five settings of V_2.

7. Draw a graph of pressure drop across the regulator versus flow rate.

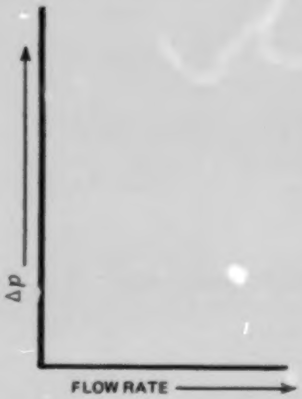
8. Set V_2 to a slightly-open position.

9. Vary output pressure of regulator to produce valves of p_3 from 0 to 10 psig. Record p_3 and flow rate for five valves. (Output pressure of V_2 is 0 psig.)
10. Draw a graph of pressure drop across V_2 versus flow rate.
11. Close V_2.
12. Set regulator for output pressure of 10 psig.
13. Open V_2 to vary flow rate. Record p_3 and flow rate for five settings of V_2.
14. Draw a graph of pressure drop across V_2 versus flow rate as V_2 is opened.
15. Explain the shape of each curve drawn.

DATA TABLE

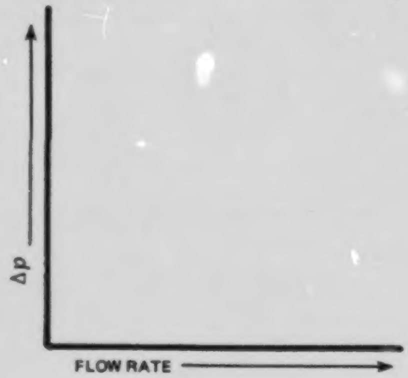
Differential pressure on regulator versus flow rate.

Δp $(p_1 - p_2)$ psig	Flow rate



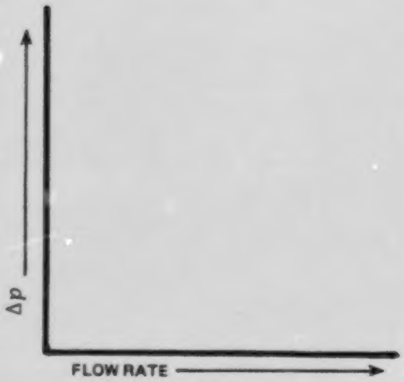
Differential pressure on valve versus flow rate at fixed valve resistance.

Δp (p_3) psig	Flow rate



Differential pressure on valve versus flow rate as valve resistance varies.

Δp (p_3) psig	Flow rate



SELF CHECK

1. Define "fluid resistance" with the use of a graph.

2. Draw graphs of differential pressure versus flow rate for each of the following:

   a. A valve at one specific setting.
   b. A pressure regulator at one specific setting.

3. Explain how the pressure drop across a valve in a fluidal system changes as the valve is opened to allow a greater flow rate.

4. Draw and label diagrams of the following valves:

   a. Gate valve.
   b. Screw-down stop valve.

5. Draw and label a diagram for a gas-pressure regulator, and explain how it functions.

6. If the fluid resistance of a regulator is 3 sec/in^2 when the flow rate is 150 lb/min, what is the fluid resistance when the flow rate is 220 lb/min? (Hint: from Figure 1 or from Module 5-0, $\Delta p = R_F Q$.)

7. What is the minimum fluid resistance of a valve if, when the valve is all the way open, the pressure differential is 0.5 lb/in^2 and the flow rate is 45 gal/min? (Express the answer both in lb/in^2/gal/min and in sec/in^2. Assume the fluid is water.)

8. The resistance of a particular circular valve has been determined to be inversely proportional to the fourth power of the diameter of the valve. When the diameter of the valve is 1 inch, the pressure differential across the valve is 1 lb/in^2 and the flow rate is 20 gal/min. What is the fluid resistance of the valve when the diameter of the valve is 2 inches?

9. The diaphragm of a regulator is designed to remain closed until the pressure against the diaphragm reaches 1 lb/in^2. What height of a vertical column of water could the diaphragm support before opening? The weight density of water is 62.4 lb/ft^3.

10. The pressure differential across a valve is 0.8 lb/in^2 when the flow rate is 2 gal/min. If the setting of the dial remains fixed, what is the pressure differential when the flow rate is 0.6 gal/min?

5E1

OHM'S LAW

In an attempt to master electricity, scientists first sought to understand its nature and its component parts. After identifying several principal components of electricity, they conducted experiments in an effort to understand better the individual nature of these components and their relationships to each other.

Georg Ohm's contribution began with experiments concerning the relationship of the potential difference, or voltage, between the ends of a metallic conductor and the current flowing through it. What resulted is one of the most important tools for working with electricity and electronics available today. In honor of the discoverer, this tool is called "Ohm's law." This module introduces Ohm's law and its relationship to voltage, current, and resistance. In the laboratory, the student will measure and calculate these three qualities.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define voltage, current, and resistance; give their units of measurement.
- Using Ohm's law, calculate resistance, current, or voltage in a circuit, given two of these three quantities.
- Given the appropriate equipment, measure voltage, current, and resistance in electrical circuits having single resistors.

DISCUSSION

PRINCIPAL COMPONENTS

The three principal components of electricity involved in Ohm's law are voltage, current, and resistance. Voltage can be an electromotive force (emf), or a potential difference. It can be described simply as electrical pressure. Voltage is the electrical component that causes electricity to flow in a circuit. Because it causes the displacement of charge, it plays the role of the forcelike quantity in the electrical system. The unit of measurement for voltage is the volt.

Current is defined as the rate of flow of electrical charge past a point in a conductor when a voltage has been applied. Current is the ratelike quantity in the electrical system. When a potential difference exists between two points (such as between points A and B in Figure 1), the electrons flow from the point of negative potential (-) toward the point of positive potential (+). [Some authors use "conventional" current flow, which is from positive (+) to negative (-), and is not the direction of electron current flow. Rather, "conventional" current flow is opposite to electron flow. It was introduced during the time of Benjamin Franklin, when it was thought that positive charges — rather than negative charges — made up electrical current in ordinary circuits. Today, we know it is the electrons that flow. Either convention can be used as long as it is carefully stated.]

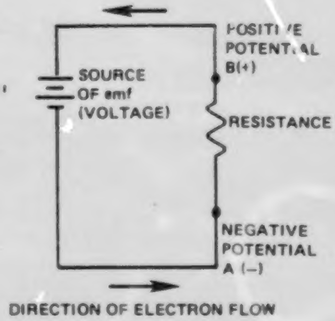


FIG . 1

The unit of measurement for current is the ampere, or amp. An ampere is the current due to one coulomb of charge flowing past a given point in one second (coulomb per second). One coulomb is equal to the charge of 6.28×10^{18} electrons.

The movement of free electrons in a conductor meets with some opposition known as resistance. Resistance can be a useful electrical component. It can be used to limit current flow, it makes the generation of heat possible, and it limits the amount of voltage that can be applied to certain devices.

The resistance of a given conductor is dependent upon four factors:

(1) Type of conductor.
(2) Length of conductor.
(3) Cross-sectional area of conductor.
(4) Temperature.

The unit of measurement of resistance is the ohm. A certain element has a resistance of one ohm when a potential difference of one volt applied to this element maintains a current flow of one ampere through it.

OHM'S LAW

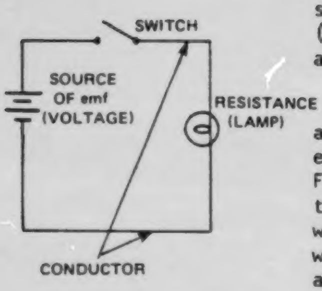


FIG. 2

The simple circuit shown in Figure 2 consists of a source of electromotive force (voltage), a resistance (such as a lamp or a resistor), a switch of some sort, and conductors to serve as circuit wiring.

In 1826, Georg S. Ohm, a German scientist, performed a series of experiments that led to an important law in electricity. Consider a circuit such as that shown in Figure 2. Ohm found that, as long as resistance and temperature remained the same, the current in the circuit was directly proportional to the applied emf. In other words, if emf across the resistance was doubled, current also was doubled. If emf was reduced to one-half the original value, the current was reduced to one-half its original value. The ratio of applied emf to current was constant. This proportionality constant now is called "resistance."

When Ohm went one step further, he found that the value of current changed when he changed the type of bulb (resistance), so long as the conducting wires remained the same. When resistance was increased, the current decreased; and when resistance was decreased, the current increased.

In honor of Georg Ohm's discovery, the unit of resistance is called the "ohm"; and the relationship between these three fundamental components of electricity is called "Ohm's law."

Ohm's law is written as a formula in Equation 1.

$$V = IR \tag{1}$$

where

V = voltage (electromotive force).
I = current.
R = resistance.

The three equivalent forms of Ohm's law are as follows:

$$V = IR$$

$$I = \frac{V}{R}$$

$$R = \frac{V}{I}$$

The third form shows resistance as the forcelike quantity (V) divided by the ratelike quantity (I) in the electrical system. Recall that such a ratio yields a measure of the resistance in other energy systems as well.

A simple tool for remembering these three forms of Ohm's law is the Ohm's-law triangle shown in Figure 3. When the unknown quantity is covered, the remaining known quantities will appear in the correct form. For example, if I and R are known, and V is unknown, cover the unknown quantity (V). The known quantities now appear as I x R. Or if V and R are known and I is unknown, cover the unknown quantity (I). The known quantities now appear as V/R. Finally, if V and I are known and R is unknown, cover the unknown quantity (R). The unknown quantities now appear as V/I. These three forms of Ohm's law are used to solve problems in Examples A, B and C.

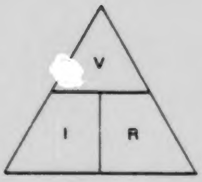
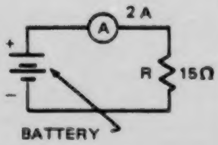


FIG. 3

EXAMPLE A: VOLTAGE PROBLEM USING OHM'S LAW



Given: The circuit shown in sketch. Circuit resistance is equal to 15 ohms, and current measured by an ammeter is equal to 2 amps.

Find: Voltage necessary for this current to flow.

Solution:

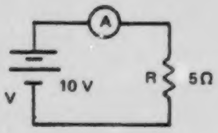
$$V = IR$$
$$= (2\ \text{A})(15\ \Omega)$$
$$= 30\ \text{A}\Omega$$
$$V = 30\ \text{volts}$$

Voltage required for a 2-A current through the 15-Ω resistor is 30 volts.

EXAMPLE B: CURRENT PROBLEM USING OHM'S LAW

Given: $V = 10\ V$, and $R = 5\ \Omega$.

Find: I.



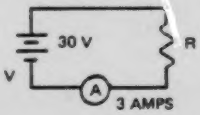
Solution:
$$I = \frac{V}{R}$$
$$= \frac{10\ V}{5\ \Omega}$$
$$= 2\ \frac{V}{\Omega} = 2\ A$$
$$I = 2\ A$$

Two amp of current flows through the circuit.

EXAMPLE C: RESISTANCE PROBLEM USING OHM'S LAW

Given: Current = 3 A; electromotive force = 30 V.

Find: R.



Solution:
$$R = \frac{V}{I}$$
$$= \frac{30\ V}{3\ A}$$
$$= 10\ \frac{V}{A}$$
$$R = 10\ \Omega$$

The resistance is equal to 10 ohms.

Basic calculations have been demonstrated using all three versions of the Ohm's law relationship. The solution of any simple circuit problem will be equivalent to one of these examples.

Because of meter inaccuracies, errors in reading the meter, temperature variations, and resistances of resistors varying from rated values, actual readings of current, voltage, and resistance obtained with a VOM or multimeter may not agree always with the mathematical values found by Ohm's law. Even so, results obtained in the following experiments should convince one of the value of the Ohm's-law relationship.

SUMMARY

Ohm's law is one of the most important tools for circuit design and analysis. This law can be stated thusly: "The current in a circuit is directly proportional to the applied voltage and inversely proportional to the resistance of the circuit."

The three equivalent forms of Ohm's law are the following:

(1) $V = IR$

(2) $I = \frac{V}{R}$

(3) $R = \frac{V}{I}$

where

V = voltage (emf).

I = current.

R = resistance.

MEASUREMENT AND CALCULATION OF VOLTAGE

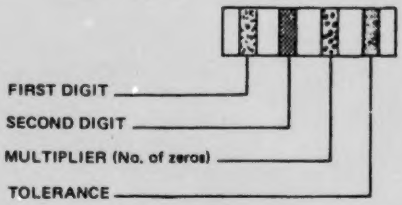
MATERIALS

Several pieces of copper wire (#22).
Variable d.c. power supply (approximately 0 to 20 volts).
Volt-ohm-milliammeter (VOM) similar to Triplett 630 or Simpson 260.
2.2-kΩ, 1/4-watt resistor.

Note: A resistor color-code chart is shown in Figure 4 for those students who are unfamiliar with determining resistor values from color codes.

0 - black
1 - brown
2 - red
3 - orange
4 - yellow
5 - green
6 - blue
7 - violet
8 - grey
9 - white

Resistor Color Code



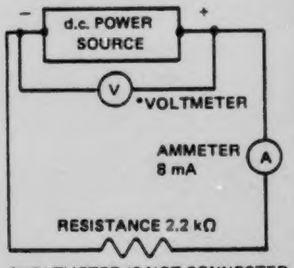
Tolerance:

No band - 20%.
Silver - 10%.
Gold - 5%.

For example, if the four bands are yellow-violet-orange-silver, resistance is 47,000 or 47 k with a tolerance of 10%.

FIG. 4

PROCEDURE



*VOLTMETER IS NOT CONNECTED UNTIL STEP 4

FIG. 5

1. Set up circuit as shown in Figure 5. Note: Value of resistor is given in kΩ, kilohms. The prefix "kilo" means 1000. The resistor can, therefore, be identified as 2.2 kilohms, 2.2 x 10 ohms, 2.2 thousand ohms, or 2,200 ohms.

2. Use V = IR to calculate amount of voltage required to cause 8 milliamperes of current to flow in circuit shown in Figure 5. Record in step 4. Note: Current has been given in mA (milliampere). This current level has been

chosen to minimize danger of electrical shock. Milli is a prefix meaning 1/1000. Therefore, current can be read as 8 mA, 8 milliamperes, 8×10^{-3} amperes, or 0.008 amperes.

3. With variable d.c. power supply set originally on 0 volts, gradually increase voltage until ammeter scale on VOM indicates 8 mA.

4. Remove VOM from ammeter position; connect it as a voltmeter across power supply. Read measured voltage; record.

   V = _______ V (calculated voltage).

   V = _______ V (measured voltage).

5. Calculate percent difference between measured and calculated voltages as follows:

$$\% \text{ difference} = \frac{(\text{calculated value} - \text{measured value})}{\text{calculated value}} (100)$$

6. Do your experimental results tend to confirm Ohm's law? Suggest a few reasons to explain why your theoretical and measured results may not have been in complete agreement.

MEASUREMENT AND CALCULATION OF RESISTANCE

MATERIALS

Several pieces of copper wire (#22).
Variable d.c. power supply (approximately 0 to 20 volts).
Volt-ohm-milliammeter (VOM) similar to Triplett 630 or Simpson 260.
0 to 10-kΩ potentiometer.

PROCEDURE

1. Set up circuit as shown in Figure 6.

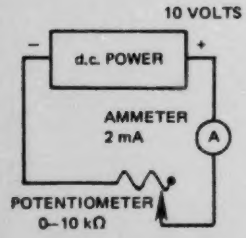


FIG. 6

2. Use Ohm's law to calculate amount of resistance required to cause 2 mA of current to flow when power supply is set for 10 V.

   R = _______ Ω (calculated value).

3. Use VOM as an ohmmeter to verify that potentiometer is set at maximum resistance. As shown in Figure 6, sliding contact should be at topmost position.

4. Set power supply voltage to 10 V, using VOM as a voltmeter across d.c. power source.

5. With VOM connected as an ammeter, reduce resistance of potentiometer until current is increased to 2 mA. (The ammeter itself contributes resistance to the circuit, but it generally is negligible.)

6. Disconnect potentiometer; use ohmmeter to measure its resistance; record below. Be sure to measure across same terminals used in circuit.

   R = _______ Ω (measured value).

7. Calculate percent difference between calculated and measured results.

MEASUREMENT AND CALCULATION OF CURRENT

MATERIAL

Several pieces of copper wire (#22).
Variable d.c. power supply (approximately 0 to 20 volts).
Volt-ohm-milliammeter (VOM) similar to Triplett 630 or Simpson 260.
4.7-kΩ, 1/4-watt resistor.

PROCEDURE

1. Set up circuit shown in Figure 7.

2. Using proper form of Ohm's law, solve for current flowing in circuit shown in Figure 7; record below.

   I = _______ A (calculated value).

3. Set power supply voltage to 15 volts, using VOM as a voltmeter.

4. Connect VOM as an ammeter; measure the current.

   I = _______ mA (measured value).

5. Calculate percent difference between calculated and measured values.

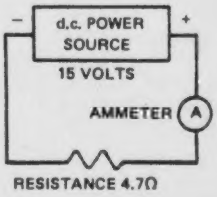


FIG. 7

SELF CHECK

1. Match the word in column A with its definition in B and its units of measurement in C by drawing a connecting line.

A	B	C
Current	Electrical opposition	Volt
Voltage	Rate of flow	Ohm
Resistance	Electrical pressure	Amp

2. What current is drawn by a 8-kΩ electric clock when operated from a 110-V line?

3. A 28-Ω soldering iron requires 5 A for proper operation. What voltage is necessary?

4. What is the effective resistance of a motor if it draws 6 A from a 110-V line?

5. The resistance of an electric percolator is 22 Ω. If it draws 5 A, what is the operating voltage?

6. A toaster draws a current of 10 Amperes when plugged into an outlet. The effective resistance of the toaster is 11Ω. What current does a 40Ω appliance draw when plugged into the same outlet?

5E2

SERIES AND PARALLEL RESISTORS

When there are several electrical devices (appliances, light bulbs, etc.) or components (resistors) to be connected together in a circuit, they are connected in either a series or a parallel manner, or in a combination of both. Resistances connected in series form a single pathway for current flow; resistances connected in parallel form branches, each of which provides a separate path for current flow. Each type of resistance circuit has its own characteristics and uses.

Measurements and calculations often are necessary to determine if a circuit is operating as it should. In the laboratory, the student will use light bulbs as sources of resistance in place of the usual resistors, in order that the affect of series and parallel resistance circuits upon electrical current flow can be observed more directly.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following:

    Series circuit.
    Parallel circuit.
    Overloaded circuit.
    Short circuit.
    Total current in a parallel circuit.
    Total voltage in a series circuit.

- Use Ohm's law to calculate resistance, voltage, and current in simple circuits made up of series and parallel resistances.

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- Given the appropriate equipment, measure resistance, voltage, and current in simple circuits.

DISCUSSION

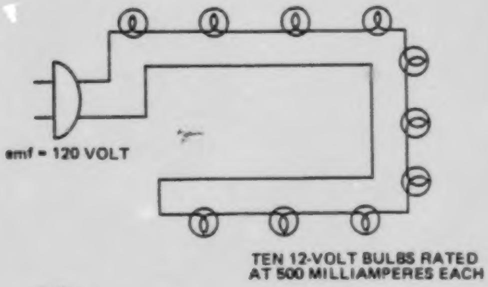


FIG. 1

A series circuit is an electrical circuit in which all components are connected in sequence, one after another, to form a single pathway for current flow. Figure 1 shows a string of Christmas lights as an example of a series resistance circuit. Notice that there is only one path through which the current can flow. Current flows from the seat of electromotive force (120-volt line source), through each bulb, and back to the source. There can be only one current, and it must be the same through each lamp. Certain general conclusions can be drawn from this example:

(1) Electrical current in a series circuit is the same through each of the components connected in series, as shown in Equation 1.

$$I_T = I_1 = I_2 = I_3 = \ldots I_N \qquad (1)$$

where

I_T = total current in a series circuit.
I_1, I_2, etc. = current through each component, including as many as there are in circuit (I_N).

(2) Applied voltage in a series circuit is equal to the sum of the voltages across each of the components connected in series. In equation form:

$$V_T = V_1 + V_2 + V_3 + \ldots V_N \qquad (2)$$

where

V_T = total applied voltage (emf) in a series circuit.
V_1, V_2, etc. = voltage across each component, including as many as there are in circuit (V_N).

(3) Ohm's law can be used to show that the total resistance (R_T) of a series circuit is equal to the sum of the resistances of the components (R_1, R_2, etc.). This relationship is given by Equation 3.

$$R_T = R_1 + R_2 + R_3 + \ldots R_N \qquad (3)$$

6

where

R_T = total resistance of a series circuit.
R_1, R_2, etc. = resistance across component, including as many as there are in circuit (R_N).

Equations 1, 2, and 3 are used to solve a problem in Example A.

EXAMPLE A: SERIES RESISTORS

Given: Each of the ten 12-volt lamps on the string of Christmas lights in Figure 1 is rated at 500 milliamperes (0.5 amperes).

Find:
(a) Resistance of each bulb.
(b) Total current.
(c) Total resistance.
(d) Total voltage.

Solution: (a) Use Ohm's law as follows:

$$R = \frac{V}{I}$$
$$= \frac{12}{0.5 \text{ amps}}$$
$$R = 24\ \Omega$$

(b) $I_T = I_1 = I_2$ . . . etc.
I_T = 500 milliamperes (0.5 amperes)

(c) $R_T = R_1 + R_2$ . . . etc.
$R_T = 240\ \Omega$

(d) $V_T = V_1 + V_2$ . . . etc
$V_T = 120$ V

The primary advantage of a series circuit is its ability to carry a large number of individual resistances without having to increase the current. Because heat generated in a resistance is related directly to current in the resistance, the advantage of a series circuit for something like Christmas lights is obvious. There is one obvious disadvantage, however, of series circuits: if any one device is burned out or tuned off, the circuit is broken and all current flow ceases.

The disadvantage described above sometimes can be turned into something useful. One switch, placed anywhere in a series circuit, can control the whole circuit. Some strings of Christmas lights arranged in a series circuit contain one bulb that acts as a flasher switch. When the flasher bulb heats up, a bimetallic strip switch opens, turning all lights off. After the bulb cools, the switch closes, turning the lights back on. A typical flasher bulb is illustrated in Figure 2.

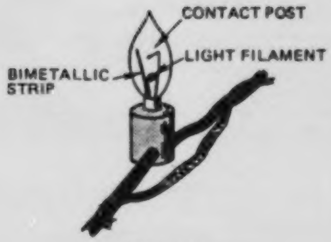


FIG. 2

PARALLEL RESISTORS

In a parallel circuit, electrical components are connected in branches, each of which provides a separate path for current flow. Figure 3 shows a typical automobile electrical system as an example of a parallel resistance circuit.

Notice that there are as many paths for current flow as there are different resistive elements connected in parallel. The current flows from the seat of electromotive force (a 12-volt car battery) simultaneously through each bulb switched into the circuit and then returns to the source. Thus, the total current in the circuit must equal the sum of the individual branch currents. Certain general conclusions can be drawn from this example:

(1) Total current (I_T) in a parallel circuit is the sum of the currents in each component branch (I_1, I_2, etc.) connected in parallel, as indicated by Equation 4.

$$I_T = I_1 + I_2 + I_3 + \dots I_N \tag{4}$$

(2) Voltage across each of the N components (V_1, V_2, etc.) connected in parallel must be equal to the applied voltage. Equation 5 gives this relationship.

$$V_T = V_1 = V_2 = V_3 = \dots V_N \tag{5}$$

(3) Electrical current through each of the component branches of a parallel resistance circuit is inversely proportional to the resistance of the branch. Note: the current may be different in each branch. It can be found by applying Ohm's law to each branch:

$$I\ (\text{headlight}) = \frac{V}{R\ (\text{headlight})}$$

and

$$I\ (\text{taillight}) = \frac{V}{R\ (\text{taillight})}.$$

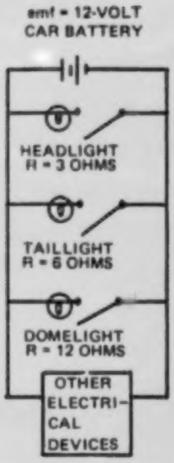


FIG. 3

Equations 4 and 5 are utilized in solving a problem in Example B.

EXAMPLE B: PARALLEL RESISTORS

Given: Voltage across each lamp in Figure 3 is 12 volts (emf), because each lamp is connected directly to the battery through a switch.

Find:
- (a) Current through each lamp.
- (b) Total current passing through the car battery.
- (c) Total effective resistance of the three-lamp circuit.

Solution: (a) Total current in the circuit is equal to the sum of the currents through each of the lamps; thus, using Ohm's law, one can determine the individual currents as follows:

(Headlight current)

$$I_1 = \frac{V}{R_1}$$

$$= \frac{12}{3}$$

$$I_1 = 4 \text{ amps}$$

(Taillight current)

$$I_2 = \frac{V}{R_2}$$

$$= \frac{12}{6}$$

$$I_2 = 2 \text{ amps}$$

(Dome light)

$$I_3 = \frac{V}{R_3}$$

$$= \frac{12}{12}$$

$$I_3 = 1 \text{ amp}$$

(b) Total current passing through car battery.

$$I_T = I_1 + I_2 + I_3$$
$$= 4 + 2 + 1$$
$$I_T = 7 \text{ amps}$$

(c) Again using Ohm's law, total effective resistance of the 3-lamp circuit, so far as the battery is concerned, is the following:

$$R_T = \frac{V}{I_T}$$

$$= \frac{12}{7}$$

$$R_T = 1.7\ \Omega$$

Thus, total resistance of the three parallel branches is less than the individual resistance of any one of the branches. The same result could be found by solving for R_T in Equation 6.

A convenient method of finding the total resistance of resistors in parallel is given in Equation 6.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (6)$$

$$\frac{1}{R_T} = \frac{1}{3} + \frac{1}{6} + \frac{1}{12}$$

$$\frac{1}{R_T} = \frac{4 + 2 + 1}{12} = \frac{7}{12}$$

$$R_T = \frac{12}{7} = 1.7\ \Omega$$

There obviously are more than three parallel branches in a typical automobile electrical system. Each additional branch causes the total resistance of the circuit to become lower, thereby causing a greater amount of total current to flow. In fact, a short circuit is basically a parallel circuit branch having negligible resistance.

Lines that carry more than a safe amount of current are said to be "overloaded." To protect against this, fuses or circuit breakers are connected in series with the supply line to "break" the circuit if the current becomes too great.

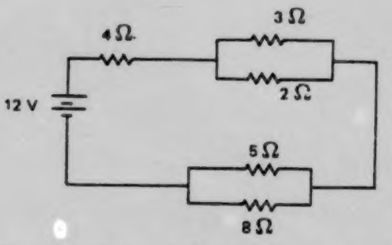
One advantage of a parallel circuit is the possibility of operating electrical devices independently of each other through the use of branch switches. This arrangment is not possible with series wiring. A disadvantage of parallel circuits is the risk of overloading the circuit.

SERIES AND PARALLEL RESISTOR CIRCUITS

Practical electrical circuits often consist of combinations of series and parallel circuits. Because such a great number of combinations of series-parallel circuits is possible, there is no definite procedure that can be followed in solving them. The method depends upon the arrangement. When possible, however, the problem should be solved by replacing each parallel branch with an equivalent series branch. This technique is demonstrated in Example C.

EXAMPLE C: REDUCING A RESISTANCE NETWORK

Given: The circuit shown at the right.



Find: Total current.

Solution: The 2-Ω and 3-Ω resistors in the upper arm of the circuit form a parallel network. By use of Equation 6, they can be reduced to a single equivalent resistance as follows:

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

$$= \frac{6}{5}$$

$$R = 1.20\ \Omega$$

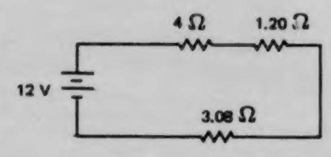
In the same way, the 5-Ω and 8-Ω resistors in the lower arm form a parallel network that can be simplified thusly:

$$\frac{1}{R} = \frac{1}{5} + \frac{1}{8} + \frac{8}{40} + \frac{5}{40} = \frac{13}{40}$$

$$= \frac{40}{13}$$

$$R = 3.08\ \Omega$$

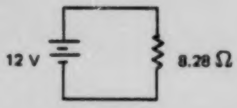
The equivalent circuit now has this appearance:



As the circuit is not reduced to one with three resistances in series, these can be combined to one equivalent resistance by use of Equation 3, as follows:

$$R = 4 + 1.20 + 3.08$$
$$R = 8.28\ \Omega$$

The circuit in its simplest form then looks like this:



Using Ohm's law, the total current in the circuit can be found.

$$I = \frac{V}{R}$$
$$= \frac{12}{8.28\ \Omega}$$
$$I = 1.45 \text{ amps}$$

SUMMARY

The electrical series circuit forms a single path through which current flows; therefore, current has the same value through each component. The parallel circuit contains at least one branch, consisting of two or more paths of current flow. The voltage in each parallel component is the same. Many electrical circuits consist of combinations of series and parallel circuits. The parallel elements often are solved when replaced (in calculations) with series circuits. Total resistance in a series circuit is equal to the sum of the individual resistances; in a parallel circuit, it is calculated either by Ohm's law or by Equation 6.

LABORATORY

SERIES AND PARALLEL RESISTANCES

MATERIALS

Five light bulbs (type #233).
Two light bulbs (type #1449).
Seven miniature screw sockets with leads.
Twelve-volt battery or d.c. power supply.
Volt-ohm-milliampere meter (VOM) similar to Triplett 630 or Simpson 260.
Several pieces of No. 22 copper wire.

PROCEDURE

1. Using volt-ohm meter, measure resistance of each #233 and #1449 lamp. Each lamp should have a resistance value within ten percent of a similar lamp. Record measurements in Data Table 1. (Notice that these are "cold" resistances of bulbs.)

2. The #233 lamp is rated at 0.27 amperes with 2.33 volts across its terminals. The #1449 lamp is rated at 0.2 amperes with 14 volts across its terminals. (Notice that these values are determined by "hot" resistance of bulb.) Using this information and Ohm's law, calculate hot resistance for each type of lamp; record results in Data Table 1.

3. Calculate percent difference between average measured cold resistance and calculate hot resistance for each type of lamp. Record results in Data Table 1.

4. Set up series circuit as shown in Figure 4.

5. With VOM meter, measure voltage across each lamp and applied voltage. Record in Data Table 2.

6. Verify that sum of voltages across each lamp is equal to applied voltage. Calculate percent difference if any exists. Record in Data Table 2.

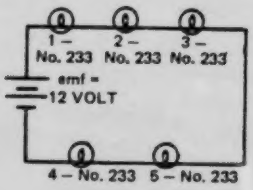


FIG. 4

7. Break circuit between any two #233 bulbs; insert VOM; measure current. Record results in Data Table 3. (Caution: Be sure that scale switch on VOM is initially set on a higher range than expected current.)

8. Using calculated resistance and applied voltage, calculate current through circuit; compare with measured value in Data Table 3.

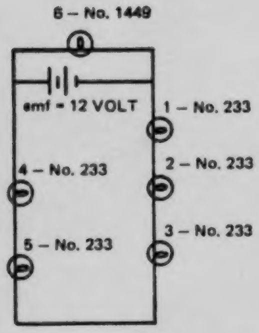


FIG. 5

9. Momentarily short circuit one set of #233 bulb leads; observe effect.

10. Without disturbing existing series circuit, connect a #1449 bulb across power supply as in Figure 5. This is now a series-parallel circuit. Branch containing #1449 bulb is in parallel with branch containing five #233 bulbs in series.

11. Momentarily short circuit one #233 bulb. Note effect of this short on #1449 bulb.

12. Add a second #1449 bulb as shown in Figure 6.

13. Measure current flow through each #1449 bulb and through branch of five #233 bulbs. From these measurements, determine total current delivered by 12-volt source. Record in Data Table 3.

14. Following Example A as a guide, calculate total theoretical current in series-parallel circuit; compare with measured value.

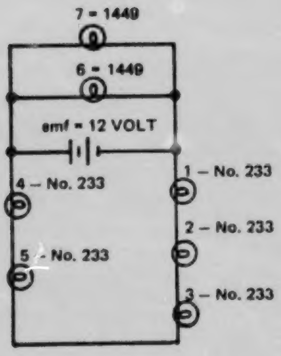


FIG. 6

15. Measure voltage across each #1449 bulb; compare with equation for parallel components in Discussion.

16. What differences would you expect to find in the collected data if commercial resistors were used in place of the light bulbs?

17. Describe and explain effects of the short circuiting in steps 9 and 11.

18. Explain the percent differences found in Data Table 1. (Hint: Recall that resistance depends upon temperature.)

DATA TABLE 1

"Cold" Resistance Measured with Ohmmeter	"Hot" Resistance Calculated from Specs.*	Percent Difference**
A. 1 - #233 ____ ohm 2 - #233 ____ ohm 3 - #233 ____ ohm 4 - #233 ____ ohm 5 - #233 ____ ohm Total ____ ohm Average #233 ____ ohm	* #233 - 0.27 A @ 2.33 V * #1449 - 0.2 A @ 14 V $R = \frac{E}{I}$ #233: ______ ohm	** For example: $\% = 2\,\frac{\{\text{average cold resistance}\} - \{\text{calculated hot resistance}\}}{\{\text{average cold resistance}\} + \{\text{calculated hot resistance}\}} \times 100$ #233: ______ %
B. 6 - #1449 ____ ohm 7 - #1449 ____ ohm Total ____ ohm Average #1449 ____ ohm	#1449: _____ ohm	#1449: ______ %

DATA TABLE 2

Measured with Voltmeter	
1 - #233	volts
2 - #233	volts
3 - #233	volts
4 - #233	volts
5 - #233	volts
Sum of voltages	volts
Applied voltage	volts
Percent difference	%

DATA TABLE 3

Measured with Ammeter	Calculated from Ohm's law	Percent Difference
A.		
1 - 5 - #233 series _____amperes	______ amperes	________ %
B.		
6 - #1449 parallel ______amperes	______ amperes	________ %
7 - #1449 parallel ______amperes	______ amperes	________ %
1 - 5 #233 parallel _____amperes	______ amperes	________ %
Total current ________ amperes	______ amperes	________ %

SELF CHECK

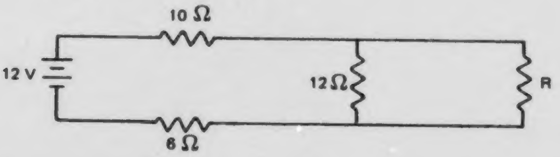
1. Define or describe the following:

    Series circuit.
    Parallel circuit.
    Overloaded circuit.
    Short circuit.
    Total current of parallel circuit.
    Total voltage of series circuit.

2. If the circuit in Figure 6 were changed from two 1449 bulbs in parallel to six bulbs connected in parallel, what would be the voltage across each of the 1449 bulbs? What would be the total current supplied by the battery?

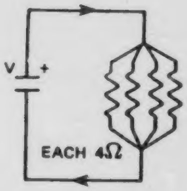
3. If the series circuit of five 233 bulbs were increased to eight 233 bulbs in series, what would be the voltage across each lamp? Does the brightness of the bulbs change, if so, why?

4. Find the value of the unknown resistor in the circuit represented by the figure below if the total current flowing out of the battery is 0.7 amp.



5. How many 10Ω appliances connected in series are necessary to reduce the current flowing in a circuit to 10 mA if the applied voltage is 12 volts?

6. If a current of 1 ampere flows through a circuit in which there is only one 10Ω resistor, what current will flow if the circuit is expanded to include four 10Ω resistors connected in parallel and the applied voltage remains the same?



5E3

INTERNAL RESISTANCE OF A BATTERY

Electrical applications of the resistance concept are familiar, but resistance of the power source in a circuit is often neglected, as it is usually a small value compared with the other elements in the system. In this module, applications will be considered in which the total circuit resistance is low enough to cause internal resistance of the battery to become significant. A method for making electrical measurements that may be used to compute the battery's internal resistance will be outlined.

OBJECTIVES

To complete this module, the student should be able to:

- Define electrical current and electromotive force.
- Write the formula for Ohm's law and explain each symbol.
- Draw a schematic diagram that includes a battery as a resistive element of an electrical circuit.
- Provided with a battery and necessary circuit components, determine internal resistance of the battery.

DISCUSSION

Concept Module 5-0, "Resistance," points out that resistance is the phenomenon that causes a ratelike quantity to decrease to zero in the absence of a forcelike quantity. In the electrical application, resistance causes current (rate of movement of charge through the circuit) to decrease to zero when the potential difference is removed from two points in the circuit.

Electrical resistance is defined by Ohm's law:

$$R = \frac{V}{I} \qquad (1)$$

where

R = resistance of a circuit element.
V = potential difference across resistive element.
I = current sustained by potential difference in the resistive element.

Values of resistance are given in ohms, potential difference in volts and current in amperes.

A battery in an electrical circuit causes current to flow because of energy stored in it. Potential difference across the electrodes of a battery is called electromotive force (emf). Measurements of emf are represented by the symbol (E) and are expressed in volts.

As a component of the circuit, the battery is also a resistor. The battery electrolyte (active chemical materials), plate separators and conductive elements all tend to restrict somewhat the flow of current. In this module, the symbol (r) will represent internal resistance of the battery. Table 1 lists units of measurement required in this module.

TABLE 1: SI UNITS FOR ELECTRICAL MEASUREMENTS

Measurement	Symbol	Unit
Potential difference	V	Volts
Electromotive force	E	Volts
Current	I	Amperes
Resistance	R	Ohms
Battery internal resistance	r	Ohms

Value of a battery's internal resistance may be found by electrical measurement. The circuit required for this procedure is shown as a schematic diagram in Figure 1.

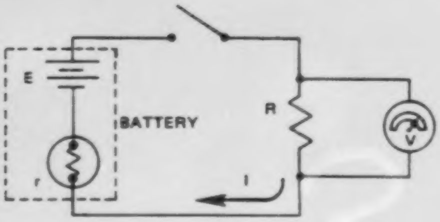


Fig. 1

Internal resistance of the battery is indicated as a resistor in series with the source of emf. When the circuit is closed, potential difference across the external resistance (R) is, by Ohm's law:

$$V = IR. \tag{2}$$

Actually, since the emf is causing current to flow through all of the elements in series, a similar statement may be written to include the battery internal resistance:

$$E = IR + Ir. \tag{3}$$

Subtracting Equation 2 from Equation 3 and solving for r gives the following expression for internal resistance:

$$r = \frac{(E - V)}{I} \tag{4}$$

where

E is measured when the circuit is open; if current is flowing, the measurement of potential difference across the battery is influenced by its internal resistance.
V is measured directly across end of the external resistance. Note that this is also the potential difference across the battery with current flow.
R is known.
I is known by the Ohm's law relationship.

A working equation for the experimental determination of the internal resistance of battery is then:

$$r = (E - V) \times (\frac{R}{V}). \tag{5}$$

Internal resistance of a battery depends upon rate of discharge of the battery because condition of the battery electrolyte is affected by discharge rate. Other factors that influence internal resistance are temperature, state of charge and age of the battery. Internal resistance data, then, may vary considerably with overall condition of the battery.

LABORATORY

MATERIALS

6-volt lantern battery with screw cap terminals.
Knife switch with screw terminals.
Four selected resistors in the range of 1 to 10 ohms. (Only resistors that have been calibrated by the manufacturer or by accurate measurement to within 1% should be used.)
Resistor power ratings should exceed $E^2/R = 36/R$ for this experiment.
d.c. voltmeter with a full-scale range of ten volts.
Three pieces of 10- to 12-gage cable with terminal lugs for series connection of the electrical circuit.

PROCEDURES

1. Measure emf of the battery; record value in column E of data table. Remember that this must be done with battery separate from any circuit and with a voltmeter that draws no current.
2. Begin with largest value resistor in set of four. Connect battery, knife switch and resistor in series. Record resistance value in column R of data table.
3. Connect voltmeter to ends of resistor.
4. Close knife switch. Watch meter needle sweep rapidly to a fairly steady value. Try to make a reading within a couple of seconds after that level is reached. Open switch; record voltage in data table, column V.
5. Refer to Equation 5; compute value of battery internal resistance. Record answer in data table, column r.
6. Repeat procedures 1 through 5, using other resistance values; complete data table.

DATA TABLE

R Resistor Value	E Volts	V Volts	r Ω

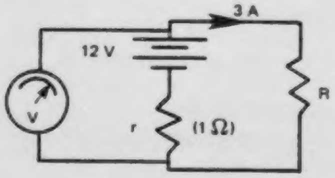
SELF CHECK

1. Draw a schematic diagram that shows a battery as a resistance element of an electrical circuit.

2. Compute internal resistance of a battery, given the following information:
   a. An automobile battery is found to have an open circuit emf of 12.2 volts.
   b. A starter motor has an "IR drop" of 5.0 volts when operating at 500 amps.
   c. When connected to the given battery, voltage across the starter motor is 6.1 volts.

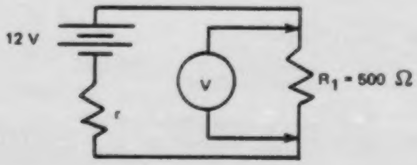
3. Write a brief discussion of the experimental procedure; explain any variations observed in results.

4. The internal resistance of the 12 volt battery shown in the circuit below is 1Ω. If a 3 Ampere current flows through the circuit, what will a voltmeter connected across the terminals of the battery read when the circuit is closed? What is the resistance of R?



5. Given the circuit of Problem 4, what is the current flowing in the circuit if the external resistance is 100Ω and the internal resistance of the 12V battery is 6Ω?

6. In the circuit shown below, a voltmeter reads 10V when placed across R_1. What is the internal resistance of the 12 volt battery?



7. Two identical batteries are connected in parallel. If the internal resistance of each battery is r what is the internal resistance of the combination?

5E4

CONDUCTORS, SEMICONDUCTORS, AND INSULATORS

One of the more important considerations when choosing materials to be used in electrical and electronic circuits is the material's ability to conduct an electrical current. Based upon their resistance to current flow, all materials fall into one of three classes:

Conductors.
Insulators.
Semiconductors.

Familiarity with each of these classes is important in the effective use of electricity.

In this module, the student will examine the electrical properties of conductors, insulators, and semiconductors. In the laboratory, the student will measure the resistance of several material samples, and will compare their characteristics.

OBJECTIVES

Upon completion of this module, the student should be able to:

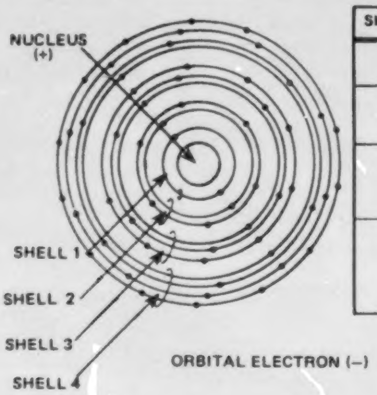
- Define:
  - Ionic bonding.
  - Covalent bonding.
  - Metallic bonding.
  - Insulator.
  - Conductor.
  - Semiconductor.
- Explain why the electrical resistance of metals increases as temperature increases.

- Explain why the resistance of semiconductors decreases as temperature increases.
- Given resistances of conductors, semiconductors and insulators, explain the difference in values for each group.
- Explain why practical electrical insulators use covalent-bonded material instead of ionic-bonded material.
- Given the appropriate equipment, measure the resistance of several materials and compare their characteristics.

DISCUSSION

Whether a material is an insulator, a conductor or a semiconductor is determined by its atomic structure.

An atom is composed of a positively-charged nucleus, or core, surrounded by negatively-charged electrons. The electrons orbit the nucleus in a definite pattern of shells and subshells. Figure 1 shows the first four electron shells of an atom. It also shows that shell 2 has 2 subshells, shell 3 has 3 subshells, and shell 4 has 4 subshells.



SHELL	SUBSHELL	ELECTRONS	TOTAL
1	S	2	2
2	S P	2 6	8
3	S P D	2 6 10	18
4	S P D F	2 6 10 14	32

FIG. 1

The energy of an electron is related to its average distance from the nucleus, with the closest electron being the least energetic. In the formation of an atom, electrons naturally fill the lower energy levels first. Helium, for example, has two electrons, both of which lie in the first shell. Copper has 29 electrons. Its first three shells are filled, with one electron left over. This remaining electron occupies the s subshell of the fourth shell (2 + 8 + 18 + 1 = 29).

The most stable configuration for any atom is to have its subshells filled. The atoms' attempt to have filled subshells causes bonding between atoms.

IONIC BONDING

Figure 2 shows an atom of sodium (Na) with 11 electrons and an atom of chlorine (Cl) with 17. Ten of the sodium electrons fill the first three subshells of sodium, leaving one extra electron. Chlorine, on the other hand, needs one more electron to fill its fifth subshell. The sodium atom gives up its electron to the chlorine atom as shown in Figure 2.

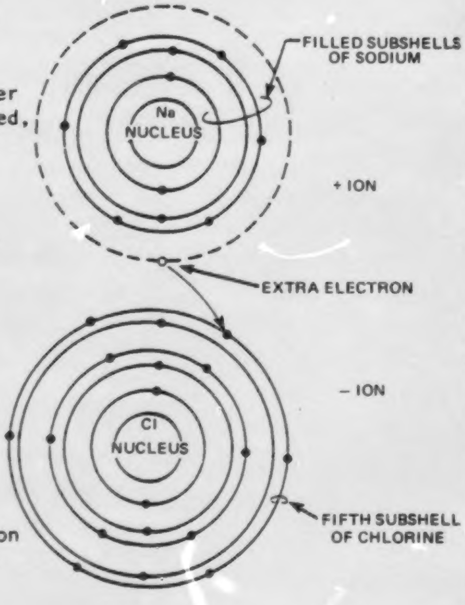


FIG. 2

Because the atoms are neutral (have no net charge) to begin with, losing one electron gives the sodium atom a charge of +1, and gaining one electron gives the chlorine atom a charge of -1.

The attraction of unlike charges holds the two charged atoms, or ions, together. This attraction is called "ionic bonding."

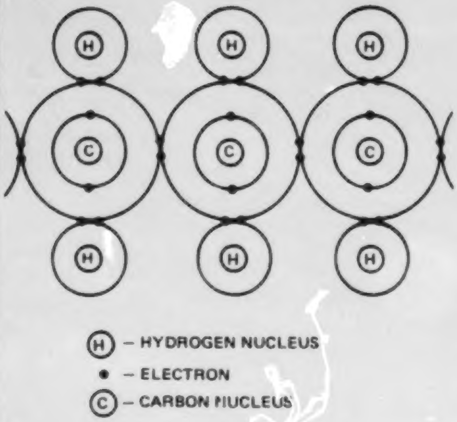


FIG. 3

COVALENT BONDING

A second way in which atoms may fill their outer subshells is shown in Figure 3, in which the structure of Polyethylene molecules is illustrated. There, each carbon atom (C) needs four additional electrons to fill its outer subshell, and each hydrogen atom (H) needs one additional electron. All these subshells are filled by sharing electrons. Each carbon atom shares one electron from two hydrogen atoms and two adjacent carbon atoms. This sharing of electrons is called "covalent bonding."

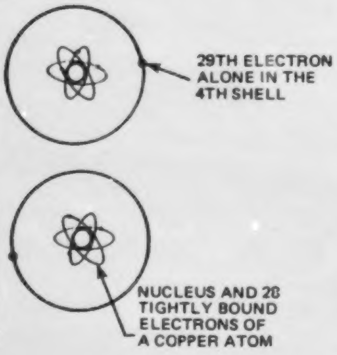

29TH ELECTRON ALONE IN THE 4TH SHELL
NUCLEUS AND 28 TIGHTLY BOUND ELECTRONS OF A COPPER ATOM

FIG. 4

METALLIC BONDING

A third method of bonding occurs in metals. Figure 4 shows two copper atoms. The single electron in the 4s subshell of each atom is very loosely bound to the atom. In a solid piece of copper, the attraction between this electron and the nucleus is too weak to hold the electron in its orbit. The electron does not really "belong" to any one copper nucleus; therefore, it is free to move throughout the piece of copper. The copper actually consists of an array of positive copper ions suspended in a "cloud" of free electrons. The repulsive forces between the positive copper ions causes them to have fixed positions within the electron cloud. The attractive forces between the positive ions and the negative electrons holds the copper together and is called "metallic bonding."

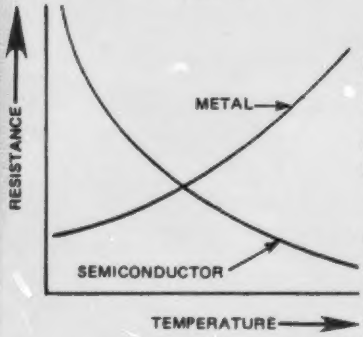
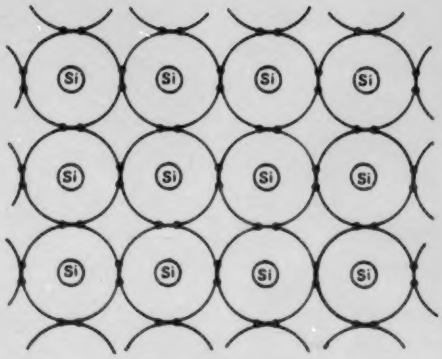


FIG. 5

INSULATORS, CONDUCTORS AND SEMICONDUCTORS

Insulators are materials whose outer electrons are bound tightly to the atoms and cannot serve as charge carriers. The bond may be either ionic or covalent. Materials commonly employed as electrical insulators are usually covalent, because this type of bonding is not broken down as easily as ionic bonding.

Conductors are materials whose outer electrons are free to move throughout the material. How well a material conducts electricity depends upon how many free electrons it has per unit volume and upon the rate at which the electrons collide with the atoms in the material. As a conductor is heated, random motion of free electrons speeds up, and the atoms themselves vibrate about their fixed positions. This increases the rate of electron collisions, thus increasing the resistance. Figure 5 shows the effect of temperature on the resistance of both metals and semiconductors. Although resistance of metals increases with temperature, resistance of a semiconductor decreases. This behavior is explained in the following paragraphs.



Si – SILICON NUCLEUS + FIRST 10 ELECTRONS

• – VALENCE ELECTRON

FIG. 6

Semiconductors are materials that are ordinarily rather good insulators but can be made to conduct by supplying a small amount of extra energy, as, for example, by raising the temperature.

Some of the best-known semiconductors are materials in which the outer subshell of atoms has four electrons and needs another four electrons to fill the subshell completely. These semiconductors include carbon (6 electrons), silicon (14 electrons) and germanium (32 electrons). Such materials form a crystalline structure through covalent bonding, as shown in Figure 6.

Because all of the outer electrons are involved in covalent bonds, there are no free electrons. These loosely-bound, or so-called "valence," electrons, however, are barely attached to the atoms. Some of them can be moved by adding only a small amount of extra energy. For this reason, the resistance of semiconductors is high at low temperature, but decreases greatly as the temperature rises.

Figure 7 gives relative resistivities of several insulators, semiconductors and conductors. The resistivity of a materal depends

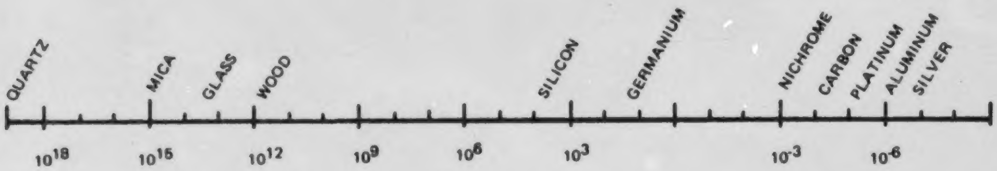


FIG. 7

upon the material, and is measured in ohm-meters. Resistance of a material is always proportional to the resistivity. For a material of given length and cross section, the higher the resistivity, the higher the resistance. As indicated in Figure 7, quartz is a good insulator (high resistivity) and, thus, a poor conductor. On the other hand, silver is a good conductor (low resistivity) and, thus, a poor insulator. Silicon and germanium, somewhat "in-between" on the resistivity scale, are classified as semiconductors.

The resistance of a material can be found by use of Equation 1.

$$R = \rho\frac{\ell}{A} \qquad (1)$$

where

R = resistance.
ρ = resistivity of material.
ℓ = length of material.
A = cross-sectional area of material.

SUMMARY

Whether a material is a conductor, an insulator, or a semiconductor depends upon its ability to conduct an electrical current. The conductor possesses low resistance to current flow; the insulator, high resistance; and the semiconductor, high resistance when at low temperature and low resistance at high temperature. Ionic, covalent and metallic bonding within atoms determines conductivity.

RESISTANCE MEASUREMENT AND CHARACTERISTICS OF MATERIALS

MATERIALS

Several meters of number 36 or 38 enameled copper wire.
Several meters of number 28 or 32 nichrome wire.
Carbon rod (refill lead for a mechanical pencil).
Silicon or germanium transistor.
Glass rod 2 mm in diameter, 5 cm long.
Matches.
Volt-ohm-milliammeter (VOM) or multimeter.

PROCEDURE

1. Make a coil of each piece of wire, being careful to leave ends exposed for electrical connections, and not allowing any part of coil to overlap.
2. Connect volt-ohm-milliammeter across each sample; measure resistance; record in Data Table.
3. Heat each sample with a match; observe change in resistance. Record resistance when heated.
4. Complete Data Table, indicating whether each material tested is an insulator, semiconductor or conductor.
5. In an electrical circuit operated at high temperature, which material in the Data Table would serve as the best conductor?

DATA TABLE

Material	Resistance at Room Temperature (Ω)	Resistance When Heated (Ω)	Type of Material (Insulator, Semiconductor, Conductor)
Copper wire			
Nicrome wire			
Carbon rod (pencil lead)			
Silicon or germanium			
Glass			

SELF CHECK

1. Define:

   Ionic bonding.
   Covalent bonding.
   Metallic bonding.
   Insulator.
   Conductor.
   Semiconductor.

2. Explain why resistance of a copper wire increases when it is heated.

3. Explain why resistance of a carbon rod decreases when it is heated.

4. Explain why copper is a better conductor than carbon.

5. Explain why glass is a poor conductor.

6. Why are materials with ionic bonding seldom employed as electrical insulators in practical circuits?

7. What is the resistivity of a material if a 10 meter wire made of the material has a resistance of 10Ω and the radius of the wire is 5 mm?

8. If a wire has a resistivity of 5×10^{-7} Ω·m, what length of the wire has a resistance of 1Ω if the cross-sectional area of the wire is 2.9×10^{-5} m^2?

9. It is known that the resistance of a conductor increases linearly with increasing temperature. It is determined experimentally that the resistance of a certain length of a particular conductor at 20°C is 1Ω and at 80°C is 1.3Ω. Write an equation which gives the resistance of the conductor as a function of temperature. What should the resistance of the wire be at 100°C?

10. The resistance of a semiconducting material as a function of temperature can be expressed as,

$$R = kR_S \left(\frac{1}{T_S} - \frac{1}{T}\right)$$

where k is a constant which depends upon the material and R_S is the resistance of the semiconducting material at some reference temperature T_S. (All temperatures are expressed in Kelvin degrees.) If the resistance of a semiconductor is 100 kΩ at 20°C, find the resistance at 150°C if k = 3 Kelvin degrees.

11. If a potential difference of 110 volts is applied across the ends of a 10 m length of wire of radius 2 mm, and the resulting current is 1×10^{-8} Amperes, should the wire be classified as a conductor, semiconductor, or insulator?

5E5

RESISTANCE OF WIRES

The resistance of an electrical conductor, such as a wire, depends upon the material from which it is made, its shape and its temperature.

In this module, the student will investigate the effect of each of these quantities. In the laboratory, the student will determine the resistance of wires at different temperatures.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate resistance of a conductor at room temperature, given its dimensions and composition.
- Calculate resistance of a conductor at any temperature, given its dimensions and composition.
- Given the appropriate equipment, determine the resistance of wires at varying temperatures.

DISCUSSION

Resistance is an inherent property of a conductor. The resistance of a conductor of uniform cross-sectional area is dependent upon four factors:

- Composition (the material from which it is made).
- Length.
- Cross-sectional area.
- Temperature.

The German physicist Georg Ohm stated that resistance of a conductor at a given temperature is directly proportional to its length, inversely proportional to its cross-sectional area and dependent upon the material from which it is made. The effect of the material itself is represented by a quantity called "resistivity," which is explained in the following paragraph. Equation 1 shows this relationship in a formula.

$$R = \rho \frac{\ell}{A} \qquad (1)$$

where

R = resistance.
ρ = resistivity.
ℓ = length.
A = cross-sectional area.

RESISTIVITY

Current in a conductor is the movement of free electrons. Resistance — the opposition to this movement — is present in varying degrees in different materials. Resistance is high when relatively few free electrons are available for conduction and when they move with difficulty through the material. This property, represented by the Greek letter ρ, is called resistivity of the material. When Equation 1 is solved for resistivity, Equation 2 is formed.

$$\rho = R \frac{A}{\ell} \qquad (2)$$

A simple meaning for the quantity ρ can be found. Imagine a sample of material with unit cross-sectional area A and unit length ℓ. When these values are substituted in the equation, ρ is simply R, the resistance of the sample. Hence, the resistivity of a material is the resistance to current offered by a unit cube of the material. A dimensional analysis of Equation 1 shows that ρ must have dimensions of resistance times length. ρ usually is expressed in units of ohm-meters or ohm-centimeters. The resistivities of several common conductors are given in Table 1, together with their temperature coefficients of resistance, to be explained later in this module. Because these quantities depend on temperature, they are given at 20°C, which is roughly "room temperature."

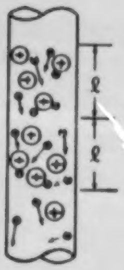


FIG. 1

TABLE 1: RESISTIVITY AND TEMPERATURE COEFFICIENTS OF VARIOUS MATERIALS AT 20° C

Material	Resistivity (ρ) (Ω - m)	Temperature Coefficient of Resistance (α)(per °C)
Silver	1.47×10^{-8}	0.00411
Copper	1.72×10^{-8}	0.00426
Aluminum	2.63×10^{-8}	0.00420
Tungsten	5.51×10^{-8}	0.0049
Constantan	4.41×10^{-7}	less than 0.00001
Nichrome	1.50×10^{-6}	0.00044

LENGTH

The longer the path the electrons are forced to travel, the more resistance they will encounter. Figure 1 shows that when the length of a wire is doubled, the number of atoms that impede electron progress doubles, thereby doubling resistance. Atoms are shown as ⊕ , free electrons shown as ⊖⟶.

A nichrome wire 10 meters long offers 10 times greater resistance than a nichrome wire 1 meter long.

CROSS-SECTIONAL AREA

The greater the cross-sectional area of a conductor, the lower the resistance. Figure 2 shows that when cross-sectional area A is doubled, twice as many electrons are allowed to move through it, thus doubling the current I. The effective resistance for the same length ℓ then is only one-half as great.

Example A illustrates the use of Equation 1 in solving a problem.

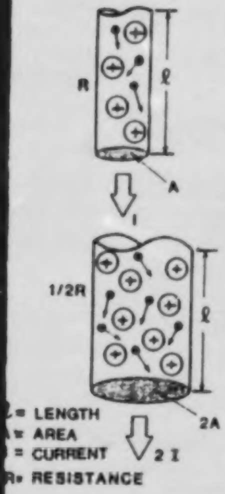


FIG. 2

EXAMPLE A: RESISTANCE OF A COPPER WIRE

Given: A copper wire at 20°C with a diameter of 0.1 millimeters and a length of 50 meters.

Find: Resistance. (Refer to Table 1 for resistivity.)

Solution: $R = \frac{\rho \ell}{A} = \frac{\rho \ell}{\pi r^2}$ (Recall: $A = \pi r^2$.)

$$= \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(50\ \text{m})}{(3.14)(5 \times 10^{-5}\ \text{m})^2} = \frac{86 \times 10^{-8}\ \Omega \cdot \text{m}^2}{(3.14)(25 \times 10^{-10}\ \text{m}^2)}$$

$$= \frac{8.60 \times 10^{-7}\ \Omega \cdot \text{m}^2}{7.85 \times 10^{-9}\ \text{m}^2}$$

$$= 1.09 \times 10^2\ \Omega$$

$$R = 109\ \Omega$$

Resistance of the wire at 20°C is 109 ohms.

TEMPERATURE

Temperature is the fourth condition which affects the resistance of a conductor. The greater the temperature of a conductor, the greater is the vibration of its atoms; and the greater is its hindrance to electron flow. Dependence of resistance upon temperature is not the same for all materials. For the most common conductors, resistivity increases as temperature increases. Change in resistance per degree of temperature change for a one-ohm sample of a material is called the "temperature coefficient of resistance" (α) of the material. Values for a few materials are given in Table 1. Units are simply reciprocal degrees of temperature.

$$R_T = R_o\ [1 + \alpha\ (T - T_o)] \qquad (3)$$

where

R_T = resistance at any temperature T.
R_o = resistance at temperature T_o, often taken to be 20°C or 0°C.
α = temperature coefficient of resistance for particular material.
$T - T_o$ = temperature change.

In Equation 3, the change in resistance resulting from the temperature change $T - T_o$ is given by $\alpha\ (T - T_o)$.

Examples B, C, and D consist of problems solved by use of Equation 3.

EXAMPLE B: EFFECT OF TEMPERATURE ON RESISTANCE

Given: A silver wire with a resistance of 100 ohms at 20°C.

Find: Its resistance at 300°C.

Solution:

$$R_T = R_o\ [1 + \alpha\ (T - T_o)]$$

$$\begin{aligned} R_{300} &= 100\ \Omega\ [1 + (0.00411)(300 - 20)] \\ &= 100\ \Omega\ [1 + (0.00411)(280)] \\ &= 100\ \Omega\ (1 + 1.15) \\ &= 100\ \Omega\ (2.15) \\ R_{300} &= 215\ \Omega \end{aligned}$$

Resistance more than doubles!

The most common conductors for which these equations give useful results are wires. The accepted standard for specifying the size or wire is the American Wire Gage. Table 2 is provided in order that problems may be worked using this nomenclature.

TABLE 2: AMERICAN WIRE GAGE VALUES AT 20°C

Gage No.	Diameter (mm)	Cross-sectional Area (mm^2)
8	3.264	8.367
14	1.628	2.082
20	0.8118	0.5176
24	0.5106	0.2048
26	0.4049	0.1288
36	0.1270	0.01267
38	0.1007	0.007964
40	0.07987	0.005010

Examples C, D and E show how to use Equations 1 and 3 to solve problems.

EXAMPLE C: RESISTANCE OF A NUMBER-36 TUNGSTEN WIRE

Given: A 4-meter length of number 36 tungsten wire is to be used at a temperature of 600°C.

Find: Its resistance.

Solution:

$$R_T = R_o\,[1 + \alpha\,(T - T_o)] \qquad \left(R_{20} = \frac{\rho \ell}{A}\right)$$

$$= \rho\frac{\ell}{A}\,[1 + \alpha\,(T - 20)]$$

$$= \frac{(5.51 \times 10^{-8}\ \Omega \cdot \mathrm{m})(4\ \mathrm{m})}{1.267 \times 10^{-8}\ \mathrm{m}^2}\,[1 + (4.9 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1})\,(600 - 20^\circ\mathrm{C})]$$

$$= (17.4\ \Omega)(1 + 2.84)$$

$$= 17.4\ \Omega\ (3.84)$$

$$R_T = 66.8\ \Omega$$

EXAMPLE D: RESISTANCE OF A NUMBER-24 COPPER WIRE

Given: No. 24 copper wire.

Find: What length will have a resistance of 1 ohm at 5°C.

Solution:

$$R_T = \rho\frac{\ell}{A}\,[1 + \alpha\,(T - 20)]$$

$$\ell = \frac{R_T\,A}{\rho\,[1 + \alpha\,(T - 20)]}$$

$$= \frac{(1\ \Omega)(2.047 \times 10^{-7}\ \mathrm{m}^2)}{(1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})[1 + (4.26 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1})(5 - 20^\circ\mathrm{C})]}$$

$$= \frac{2.047 \times 10^{-7}\ \Omega \times \mathrm{m}^2}{(1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})[(1 + 4.26 \times 10^{-3}\ {}^\circ\mathrm{C}^{-1})(-15^\circ\mathrm{C})]}$$

$$= \frac{2.047 \times 10^{-7}\ \Omega \cdot \mathrm{m}^2}{(1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})(0.937)}$$

$$\ell = 12.7\ \mathrm{m}$$

The required length of copper wire is 1.27 m.

Note: The negative indicates a decrease in temperature from 20°C.

EXAMPLE E: RESISTANCE OF NICHROME WIRE

Given: A piece of nichrome wire with a resistance of 25 ohms at 20°C.

Find: The temperature at which it will have a resistance of 28 ohms.

Solution: $R_T = R_o [1 + \alpha (T - T_p)]$

Solving for T,

$$T = 20°C + \frac{(R_T/R_o - 1)}{\alpha}$$

$$= 20°C + \frac{(28\ \Omega/25\ \Omega - 1)}{4.4 \times 10^{-4}\ °C^{-1}} = 20°C + \frac{0.12}{4.4 \times 10^{-4}}\ °C$$

$$= 20°C + 273°C$$

$$T = 293\ °C$$

Thus, nichrome wire increases its resistance from 25 ohms to 28 ohms when temperature increases 273°C above room temperature of 20°C.

SUMMARY

The degree of resistance of a wire depends upon its composition, length, cross-sectional area, and temperature. A wire possessing good conductance has low resistance. A wire having strong resistance will rise in temperature and in resistance when current is increased through it.

RESISTANCE OF WIRES AT DIFFERENT TEMPERATURES

MATERIALS

Dry cell (1.5 volt).
Switch (knife or key).
Ammeter.
Nichrome wire (#36 or #38).
Copper wire (#38 or #40).

PROCEDURE

1. Use a piece of nichrome wire 50 centimeters long, to connect a circuit as shown in Figure 3.
2. Close circuit; record immediate "cold" current in Data Table.
3. Allow current to flow for 30 seconds to heat nichrome wire. Record "hot" current that seems to remain constant.
4. Open circuit; allow nichrome wire to cool. Remove wire from circuit; fold it in two; twist it so it forms a piece of wire 25-cm long having twice the cross-sectional area.
5. Insert this resistance into circuit; record "cold" current.
6. Allow wire to heat as before; record "hot" current that remains constant.
7. Remove this wire; replace it with a 25-cm length of nichrome wire of same gage. Record "cold" and "hot" currents as before.
8. Repeat steps 1 through 7, using 10-meter and 5-meter lengths of copper wire.
9. Using Ohm's law, R = V/I, calculate "cold" and "hot" resistance for each trial; record.
10. Assume temperature at "cold" resistance to be 20°C. Use tables and equations in this module to calculate temperature of wire at "hot" resistance for each trial; record.
11. Explain how the results of this experiment show the applications of Equations 1 and 2.

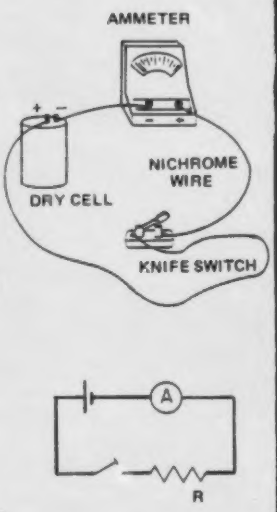


FIG 3

DATA TABLE								
				Current		Resistance		
Trial	Material	Length	Area	Cold	Hot	Cold	Hot	Temperature
1	Nichrome	50 cm	A					
2		25 cm	2A					
3		25 cm	A					
4	Copper	10 m	A					
5		5 m	2A					
6		5 m	A					

SELF CHECK

1. Calculate the resistance of each of the following:

   20 m of number 8 copper wire at 20°C.
   100 m of number 14 silver wire at 135°C.
   6 m of number 40 constantan at 20°C.
   1 kilometer of number 40 nichrome wire at 500°C.

2. A piece of copper wire has a resistance of 0.84 Ω at 20°C. If it is 10 m long, what gage is it?

3. A tungsten lamp filament has a resistance of 2.65 Ω at 20°C. At what temperature will its resistance be 8.0 Ω?

4. At 20°C a number 26 wire 4.2 m long has a resistance of 0.858 Ω. Of what material is it made?

5. A resistor is heated from 26°C to 88°C; its resistance changes from 138 Ω to 180 Ω. Of what material is it made?

6. Write a paragraph or two on the conclusions that can be drawn from this module regarding the effect of conductor material, length, cross-sectional area and temperature on the resistance of a conductor.

5E6

RESISTANCE OF SEMICONDUCTOR JUNCTIONS

Conventional resistors are "fixed" in value (except for temperature changes); therefore, their control of current flow in a circuit also is fixed. Some circuit applications, however, call for a device that has variable control over circuit current, meaning that the resistance of the device must be variable. This module examines the properties of the semiconductor P-N junction diode that make it useful as a voltage-variable resistor. The NPN transistor is analyzed also to demonstrate how resistance of the output circuit is varied, or controlled, by input bias voltage. In the laboratory, the student will measure the resistances of several diodes and transistors.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define:
  - Intrinsic material.
  - Extrinsic material.
  - Doping.
  - P-type material.
  - N-type material.
  - Depletion region.
  - Barrier potential.
  - Forward bias.
  - Reverse bias.
  - Knee voltage.
  - Junction resistance.
  - Bulk resistance.
  - Zener point.
- Explain with the use of a diagram how an NPN transistor is constructed and connected to bias batteries.
- Explain with the use of diagrams how a diode conducts current in one direction but not in the other.

- Explain how changing the current in the emitter-base circuit of an NPN transistor changes the resistance of the base-collector circuit.

- Given the appropriate equipment, measure the resistances of diodes and transistors in a variety of conditions.

DISCUSSION

The current through most materials is inversely proportional to their resistance when an external voltage is applied. The ease with which current flows depends upon availability of the electrons which are to move through the material. In pure (intrinsic) germanium or silicon, electrons are locked into place in each atom in such a manner that conduction cannot occur.

Normally, each carbon, silicon and germanium atom has four electrons in its outer orbit and available for current flow; but in carbon atoms, some of the electrons are not tightly bound but are free to shift from atom to atom. Carbon, therefore, is a better conductor because its valence electrons are more easily moved. As ambient temperature increases, however, some of the electrons in silicon or germanium are energized by heat and consequently break away from their atoms in the crystalline structure, thus becoming more easily available for current flow. The resistance of silicon or germanium, therefore, is lowered as temperature increases and these materials function like conductors. These materials are classified as "semi-conductors," because at low temperatures they are insulators (high resistance), and at higher temperatures conductors (low resistance).

Intrinsic germanium or silicon can have its natural resistance altered by the addition of "impurities" in a process called "doping." Doped germanium or silicon is then called "extrinsic." As mentioned before, germanium or silicon atoms normally have 4 electrons in their outer orbit. These atoms are locked to neighboring atoms in the crystalline structure of the material. Elements such as phosphorous, arsenic and antimony have 5 electrons in their outer orbit. If germanium or silicon is "doped" with any of these three "impurities," then, proportional to the amount of doping, the impurity atoms replace some of the germanium or silicon atoms. At each place in the crystalline structure where this process happens, four of the five electrons of the impurity atom lock into the crystalline structure, and, as before, leave the fifth electron available for conduction, as shown in Figure 1. The heavier the doping, the greater the number of "fifth" electrons available for current flow, and the lower the resistance of the "extrinsic" material. This new, doped, extrinsic material is called "N-type" because electrons (negative) are available as the main, or majority, carrier of current.

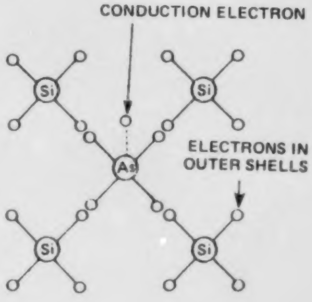


FIG. 1

Boron, aluminum and gallium are examples of elements that have 3 electrons in their outer orbit. When these elements are used to dope germanium or silicon, a "hole" is created where each impurity atom locks into the crystalline structure. The crystalline structure can accomodate four electrons per atom, but only three are present. The space at which the fourth electron is missing is called a "hole," as shown in Figure 2. A hole has the property of a positive charge (absence of

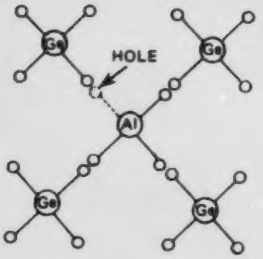
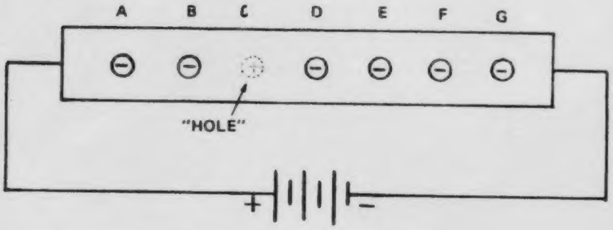


FIG. 2

a negative charge); and it can move or flow in the same manner as an electron moves, although holes flow in the opposite direction from electrons because of their positive charge.

The idea that a hole (being "nothing") can move may seem mystifying at first, but it is a simple concept. In Figure 3, a piece of P-type material doped with impurity atoms to produce "holes" is connected to an external battery to produce current flow. The electrons, being negative charges, are attracted to the positive terminal of the battery, and will move to the left. The holes, being positive charges, should move to the right. Electron D moves to the left and replaces the hole at C. The hole, therefore, is not to the right, at the previous location of electron D. Next, electron E fills the hole at D, and the hole has "moved" another position to the right, to E. As electrons move to the left, holes move to the right. Holes can be represented as stepping stones that allow electrons to travel more easily the distance across the material.



"HOLE" FLOW IN P-TYPE MATERIAL

FIG. 3

P-type and N-type materials, by themselves, are nothing spectacular. They are merely conductors whose resistance is controlled by the amount of doping. In N-type material, electrons are the majority carrier (of current); whereas, in P-type material, holes are the majority carrier. The combination of these two materials in a special way results in a device with properties of voltage-controlled resistance.

When N-type material is joined to P-type material, a junction is formed, as shown in Figure 4a. Immediately, the holes at one side of the junction combine with the electrons at the other side. Where this happens, the majority carriers have been "depleted," and a "barrier potential" is created. The P-type material has become slightly negative because it has lost some positive charge, and the N-type material has become slightly positive because it has lost some electrons, as shown in Figure 4b and c. Recombination may continue as more holes recombine with electrons, but the barrier potential created by the first level of recombination slows down the process. The depletion region widens as recombination continues. Finally, the depletion region becomes so large, and the opposing barrier potential becomes so great, that holes from the P-type material no longer can cross over the depletion region to recombine with the electrons from the N-type material. Recombination then ceases.

The device so formed is called a P-N junction diode. The barrier potential created in a germanium diode is about 0.3 volts; whereas, the barrier potential for a silicon diode is about 0.7 volts. The resistance of the P and N materials themselves is a function of the percentage of doping, and is generally very low (a few ohms). But the resistance of the depletion region, because there are no majority carriers, is very high, and is inversely proportional to the width of the depletion region. Because the width of this region, and hence the resistance of the diode, can be controlled by an external, or "bias," d.c. voltage, the diode is called a "voltage-controlled" resistor.

When a d.c. voltage is applied in the forward direction (plus to P, minus to N), the holes and electrons, respectively, are pushed by the battery voltage toward the depletion region - electrons toward electrons and holes toward holes, as shown in Figure 5a. Since like charges repel, this process shrinks the depletion region, thus lowering the resistance of the diode. When the applied voltage exceeds the 0.3 volts for germanium or 0.7 volts for silicon, the barrier potential is overcome completely; and the depletion region disappears. The resistance of the diode at this point and beyond is very low

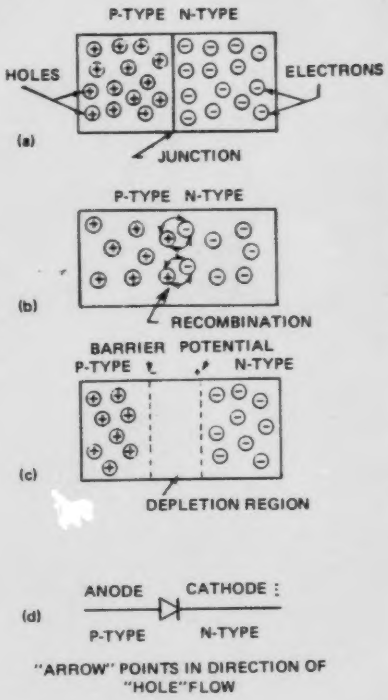

P-TYPE N-TYPE
HOLES
ELECTRONS
(a)
JUNCTION
P-TYPE N-TYPE
(b)
RECOMBINATION
BARRIER POTENTIAL
P-TYPE N-TYPE
(c)
DEPLETION REGION
ANODE
CATHODE
(d)
P-TYPE
N-TYPE
"ARROW" POINTS IN DIRECTION OF "HOLE" FLOW

FIG. 4

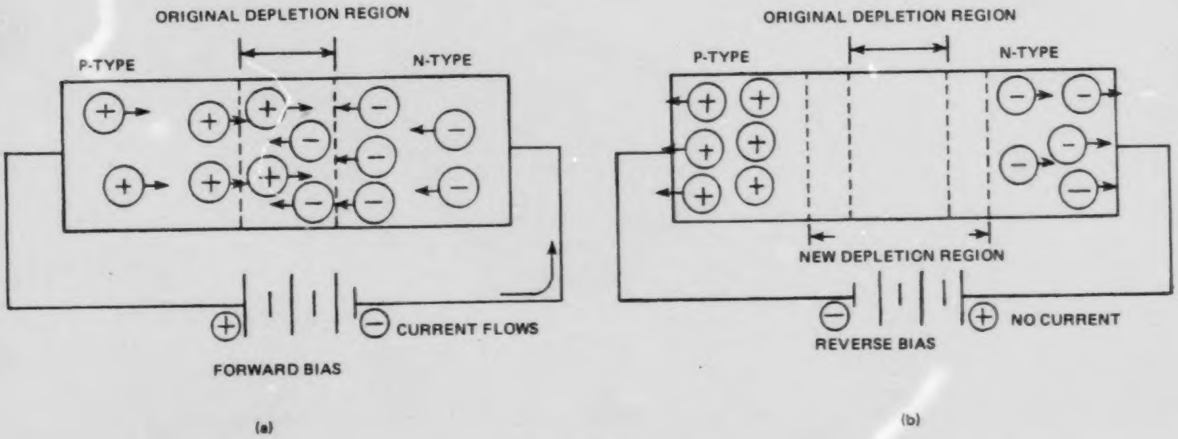

ORIGINAL DEPLETION REGION
P-TYPE
N-TYPE
CURRENT FLOWS
FORWARD BIAS
(a)
ORIGINAL DEPLETION REGION
P-TYPE
N-TYPE
NEW DEPLETION REGION
REVERSE BIAS
NO CURRENT
(b)

FIG. 5

(a few ohms), and the diode behaves like a short circuit or a closed switch.

In Figure 5b, a reverse-bias voltage will cause holes and electrons to be attracted toward the battery terminals, thus widening the depletion region and causing the diode to exhibit an even larger resistance than in its normal state. Under this condition, the diode exhibits very high resistance and behaves like an open circuit or like an open switch.

Figure 6 shows the current-voltage relationship of a P-N junction diode for various amounts of forward and reverse bias. Resistance at any point on the curve can be found simply by dividing the voltage by the current. As forward bias is applied, starting from zero volts, the depletion region is still large and the diode resistance therefore high. Diode resistance at this point is predominantly the junction resistance, or the resistance of the depletion region. As the forward-bias voltage is increased toward the barrier potential (known as the "knee voltage"), the junction resistance decreases because the depletion region narrows. When the knee voltage is reached, the depletion region disappears; and the curve takes a sharp turn upward, indicating low resistance. Ideally, the diode is considered to have a resistance of 0 ohms when operated beyond its barrier potential, or to appear to be a short circuit (indicated by the vertical dotted line above the knee voltage). But practically all P or N material has some resistance, called bulk resistance, of a few ohms, shown by the sloped line labeled "bulk resistance." Any device has a power limitation (such as a 1/4 watt resistor); diodes are no exception. If too much forward current is forced through the diode, the I^2R product will exceed the power rating of the diode, and burnout will occur.

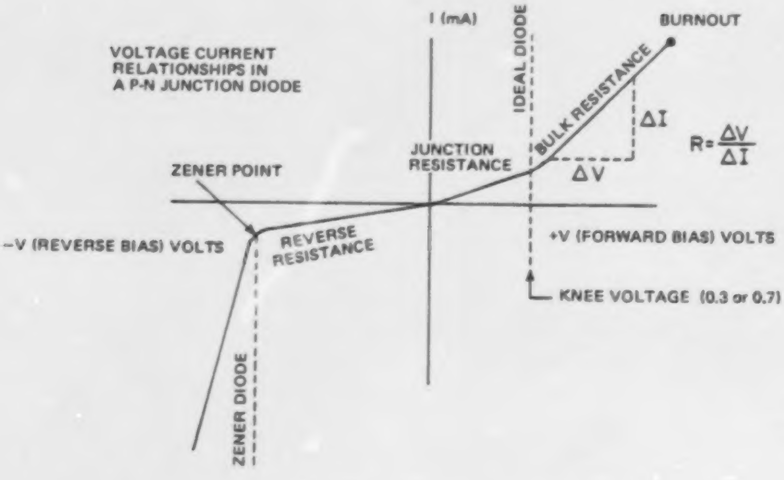


FIG. 6

As reverse bias is applied, the diode exhibits some "reverse" resistance, because the widened depletion region has a very high, but measurable, resistance (on the order of megohms). As reverse bias continues to be applied, a point is reached at which breakdown occurs. This is called "zener point," and is the point at which the applied voltage is so great that it forces carriers across the depletion region. This situation is similar to "arc-over" in a switch. With the depletion region broken down, the resistance of the diode drops drastically; and the diode conducts heavily, as the sharp, downward turn of the curve shows. This condition is not necessarily bad, as long as the burnout point is not exceeded. Special diodes, called "zener diodes," are built specifically to operate in this zener region, and are useful as voltage regulators. The dashed line labeled "zener diode" on the curve shows that in this region of operation the current flow through the diode can vary. The voltage across the diode remains constant, making it valuable as a voltage regulator.

TRANSISTORS

The fact that a P-N junction diode is a voltage-controlled resistor whose resistance is a function of the bias voltage applied to it, provides a clue about how a transistor operates. Essentially, a transistor is nothing more than two diodes constructed back-to-back.

Figure 7 shows the two possible variations of transistor construction. Figure 7a shows a P-N junction (emitter-base) connected to an N-P junction (base-collector), with the base (N) being common to both. The reverse is shown in Figure 7b. The terminal with the arrow is called the "emitter"; and the arrow points in the direction of conventional current flow (hole flow), which is the opposite direction of electron flow. In normal operation of the transistor, when properly biased, majority carriers (either holes for PNP type or electrons for NPN type) are "emitted" by the emitter and "collected" by the collector. The amount of this current flow is regulated by the voltage-controlled resistance properties of the transistor.

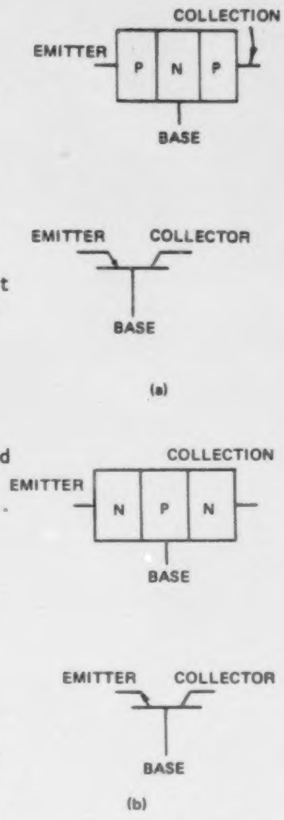


FIG. 7

The biasing of a transistor is important to its proper operation. Figure 8 illustrates how an NPN transistor is connected to a battery supply for correct operation. The emitter-base junction is always forward-biased; therefore, an NPN transistor will have its emitter connected to the minus terminal of the battery, and the base connected to the plus terminal. The resistance of the corresponding diode is low, and current flows very easily in the emitter-base circuit loop. The base-collector junction is always reverse-biased; and, as shown in Figure 8, the collector, which is N-type material, is connected to the plus terminal of a battery, and the base is connected to the minus terminal.

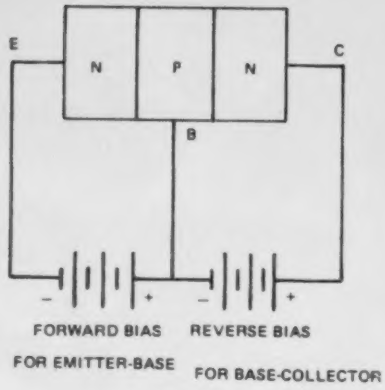


FIG. 8

The resistance of this base-collector loop is thus very high and, theoretically, no current flows in this loop. There is, however, one important requirement in transistor construction which makes the transistor work. This same requirement explains why transistor action cannot be achieved merely by placing two diodes back-to-back: In transistor construction, the base region is of very thin construction. The depletion regions of each diode practically touch each other. Under these conditions, the following happens: Majority carriers (in the case of the NPN transistor, this would be electrons injected by the forward-bias battery into the emitter) flow from the emitter to the base very easily, because of the low resistance to the forward-biased loop. When these majority carriers enter the base region, they "punch through" the base and end up in the collector region, where they are attracted by the plus voltage on the collector terminal. The percentage of electrons traveling through the base to the collector (punching through) is very large - about 90 to 99 percent.

The significance of this punch-through action is this: Normally the resistance of the base-collector junction is very high, and no current would flow in this circuit. But because of the punch-through, the resistance of the base-collector junction is altered, and current flow in the collector circuit is inversely proportional to the altered resistance. In effect, the resistance of the collector (output) circuit is being controlled by the resistance of the emitter (input) circuit. The resistance of the input circuit is transferred to the output circuit; hence, the origin of the name "tran-sistor" (meaning "TRANsfer-reSISTOR").

SUMMARY

Many diodes and transistors have the capacity to vary their resistance to current flow. Such variable resistance is a vital part of some circuits. The small size, great reliability and low cost of these modern devices have been responsible for great strides in electronics.

DIODE RESISTANCE

MATERIALS

1000 ohm resistor, 1/4 watt (current limiter).
1N270 Germanium Diode.
1N914 Silicon Diode.
1N758A Zener Diode.
0-20 volt d.c. power supply.
d.c. milliammeter.
d.c. micro-amp meter.
d.c. voltmeter.
Graph paper.

PROCEDURE

1. Connect circuit as shown in Figure 9.

2. Starting with 0 volts, change voltage in 0.1-volt steps, and record current in Data Table 1. Do not exceed 10 mA.

3. Reverse diode and repeat, except vary voltage in 1-volt steps; record results. Do not exceed 20 volts or 10 mA, whichever comes first. (Remember that current flow before zener point is reached will be in the order of micro-amps.)

4. Repeat 1-3 with the other two diodes, recording results.

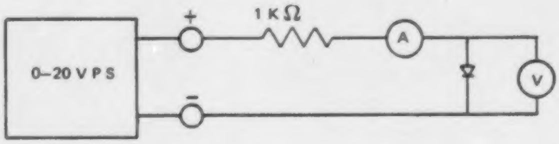


FIG. 9

5. Plot your results on graph paper labeled "volts" on the horizontal axis and "current" on the vertical axis. The graph should look like Figure 6. The germanium diode will have gradually-changing slopes. Knee-voltage may not be immediately recognized, and there may not be a definite "zener" point. The silicon diode should have a

noticeable knee-voltage at about 0.4 to 0.7 volts, and a noticeable zener point. The zener diode should have the same shape as the silicon diode in the forward region, but have a dramatic zener point at 9.6 volts reverse bias.

6. Calculate resistance of each diode at various points on the curve, using the formula R = V/I. Record in Data Table 2. Calculate resistance at the following points:
    a. Midpoint of junction resistance region.
    b. Knee-voltage.
    c. Midpoint of bulk resistance region.
    d. Midpoint of reverse resistance region.
    e. A point past the zener point.
7. How does the resistance of the three diodes differ?

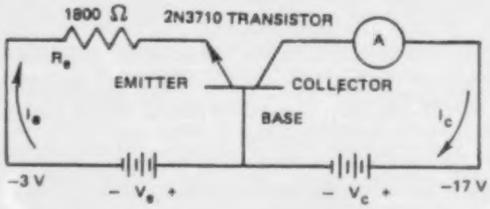
TRANSISTOR RESISTANCE

MATERIALS

2N3710 silicon NPN transistor.
0-20 volt d.c. dual power supply.
d.c. milliammeter.
1800 ohm resistor.
3900 ohm resistor.
6800 ohm resistor.
VTVM.
Graph paper.

PROCEDURE

1. Connect circuit as shown.
2. Calculate I_e (emitter current) from $I = \frac{V_e}{R_e}$. Record in Data Table 3.
3. Measure I_c (collector current); and compare with I_e; and record.
4. Calculate and record resistance of output



circuit from $R_c = \frac{V_c}{I_c}$.

5. Repeat 1-4 with R_e of 3900 ohms and then 6800 ohms. Record.

6. With output voltage set at 17 volts and R_e of 1800 ohms, vary input voltage from 1 to 5 volts in 0.5-volt steps; and record output current in Data Table 4.

7. Calculate and record output resistance R_c for each data point, and draw a graph of output resistance vs. input voltage.

8. What are the basic differences between transistor resistance and diode resistance?

DATE TABLE 1: DIODE RESISTANCE

Germanium Diode		Silicon Diode		Zener Diode	
Voltage (V)	Current (ma)	Voltage (v)	Current (ma)	Voltage (v)	Current (ma)

DATE TABLE 2: CALCULATED DIODE RESISTANCE

Resistance (Ω) of	Germanium	Silicon	Zener
a. Junction resistance region			
b. Knee voltage			
c. Bulk resistance region			
d. Reverse resistance region			
e. Past zener point			

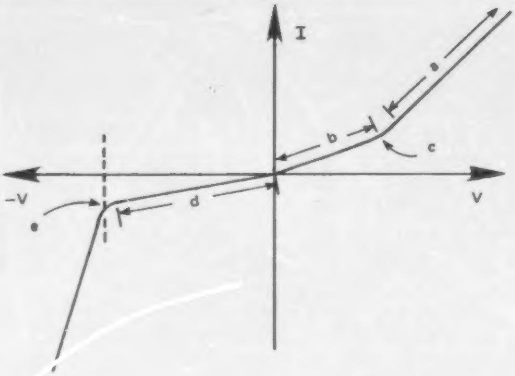
DATA TABLE 3: TRANSISTOR RESISTANCE

R_e (Ω)	V_e (V)	I_e (ma)	V_c (V)	I_c (ma)	R_c (Ω)
1800					
3900					
6800					

DATA TABLE 4: CALCULATED TRANSISTOR OUTPUT RESISTANCE

Steps (0.5 volts)	V_e (V)	I_c (ma)	V_c (V)	R_c (Ω)

SELF CHECK



1. Define the following terms:
   - a. Intrinsic material.
   - b. P-type material.
   - c. N-type material.
   - d. Depletion region.

2. On the curve at the right identify the following:
   - a. ____________________
   - b. ____________________
   - c. ____________________
   - d. ____________________
   - e. ____________________

3. Draw and label a diagram showing the parts of an NPN transistor and the proper connections for bias batteries.

4. Draw diagrams showing the movement of electrons and holes in a diode for:
   - a. Forward bias.
   - b. Reverse bias.

5. Explain how the resistance of the base-collector circuit of an NPN transistor is controlled by current in the emitter-base circuit.

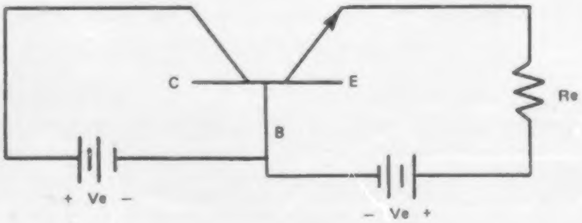
6. If the burnout of a diode occurs at 0.07 watt, what is the maximum current that can flow through the diode before burnout occurs? The bulk resistance of the diode is 1.5Ω.

7. What is the emitter current if the emitter voltage is 1.8 volts and the emitter resistance is 1500Ω?

8. What is the collector resistance if $V_c = 15$ V and $I_c = 1.9 \times 10^{-6}$ A?

9. What is the knee voltage of a diode if at this point the resistance is 40Ω and the current is 10 mA?

10. What is wrong with the way the transistor circuit shown below is connected?



RESISTANCE OF GAS LASER TUBES

During the relatively brief time since the construction of the first laser, modern technology has found a myriad of applications for lasers of many types. These applications embrace many different fields: electronics, manufacturing, communications, surveying, medicine, energy and many others.

In this module, the student will become familiar with the electrical characteristics of one class of lasers — the gas laser. These principles will be applied to delineate the general requirements for a gas-laser power supply.

In the laboratory, the student will measure the electrical properties of a gas discharge tube.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:

    Ion.
    Breakdown voltage.
    V-I characteristic curve.
    Sustaining voltage.

- State two ways in which neutral atoms in a gas laser tube can be ionized. Briefly explain each process.
- Describe the behavior of the voltage across a laser tube relative to the current through it for a tube operating in the negative resistance region.

- List the three elements of a gas-laser power supply, and briefly explain the function of each.
- Given the running supply voltage, the desired operating current, and the V-I characteristic of the tube, calculate the necessary ballast resistance.
- Given the V-I characteristic of a laser tube, determine the values of the breakdown voltage and of the sustaining voltage; and label the negative resistance region.
- Given the appropriate equipment, collect the data required to plot the V-I characteristic of a neon bulb.

DISCUSSION

Figure 1 depicts a gas-filled tube; it is similar to an ordinary gas laser tube. A voltage is maintained across the tube by connecting each electrode to one of the terminals of a battery or power supply. The electrode connected to the positive terminal is called the "anode." The electrode connected to the negative terminal is termed the "cathode."

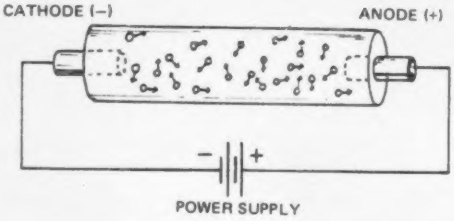


FIG. 1

Under ordinary circumstances, the atoms of the gas are electrically neutral and, therefore, are not attracted to either the anode or the cathode. No conduction occurs along the tube, and the resistance of the gas is said to be "infinite."

As the voltage across the tube increases, the pull exerted upon the electron of the gas atoms by the anode also becomes greater. The electrons, being negatively charged, are pulled toward the anode. The remaining atoms, less several electrons, now are positive ions and are pulled toward the cathode. Because the electrons in the atom are tightly bound to the positive nucleus, the initial pull exerted by the electrodes has no net effect. At some point, however, the pull becomes so great that an electron is torn away from a neutral atom, leaving the atom with one less electron than proton. This process is called "ionization," and the atom is called a "positive ion" due to an excess proton in the nucleus. In general, any atom not electrically neutral is called an "ion"; its charge depends upon whether it has an excess or a deficiency of electrons. The value of the voltage across the tube when ionization occurs is referred to as the "breakdown voltage."

When the voltage across the tube reaches the breakdown voltage, the electrons torn away from the neutral atoms move toward the anode; and the positive ions move toward the cathode (Figure 2). During passage to their respective electrodes, these charge particles undergo collisions with neutral atoms. The collisions can occur with sufficient force to knock electrons loose from other neutral atoms, creating more free electrons and ions in a process called "ionization by collision." This further increases the number of charge carriers and makes even more collisions occur, freeing even more electrons and ions in an avalanche effect. This rapid increase in the number of charge carriers causes a corresponding increase in the

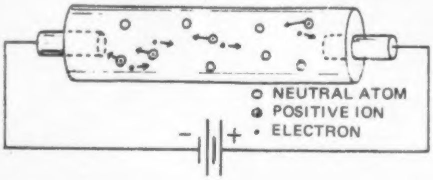


FIG. 2

current through the tube and decreases the voltage across it. These effects combine to decrease the resistance of the tube. When the tube operates in the region through which any increase in current causes a decrease in voltage across the tube, the tube is said to exhibit "negative resistance."

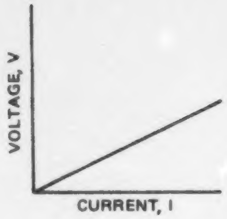


FIG. 3

In order to gain some insight into the concept of negative resistance, the student should examine some devices that exhibit more ordinary behavior: Recall from Module 5E1, "Ohm's Law," that many common materials (metals in particular) obey Ohm's Law. For these materials, called "ohmic" or "linear," the voltage increases in proportion to the current according to the relation presented by Equation 1.

$$V = IR \tag{1}$$

where
V = voltage across the device.
I = current through the device.
R = resistance.

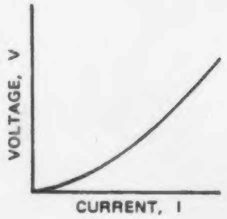


FIG. 4

A graph of voltage plotted against current (called the "V-I characteristic") for an ohmic device is always a straight line as in Figure 3. The resistance is calculated as the slope of the line in the V-I characteristic; and since the slope is constant, so is the resistance. Because this slope is positive for ohmic devices, the resistance also is positive. Devices that exhibit a positive resistance always operate in a stable fashion. Thus, if a voltage is established across the device, the current through it will remain constant.

Many other devices fall into the non-linear class. For these devices (semiconductors and vacuum tubes, for example), the V-I characteristic is not a straight line (Figure 4). The slope of the line, however, is everywhere positive, indicating that semiconductors and vacuum tubes are non-linear devices that exhibit a positive, though changing, resistance and, hence, that operate in a stable manner.

A gas discharge tube (e.g., a gas laser tube) is a non-linear device that exhibits negative resistance (Figure 5). The negative resistance region is recognizable on the V-I characteristic as the portion with a negative slope. In this region, increases in the current through the tube are accompanied by decreasing values of the voltage across the tube. The negative resistance region is very unstable. If the tube is operating in this region, the current will continue to increase, and will be accompanied by decreasing voltage. This process continues until the flat portion of the V-I characteristic is reached, by which time the current may have become large enough to damage the tube seriously.

The voltage corresponding to the flat portion of the curve is called the "sustaining voltage" and is the smallest value to which the voltage across the tube can fall while the discharge is maintained. The current still can be increased, even after the tube voltage has fallen to the sustaining value. The tube voltage, however, no longer will decrease as current is increased. The region of negative resistance always lies in the range of currents beginning with the breakdown voltage and terminating when the sustaining voltage is reached.

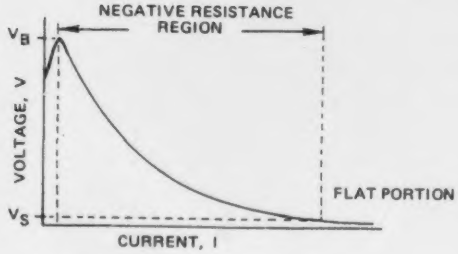


FIG. 5

Figure 6 is a block diagram of a typical gas laser power supply that consists of the following elements: (a) starter, (b) running supply, and (c) current limiter (also called a "ballast resistor").

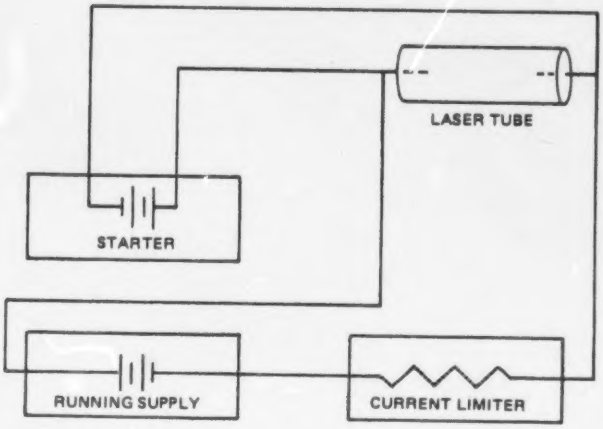


FIG. 6

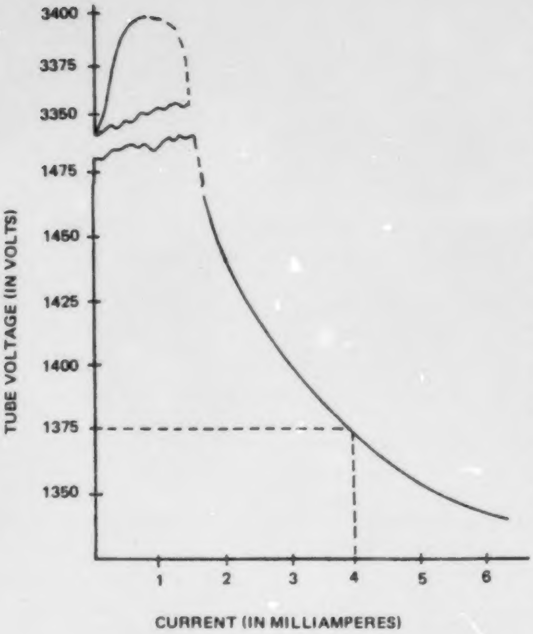


FIG. 7

The purpose of the starter is to generate a d.c. pulse of sufficiently-high voltage to cause ionization of the gas in the laser tube. For this reason, the voltage of the pulse provided by the starter must exceed the breakdown voltage of the gas. Figure 7 is the V-I characteristic of a laster tube containing a mixture of helium and neon gases. For this helium-neon laser, the starter-voltage pulse must be in excess of 3400 V.

Once ionization occurs, however, the high voltage provided by the starter is excessively large for maintaining the discharge. Hence, gas laser power supplies are designed in such a way that the starter element is de-energized when ionization occurs in the tube. The discharge then is maintained by the lower voltage-running supply.

The major function of the running supply is to provide sufficient current and voltage to maintain a continuous discharge through the gas at the desired current level. To accomplish this function, the running supply operates at a lower voltage than the starter and remains on as long as the discharge continues.

It should be noted, however, that, of itself, the running supply cannot maintain a stable discharge through the laser tube because gas lasers are operated in the negative resistance region, where the voltage across the gas depends upon the current through it. Hence, to insure a stable discharge, the current through the tube must be held at some constant value. The method used to achieve constant current employs a current limiter (also called a "ballast resistor") in series with the tube and the running supply.

The function of the ballast resistor is to restrict the current in the circuit to a safe and constant value. The appropriate value of the ballast resistor depends upon three factors:

(a) the desired operating current of the laser.
(b) the voltage output of the running supply.
(c) the V-I characteristic of the laser tube.

The ballast-resistor value is estimated easily from Equation 2.

$$R_B = \frac{V - V_t}{I} \qquad (2)$$

where

R_B = ballast resistance.
V = running supply voltage.
V_t = tube voltage when current is I.
I = current.

EXAMPLE A: BALLAST RESISTANCE FOR A He-Ne LASER

Given: The helium-neon laser described by the V-I characteristic of Figure 7 is to be operated from a 3000-V running supply at a current level of 4mA.

Find: The ballast resistance required.

Solution: $R_B = \frac{V - V_t}{I}$

Use the V-I characteristic (Figure 7) to find V_t when I = 4mA:

V_T = 1375 V

Then:

$R_B = \frac{3000\ \text{V} - 1375\ \text{V}}{4 \times 10^{-3}\ \text{A}}$ (Using I in amps)

$R_B = 406{,}250\ \Omega \approx 406\text{k}\Omega$

The ballast resistance required to stabilize the current at 4 mA is approximately 406 kΩ.

SUMMARY

The discharge that takes place in a gas laser tube is initialized by a high-voltage pulse that ionizes the gas in the tube. Collisions between neutral atoms and ions increase the conduction through the tube and give rise to the negative resistance region of the laser tube's V-I characteristic. Hence, gas laser power supplies initially must provide a voltage pulse in excess of the breakdown voltage and must limit the current provided by the running supply by means of a ballast resistor. The value of the ballast resistor can be estimated from a knowledge of the running supply voltage and the laser's characteristics.

ELECTRICAL PROPERTIES OF A GAS DISCHARGE TUBE

MATERIALS

Variable d.c. power supply, 0-150 V, 0-1 mA.
Neon bulb.
Microammeter, 0-50 µA.
Vacuum tube voltmeter (VTVM) or electronic digital multimeter (DMM), 0-150 V.
Variable resistor 0-5 MΩ, 0.1 W.

PROCEDURE

1. Connect the circuit shown in Figure 8. Before placing variable resistor in circuit, use ohmmeter on DMM or VTVM to adjust its value to 1MΩ (1×10^6 Ω).

2. Adjust output of power supply to its maximum value of 150 V. Measure and record voltage across power supply V_{max}. Note: The neon bulb should be glowing, indicating that a discharge is taking place.

3. In the Data Table, record the values of current I through the bulb and the voltage V_b across the bulb.

4. Reduce output of power supply until current has decreased approximately 5µA. Record new values of I and V_b in Data Table.

5. Plot this value of V_b versus I on a suitable graph.

6. Repeat Steps 4 and 5 until the current has been reduced to zero and entire V-I characteristic has been plotted. You may need to take data in smaller increments as current is decreased. Draw a smooth curve thorugh points on the V-I characteristic.

7. From the V-I characteristic, determine values of breakdown voltage V_B and of the sustaining voltage V_s. Record these values.

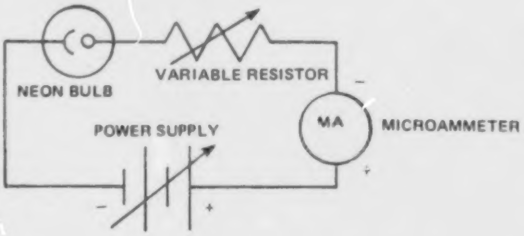


FIG. 8

8. Choose a point on V-I characteristic near center of negative resistance region. Record V_b and I for this point.

9. Estimate ballast resistance R_B necessary to stabilize current through tube at value chosen in Step 8 when power supply voltage is adjusted to its maximum value. Use Equation 2 with values of $V_t = V_b$, I from Step 8, and $V = V_{max}$ as measured in Step 2.

10. To test your prediction, turn off power supply, and remove variable resistor from circuit. Adjust its resistance to equal R_B (as calculated in Step 9), and replace it in circuit.

11. Turn on power supply and adjust its output to maximum. Measure and record the current through the bulb, and compare it to desired value (from Step 8).

12. Explain how this experiment demonstrates the principles explained in the Discussion section.

DATA TABLE

V_{max} = ________ volts

I(μA)	V_b(V)

V_B = ____________ V

V_s = ____________ V

Point in negative resistance region:

I = ____________ μA

V_b = ____________ V

$R_B = \frac{V_{max} - V_b}{I}$ = __________ Ω

Measured current = ______________ μA

SELF CHECK

1. Define:

   Ion.
   Breakdown voltage.
   V-I characteristic.
   Sustaining voltage.

2. State two ways in which neutral atoms in a gas laser tube can be ionized. Briefly explain each process.

3. Describe the behavior of the voltage across a gas laser tube relative the current through it when the tube is operating in the negative resistance region.

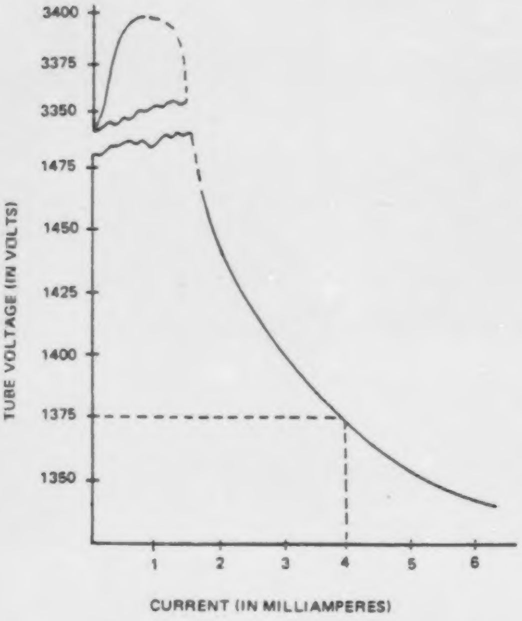


FIG. 9

4. List the three elements of a gas laser power supply, and briefly explain the function of each.

5. A laser described in the adjoining V-I characteristic graph is to be operated from a 2500-V power supply. The desired operating current is 3.5 mA. Find the ballast resistance necessary to stabilize the current at that value.

6. Determine the breakdown voltage and sustaining voltage for the laser which has the V-I characteristic shown in the adjoining graph. Label the negative resistance region on the graph.

7. If the ballast resistance necessary to stabilize an operating current of 5 mA is 400 kΩ, find the supply voltage. Assume that the V-I characteristics of the laser are described in Figure 7.

8. If the tube voltage is related to the current by the relationship $V_t = 4/I$ in the negative resistance region, what is the tube voltage when the current is 3.5 mA?

9. Find the tube voltage if the ballast resistance is 275 kΩ, the supply voltage is 3200 V and the current is 4.3 mA.

5E8

VOLT-OHM MILLIAMMETER

Module 5-0, "Resistance," discusses the relationship of voltage, current and resistance known as Ohm's Law. A volt-ohm-millammeter (VOM) is a device used to measure all three of these electrical quantities. It is one of the most versatile and commonly-used test instruments in technical fields related to electrical and electronic circuits. It is easy to use, rugged, portable, and inexpensive.

Commercial VOM's are available at prices that range from twenty to several hundred dollars. Besides the quality of the components and workmanship that goes into construction, sensitivity is an important criteria in selecting a VOM. Applications determine the sensitivity needed. A meter with a sensitivity range of 20,000 Ω/V is adequate for measurement on most practical circuits, while very high-resistance circuits require meters with a sensitivity of 50,000 to 100,000 Ω/V. On the other hand, some circuits permit the use of a meter with a sensitivity of 1000 Ω/V. Meter sensitivity is determined largely by construction of the movement used to cause needle deflection.

This module deals with the basic elements of the meter and its measurement of the most common electrical quantities. Proper use and precautions to be taken with the instrument also are discussed.

OBJECTIVES

Upon completion of this module, the student should be able to:

- List four electrical quantities measured by a VOM.
- Given an actual application requiring the use of a VOM, determine proper adjustments to the range and function switches (when power load is known or unknown) to measure the following:

Voltage.
Current.
Resistance (including zeroing the scale).

- Use a VOM to determine —

    a.c. or d.c. voltage.
    d.c. current.
    Resistance.

DISCUSSION

There are many brands of volt-ohmmeters (VOM's) having different sizes, shapes, and styles. Most of them measure the same basic electrical quantities, which are —

- d.c. voltage.
- d.c. current.
- a.c. voltage.
- Resistance.

The VOM consists of a meter movement, a voltage source (battery), a rectifier circuit, and a collection of resistors that can be switched into the circuit. The position of the needle of the meter is proportional to the direct current through the coils of the meter movement. The additional circuit components are arranged to make meter current proportional to some other quantity being measured. The basic circuits for each type of measurement are described below.

D.C. VOLTAGE

Figure 1 shows the basic current used for d.c. voltage measurement. The value of the resistor is chosen to allow a current to flow through the meter to produce a full-scale needle deflection when a specific voltage is applied to the input. The voltage calibration scale of the meter is changed when one resistor is switched to another.

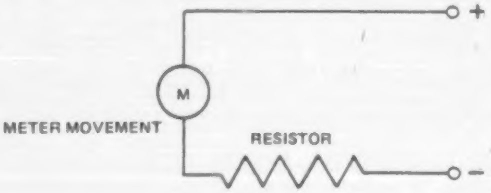


FIG. 1

A.C. VOLTAGE

The circuit used to measure a.c. voltage is the same used for d.c. voltage measurement, except that a rectifier circuit is added. The rectifier changes the a.c. current to a d.c. current to drive the meter movement. Rectifier circuits will be discussed in a later module.

D.C. CURRENT

Figure 2 shows the basic circuit employed in the measurement of d.c. current. The value of the resistor, called a "shunt," is chosen to allow only a small fraction of the current to flow through the meter movement. Most of the current flows through the lower resistance shunt. Current calibration of the meter is changed when one resistor is switched to another.

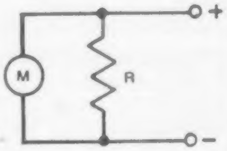


FIG. 2

RESISTANCE

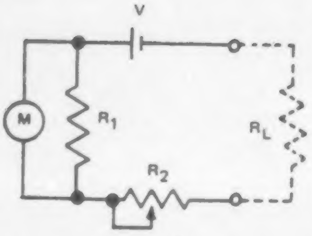


FIG. 3

Figure 3 shows the basic circuit used for resistance measurement in a VOM. The meter responds to a current produced by a battery in the VOM. This current passes through the resistance being measured R_L, and part of it flows through the meter movement to produce needle deflection. Any change in the value of R_1 changes the resistance scale of the meter. R_2 is a variable resistor used to zero the meter.

METER FACE

The front portion or face of the meter includes a function switch and a range switch (Figure 4). The function switch selects the variable to be measured, and is commonly a round knob with a pointer that indicates the selection of measurement to be made. It should point to a.c. for a.c. voltage measurement and +d.c. or -d.c. for d.c. voltage, current and resistance measurements. The range switch is used to select the full-scale value that can be measured in current, voltage, or resistance. For example, if the function switch were set to indicate +d.c. and the range switch were set for 250 V, the pointer would show full-scale deflection when a 250 V d.c. source is connected to the test leads of the VOM. The best way to become familiar with a VOM is to use the instrument to perform the measurements described in the laboratory.

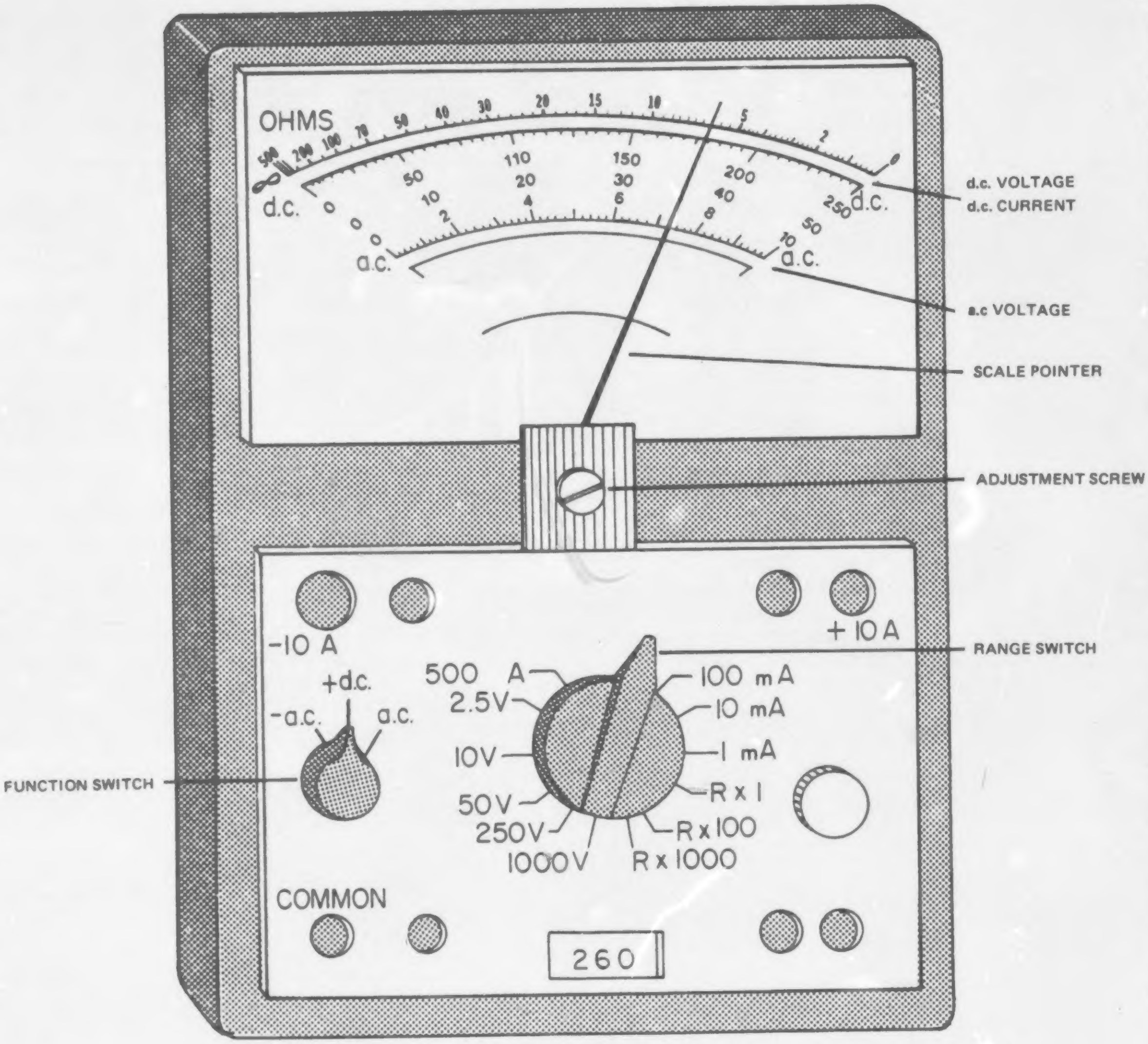

OHMS
d.c.
a.c.
d.c.
a.c.
d.c. VOLTAGE
d.c. CURRENT
a.c VOLTAGE
SCALE POINTER
ADJUSTMENT SCREW
-10 A
+10 A
RANGE SWITCH
500 A
2.5V
10V
50V
250V
1000V
100 mA
10 mA
1 mA
R x 1
R x 100
R x 1000
+d.c.
-a.c.
a.c.
FUNCTION SWITCH
COMMON
260

FIG. 4

LABORATORY

MEASUREMENTS WITH A VOLT-OHM MILLIAMETER

MATERIALS

a.c./d.c. power supply, preferably with a variable output from 0 to 10 volts d.c. and 0 to 12 volts a.c.
Four resistors (one 100 Ω, one 50 Ω and two 25 Ω).
Two test leads from the power supply, preferably one coded red and one black.
A VOM capable of measuring the four electric variables discussed (Simpson type 260 or equivalent).

PROCEDURE

The remainder of this module is a self-paced student exercise in the proper methods for making voltage, current, and resistance measurements with a VOM.

TEST CIRCUIT CONSTRUCTION

Make sure all items in the materials list (or acceptable substitutes) are available. Carefully follow these steps:

1. Make sure power supply switch is off.
2. Connect the resistors (listed in the materials section) in series to create a circuit (Figure 5).
3. Attach positive lead (usually red) from power supply (Figure 6) to the end of test circuit nearest R_1.
4. Attach negative lead (usually black) from power supply to end of test circuit nearest R_4. Use test circuit (attached to power supply) to check basic VOM functions described below.

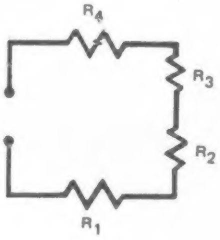


FIG. 5

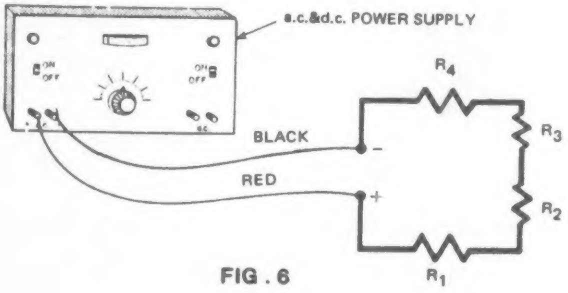


FIG. 6

PROPER USE OF A VOM

Precautions taken to avoid excessive error of measurement and to prevent meter damage can be categorized in accordance with variables being measured.

VOLTAGE MEASUREMENT PRECAUTIONS

When approximate voltage is not known, set range switch to highest setting available. This higher range prevents overranging (damage caused by more energy being sent through the meter than that circuit is capable of recording). After this reading is made, select a lower setting if needed for a more accurate reading.

1. The scale selected should allow readings to be taken from the upper 50% of the scale, when possible, to ensure the most accurate reading.
2. If meter reading changes significantly when switched from one range to another, one probable cause is meter loading. Use a more sensitive VOM if this occurs. Also, some meters need to be rezeroed on each scale before giving consistent readings.
3. To measure voltage, the function switch must be set on voltage — not on current or resistance.

DIRECT CURRENT (D.C.) VOLTAGE MEASUREMENT

Use previously constructed test circuit (Figure 6) and follow these steps:

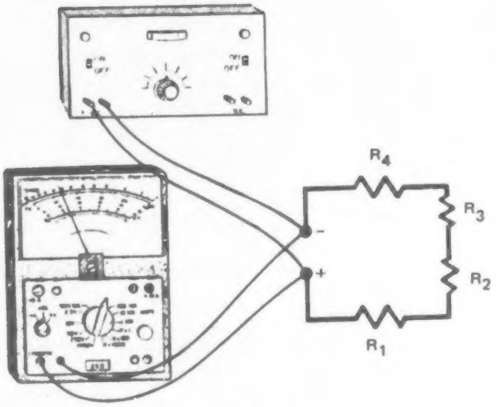


FIG . 7

1. Make sure power supply switch is "off."
2. Set VOM function switch to +d.c.; set range switch to highest voltage scale value (1000 V for VOM shown in Figure 4).
3. Connect positive lead from VOM (Figure 7) to test circuit at the point closest in test circuit to the positive source (from power supply).
4. Connect negative lead to test circuit at the point in test circuit nearest negative source (from power supply).

5. Stop. Request a circuit check from instructor before proceeding.

6. Turn on power supply.

7. The voltmeter reading should be essentially zero at this high range scale.

8. Switch to a lower voltage range scale; the needle should move toward center of scale.

9. Find lowest scale usable without overranging meter; read the meter.

10. Record in Data Table the reading from voltage scale that corresponds to selected value of range switch. Example: reading shown in Figure 8 indicates 6.8 volts, 34 volts or 170 volts on d.c. voltage scales, depending on which voltage scale was selected by range switch.

11. Turn power supply off.

12. Disconnect power supply (only) from circuit.

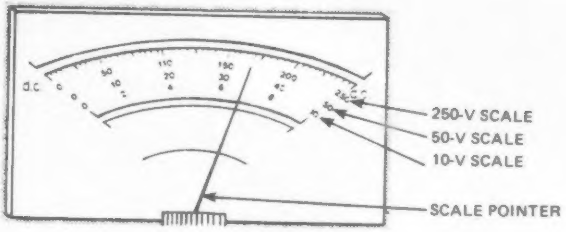


FIG. 8

ALTERNATING CURRENT (A.C.) VOLTAGE MEASUREMENT

1. Make sure power supply switch for a.c. and d.c. is turned off.

2. Connect test circuit lead wires to a.c. outlet of power supply.

3. Set VOM function switch to a.c.

4. Set range switch to highest voltage range.

5. Stop. Request a test circuit check before turning power supply on.

6. Turn power supply on.

7. Reduce range switch setting until scale pointer deflection is in upper half of meter range.

8. Take reading from voltage scale that corresponds to selected value (of range switch); record reading here: _________.

Example reading (Figure 9) indicates 6.8, 34 or 170 volts a.c., depending on voltage scale selected by range switch.

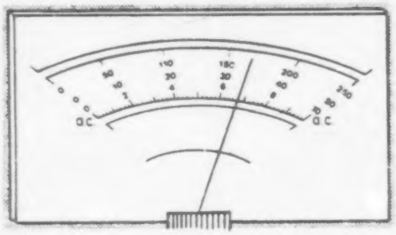


FIG. 9

9. Turn power supply off.

10. Disconnect power supply from circuit.

CURRENT MEASUREMENT PRECAUTIONS

1. Always select a range high enough to prevent meter from overranging.

2. Connect VOM leads in such a manner that meter is in series with circuit under test.

DIRECT CURRENT (D.C.) MEASUREMENT

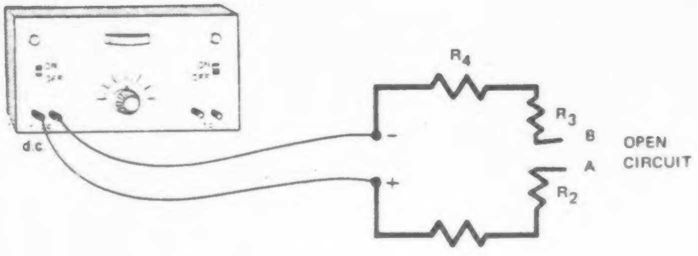


FIG. 10

1. Make sure that main switch of power supply is turned off. Connect power supply leads to test circuit.

2. Use circuit previously created; interrupt or break circuit as shown in Figure 10.

3. VOM function switch should point to +d.c.

4. Set range switch to highest current range scale available [500 mA (milliampere) for VOM shown in Figure 4].

5. Connect positive lead from VOM to point A (Figure 10).

6. Connect negative lead to point B.

7. Turn on power supply. Note: If scale pointer moves off scale, polarity (positive, +, and negative, -) is reversed and may be corrected by turning function switch from +d.c. to -d.c. or vice versa.

8. Reduce scale value with range switch until reading can be made in upper half of scale without overranging meter.

9. Reading should be made from d.c. scale by using 0-10 values on that scale. Example reading shown in Figure 11 indicates a d.c. current reading of 9 with function switch at 10 mA.
10. Record reading here: ________.
11. Turn power supply off; disconnect VOM; and rejoin break in circuit.

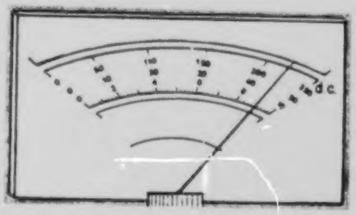


FIG. 11

RESISTANCE MEASUREMENT PRECAUTIONS

1. Make sure power supply is removed from circuit.
2. To ensure accurate measurement, disconnect (from circuit) one end of component to be measured.
3. Each time a new resistance scale is selected, "zero" meter. The meter is "zeroed" by touching together the positive and negative leads and simultaneously turning "zero ohms" selector (Figure 4) until indicator reads zero.

RESISTANCE MEASUREMENT

1. Check each component in test circuit for its resistance value.
2. Set VOM function switch to either +d.c. or -d.c. (polarity is within meter, not circuit).
3. Set range switch to R x 100.
4. Zero meter by touching leads and turning "zero ohms" knob (Figure 4) until meter reads zero ohms on top scale. Note: Needle deflection is opposite for resistance since the zero end of scale is at far right of meter.
5. Disconnect one end of resistor R_1.
6. Attach VOM leads to each side of resistor under test, and read resistance from top scale. Figure 12 indicates a 100 Ω resistance when range switch is set at R x 100.
7. Change range switch to R x 1, and read again; record both readings here: R x 100 = ______ and R x 1 = ______.

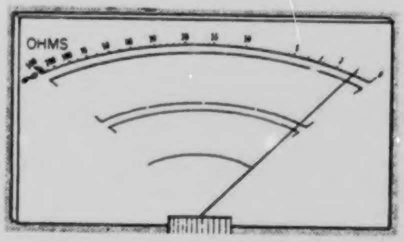


FIG. 12

8. Repeat these steps for each of other three resistors.

9. Record your readings in Data Table.

10. Reconnect all resistors in series.

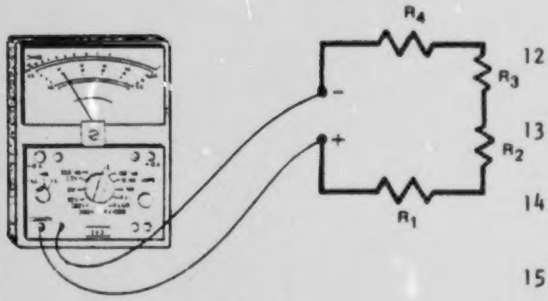


FIG. 13

11. Measure resistance of entire circuit as in Figure 13. Record reading here: _______.

12. Compare this reading with totals from the Data Table.

13. If more than a 25% difference is observed, check with instructor.

14. Disassemble circuit and return all equipment.

15. Describe how a VOM can be used in troubleshooting a circuit.

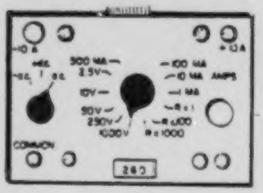
DATA TABLE				
		Range Switch Scale		
		R x 1	R x 100	
Resistor	R_1			
	R_2			
	R_3			
	R_4			
		= Total Resistance		

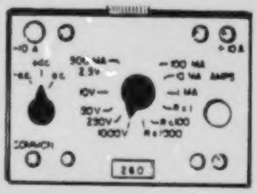
SELF CHECK

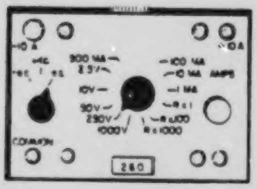
1. Check functions commonly measured by a VOM.

_____ d.c. voltage.
_____ a.c. voltage.
_____ watts.
_____ d.c. current.
_____ a.c. current.
_____ resistance.

2. Circle the meter that illustrates proper settings to be used to measure an approximate 500 V a.c. source.

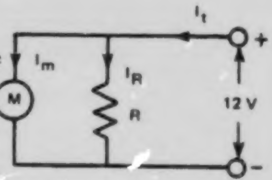

a.


b.


c.

3. The maximum current which flows in a meter used to measure d.c. voltage is 1 mA. What resistance should the meter have if the full scale needle deflection of the meter corresponds to 5 V?

4. In the circuit shown at right, the current flowing through the meter is 0.1 mA and the applied voltage is 12 volts. If the total current flowing out of the voltage source is 10 mA, find the value of the shunt resistance.



5. Consider the circuit of Figure 3 in the module. If R_1 = 1000Ω, R_2 = 10Ω, and V = 12 V, find the value of R_L if a current of 2 mA flows out of the battery and the current through the meter is 0.5 mA.

6. What is the current which flows through a d.c. voltage meter if the meter reads 7 volts and has a resistance of 10 kΩ? (See Figure 1).

7. What is the maximum voltage which should be applied to a d.c. current meter (Figure 2) if full scale deflection of the meter occurs when the current through the meter is 0.3 mA and the resistance of the meter is 30 kΩ?

5T1

THERMAL CONDUCTIVITY

The ability of various materials to conduct heat varies considerably. Materials that conduct heat are called "conductors" or "heat conductors." The best heat conductors are metals. The poorer conductors, such as wood, asbestos, etc., are called insulators. The ability of a substance to transmit heat by conduction is a physical property of the particular material and is characterized by its "thermal conductivity." In this module, we shall investigate heat flow rate of several materials and examine how heat flow rate varies with material, thermal conductivity, and material thickness.

OBJECTIVES

To complete this module, the student should be able to:

- Define:
  - Heat conductor.
  - Insulation.
  - Thermal conductivity.
  - Temperature gradient.
  - Heat flow rate.
- Explain how heat flow rate depends on the thermal conductivity, temperature gradient and geometry of a substance.
- Calculate typical heat flow rate problems, given formulas, materials and properties.
- Demonstrate how heat flow rate varies with thermal conductivity and material thickness.
- Given a list of thermal conductivities for various materials, identify good heat conductors and good heat insulators.

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DISCUSSION

A metal surface feels cooler than a wooden surface even though they are both at room temperature. This effect is caused by the higher thermal conductivity of the metal. Heat from the hand is conducted away by the metal much more quickly than by the wood. All materials possess varying abilities to conduct heat. The heat transferred through a material can be calculated with the following equation:

$$H = \frac{k\ A\ (T_2 - T_1)}{x} \qquad (1)$$

where

H = heat flow rate (Btu/h) or (cal/s).
k = thermal conductivity ($\text{Btu} \times \text{in/h} \times \text{ft}^2\ \text{F}^\circ$) or ($\text{cal} \times \text{cm/s} \times \text{cm}^2\ \text{C}^\circ$).
A = area (ft^2) or (cm^2).
x = thickness (in) or (cm).
$(T_2 - T_1)$ = difference in temperature between the two surfaces "x" inches apart (F°) or (C°).
$(T_2 - T_1)/x$ = temperature gradient in F°/inch or C°/cm.

The terms used here are shown in Figure 1.

From Equation 1, the rate of heat flow through a particular material depends on several factors. When the area A is doubled, the flow rate also doubles. Since the thickness x is in the denominator, a doubling of the thickness will cut the flow rate in half. An increase in the temperature difference will also increase the heat flow rate. The remaining term in Equation 1 is the thermal conductivity k. A large value of k gives a high conduction rate. Every material has its own coefficient of thermal conductivity. For example, concrete has a thermal conductivity of:

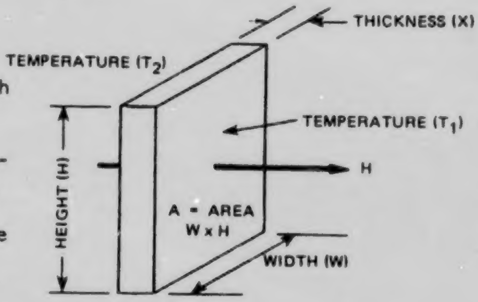


Fig. 1

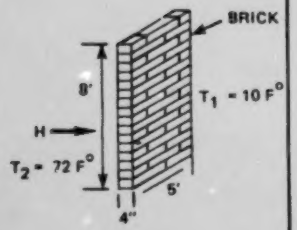
$$\frac{12\ \text{Btu} \times \text{in}}{\text{h} \times \text{ft}^2\ \text{F}^\circ}.$$

The thermal conductivities of several common materials are given in Table 1.

TABLE 1. MATERIAL THERMAL CONDUCTIVITIES

Material	Conductivity (Btu x in/h x ft² F°)
Plywood	0.80
Glass fiber insulation	0.25
Concrete	12.0
Brick	9.0

To emphasize this concept further, consider Examples A and B.



EXAMPLE A: HEAT FLOW THROUGH BRICK WALL

Given: An old, broken, 5 ft x 8 ft window is removed and replaced with 4" thick brick. On a particular winter day, the outside temperature is 10° F and the inside temperature is 72° F.

Find: Heat flow through bricked-up opening.

Solution: From Table 1, the thermal conductivity of 4" brick is:

$$k = \frac{9\ \text{Btu} \times \text{in}}{\text{h} \times \text{ft}^2\ \text{F}^\circ}$$

$$A = 5\ \text{ft} \times 8\ \text{ft} = 40\ \text{ft}^2$$

$$x = 4\ \text{in}$$

$$(T_2 - T_1) = 72^\circ\ \text{F} - 10^\circ\ \text{F} = 62\ \text{F}^\circ$$

$$H = \frac{kA\ (T_2 - T_1)}{x}$$

$$= \frac{9\ \text{Btu} \times \cancel{\text{in}}}{\text{h} \times \cancel{\text{ft}^2}\ \cancel{\text{F}^\circ}} \times \frac{40\ \cancel{\text{ft}^2} \times 62\ \cancel{\text{F}^\circ}}{4\ \cancel{\text{in}}} = 5580\ \frac{\text{Btu}}{\text{h}}.$$

From Examples A and B, and Equation 1, materials with higher coefficients of thermal conductivity will have higher heat flow rates if all other parameters are the same. Materials with large values of thermal conductivity have little resistance to heat flow. On the other hand, materials with low thermal conductivities ($k = 0.25$ Btu x in/h x ft^2 F°) are the most efficient insulators.

EXAMPLE B: HEAT FLOW THROUGH CONCRETE AND PLYWOOD

Given: All conditions being equal.

Find: Thickness of concrete that will have the same heat flow as 1 square foot of 1 inch plywood.

Solution:

$$H_c = \frac{k_c\ A\ (T_2 - T_1)}{x_c} = \frac{12\ A\ (T_2 - T_1)}{x_c}$$

$$H_p = \frac{k_p\ A\ (T_2 - T_1)}{x_p} = \frac{0.80\ A\ (T_2 - T_1)}{x_p}$$

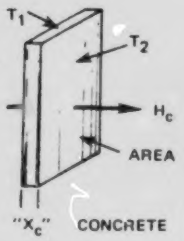
Equating the heat flows:

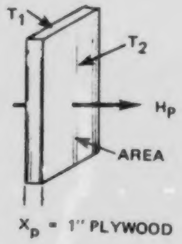
$$\frac{12\ \cancel{A}\ \cancel{(T_2 - T_1)}}{x_c} = \frac{0.80\ \cancel{A}\ \cancel{(T_2 - T_1)}}{x_p}$$

$$x_c = \frac{12}{0.80} \times p$$

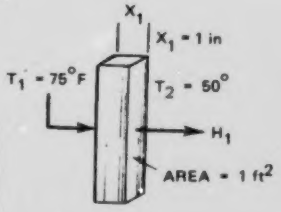
$$x_p = 1 \text{ inch}$$

$$x_c = 15 \text{ inches.}$$





Equation 1 indicates that if heat is conducted through two different thicknesses of the same material, the thicker sample would conduct less heat per second. This effect is illustrated in Example C.



EXAMPLE C: HEAT FLOW THROUGH DIFFERENT THICKNESSES OF PLYWOOD

Given: Plywood 1" thick and 12" thick.

Find: Heat flow through each thickness.

Solution:

(a)

$$H_1 = \frac{k\ A\ (T_2 - T_1)}{x_1} = \frac{0.80\ \text{Btu} \times \cancel{\text{in}}}{\text{h} \times \cancel{\text{ft}^2}\ \cancel{\text{F}}} \times \frac{1\ \cancel{\text{ft}^2} \times 25\ \cancel{\text{F}}}{1\ \cancel{\text{in}}}$$

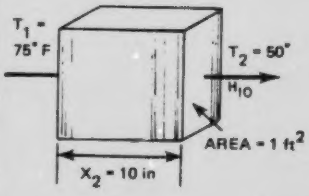
$$H_1 = 20.0\ \frac{\text{Btu}}{\text{h}}.$$

(b)

$$H_{10} = \frac{k\ A\ (T_2 - T_1)}{x_2} = \frac{0.80\ \text{Btu} \times \cancel{\text{in}}}{\text{h} \times \cancel{\text{ft}^2}\ \cancel{\text{F}}} \times \frac{1\ \cancel{\text{ft}^2} \times 25\ \cancel{\text{F}}}{10\ \cancel{\text{in}}}$$

$$H_{10} = 2\ \frac{\text{Btu}}{\text{h}}.$$

Note: The heat flow rate is 10 times less for a sample 10 times thicker.



From these examples, it is shown that the resistance to heat flow can be increased either by choosing a material with a lower thermal conductivity, k, or by increasing the thickness, x, of the material.

EFFECT OF THERMAL CONDUCTIVITY ON HEAT FLOW RATE

MATERIALS

Three heat lamps.
Three oven thermometers.
Container with trays for holding samples.
Three samples of materials with the same area and thickness. (Wood, concrete, and steel are used in this example, but any three materials with markedly different thermal conductivities will be sufficient.)

PROCEDURE

1. Place three different materials such as wood, concrete and steel in trays A, B and C. The geometry of each tray must be identical.
2. Place an oven thermometer under each of the materials or in appropriately drilled holes near the bottom of each sample as shown in Figure 2. Record the temperature of each thermometer. (Each should read the same room temperature.)
3. Turn on heat lamps and allow a sufficiently long period of heating time to insure that each oven thermometer has reached equilibrium, that is, has reached its highest temperature value.
4. Record temperatures in Data Table 1.
5. From appropriate handbooks, look up conductivities (k) for the materials used and record in Data Table 1.
6. From the recorded data, determine which material permitted the largest and the smallest heat flow rate. Is it true that, other things being equal, the heat flow rate depends directly on the thermal conductivity? What measurement made above was used as an indication of the heat flow rate?

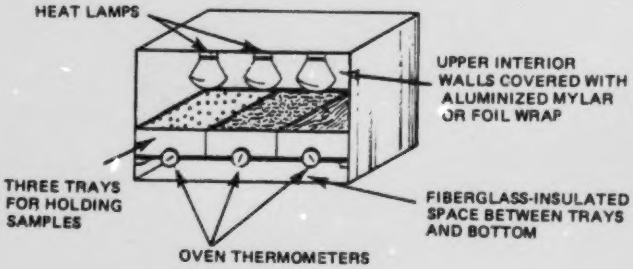

HEAT LAMPS
UPPER INTERIOR WALLS COVERED WITH ALUMINIZED MYLAR OR FOIL WRAP
THREE TRAYS FOR HOLDING SAMPLES
FIBERGLASS-INSULATED SPACE BETWEEN TRAYS AND BOTTOM
OVEN THERMOMETERS

Fig. 2

DATA TABLE 1.			
Sample	Material	k $\frac{Btu \times in}{h \times ft^2\ F^\circ}$	T (°F)
A	Wood		
B	Concrete		
C	Steel		

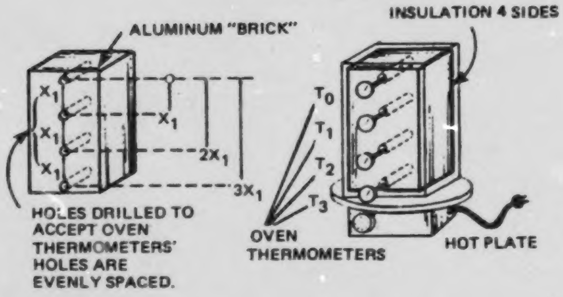
EFFECT OF THICKNESS ON HEAT FLOW RATE

MATERIALS

Four oven thermometers.
One piece of aluminum or steel about the size of a "house brick," ~ 2" x 4" x 8".
Ordinary hot plate (110 V).
Sufficient insulation (glass wool or pipe-wrap cork) to cover sides of aluminum "brick" described above.

PROCEDURE

1. Drill holes in aluminum (or steel) brick to just permit insertion of the oven thermometers stems (Figure 3a).
2. Wrap aluminum with appriate insulation so that only top and bottom remain uncovered. Place brick on hot plate, insert oven thermometers, and turn on hot plate. Allow sufficient time for each thermometer to reach equilibrium temperature (Figure 3b).
3. Record temperature of each thermometer and fill in Data Table 2.
4. Plot ΔT versus thickness (ΔX) on a graph


ALUMINUM "BRICK"
X_1
X_1
X_1
X_1
$2X_1$
$3X_1$
HOLES DRILLED TO ACCEPT OVEN THERMOMETERS' HOLES ARE EVENLY SPACED.
INSULATION 4 SIDES
T_0
T_1
T_2
T_3
OVEN THERMOMETERS
HOT PLATE

(a) (b)

FIG. 3

as shown in Figure 4.

5. What does the graph indicate concerning the heat flow rate as a function of thickness?

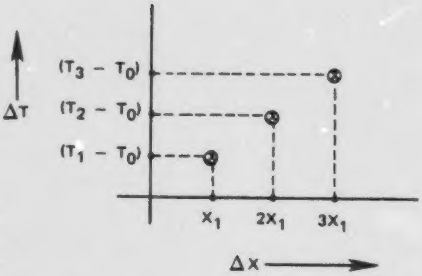


FIG. 4

DATA TABLE 2.				
Thickness	(Top) o	X_1 = __inches	$2X_1$ = __inches	$3X_1$ = __inches
Temperature	T_o = __°F	T_1 = __°F	T_2 = __°F	T_3 = __°F
ΔT		(T_1-T_o) = __F°	(T_2-T_o) = __F°	(T_3-T_o) = __F°

SELF CHECK

1. Define:
    - Heat conductor.
    - Insulation.
    - Thermal conductivity.
    - Temperature gradient.
    - Heat flow rate.
2. The heat flow rate through a plywood wall is 700 Btu per hour. The wall is 8 ft x 12 ft, and the plywood is 3/4"

thick. The temperature on the cool inside surface of the wall is 72°F. What is the outside temperature?

3. What thickness of fiberglass insulation is required for a heat flow rate through one square foot of insulation of 5 Btu/h if the temperature difference is 50 F°?

4. What is the thermal conductivity of a type of wood if a wall made of the wood is 6 inches thick and heat flows through the wall at a rate of 200 Btu/hr when the temperature difference on opposite sides of the wall is 30 F° and the wall has an area of 40 ft^2?

5. If the heat flow through one outside wall (area 60 ft^2) is 5000 Btu/hr on a cold day, what is the heat flow out of another outside wall of the house if the second wall is made of the same material and is the same thickness as the first but has an area of only 40 ft^2?

6. The inside temperature of a house is maintained at 72°F. If the outside temperature is 40°F, the heat flow through a particular wall is 1000 Btu/hr. What is the outside temperature if the heat flow through the same wall is 2000 Btu/hr?

POWER

APPLICATION MODULES

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6F1

HYDRAULIC POWER

Fluid power is power transmitted by a pressurized liquid or gas. It may be used to power machines or to control or regulate machines and power sources. Applications of fluid power are as diverse as hydro-electric power generation, metal forming, paper making, materials conveying, locomotion, space exploration or computing. Fluid power equipment ranges in size from huge presses to miniature digital components; fluids range from super-heated steam to liquid nitrogen.

When the fluid used is an incompressible liquid, the system is known as hydraulic. This module will discuss the applications, advantages and disadvantages of hydraulic power. In the laboratory, the student will measure power in a hydraulic system.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define the following terms:
  - Hydraulic system.
  - Pneumatic system.
  - Pressure head.
- Given the depth of fluid in a tank and the specific gravity of the fluid, determine the pressure at the bottom of the tank.
- Determine the horsepower required to pump water into a tank of a given height at a given rate.
- List three disadvantages of using fluid power and five advantages of using hydraulic power.
- Given the appropriate equipment, determine the horsepower requirements for a fluid pump in a hydraulic system.

DISCUSSION

Fluid systems are divided naturally into two broad categories. Pneumatics is the study of gases (compressible fluids) in motion. Hydraulics is the study of liquids (incompressible fluids) in motion. The behavior of a hydraulic and pneumatic system varies widely because of the differences in compressibility between the two fluids. This module discusses power in hydraulic systems.

Figure 1 is an example of a simple hydraulic system. The pump provides power to move the liquid through the system. Control of fluid flow is provided by the valves. Output work appears in the form of motion of the piston. In other hydraulic systems, work may appear as rotation of a fluid turbine or increased potential energy of the fluid as it is moved to a greater height.

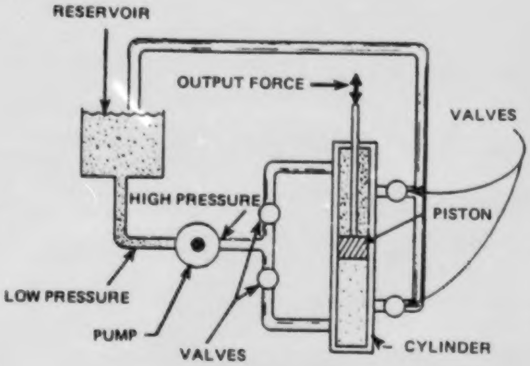


FIG. 1

In order to move fluid through any system, work must be done on the fluid. Moving 55 pounds of fluid through a vertical distance of 10 feet requires the expenditure of 550 foot-pounds of work. If this is done in one second, the power required is 550 foot-pounds per second, or one horsepower.

Conditions are somewhat different in fluid-power systems compared to mechanical systems, since the immediate objective of input work in the former is to increase the potential energy of the system, rather than to impart motion. The observable conditions at the pump show a mass of fluid entering and leaving at a constant volumetric rate, but at a different pressure, with the exit pressure always being the greater. If this pressure differential is considered as a difference in head, it becomes clear that increasing the pressure of a mass of liquid is the same as lifting this mass of fluid through the equivalent head distance. In other words, if the same 55 pounds of fluid previously considered has its head increased by 10 feet, 550 foot-pounds of work will have been done on the fluid. The power required to do this amount of work in one second is one horsepower. The same power will also be expended by pumping 55 pounds of fluid per second across a pressure differential equivalent to a head of 10 feet.

The situation depicted in Figure 2 puts these concepts into perspective and develops the formula for the hydraulic horsepower required to move a fluid through a system. This figure shows a cylinder of fluid flowing through a pipe with cross sectional area A. A cylinder of fluid of length ℓ is moving to the right in this figure. Force F_1 is causing the cylinder of fluid to move to the right. Force F_2, which is smaller than F_1, represents the back

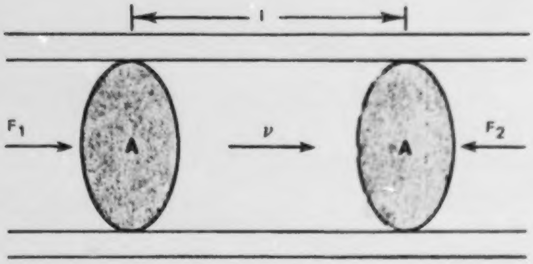


FIG. 2

pressure, or the resistance to this motion. The fluid is flowing at a constant velocity v. The net work done on the cylinder of fluid after it has moved a distance ℓ is calculated as follows: After the sylinder of fluid has been moved a distance ℓ, the work done by forces F_1 and F_2 are given in Equation 1. Note that the work done by F_2 is negative since it opposes the motion of the cylinder of fluid.

$$W = F_1\,\ell - F_2\,\ell = (F_1 - F_2)\ell = \Delta F\,\ell \tag{1}$$

where

W = work.
F_1 = force 1.
F_2 = force 2.
ℓ = distance.
ΔF = force differential across the column of fluid.

Since ΔF is the force differential across the column of fluid in Equation 1, the total amount of work done in moving the cylinder of liquid through a distance ℓ is expressed by Equation 2.

$$W = \Delta F\,\ell \tag{2}$$

The right side of Equation 2 may be multiplied and divided by the area to obtain the following relationship:

$$W = \Delta F\,\ell\,\frac{A}{A} = \frac{\ell F}{A}\,\ell\,A \tag{3}$$

where
A = area.

Since the pressure differential across the cylinder of fluid is

$$\Delta p = \frac{\Delta F}{A}$$

and the volume of the fluid cylinder is

$$\Delta V = \ell\,A$$

Equation 3 becomes Equation 4.

$$W = \Delta p \, \Delta V. \qquad (4)$$

If Equation 4 is divided by the time t, the result is

$$\frac{W}{t} = \Delta p \, \frac{\Delta V}{t};$$

and since work per unit time $\frac{W}{t}$ is power, and $\frac{\Delta V}{t}$ is the volume flow rate, this equation becomes Equation 5.

$$P = \Delta p Q \qquad (5)$$

where

P = power = $\frac{\text{work}}{\text{time}}$.

Q = flow rate = $\frac{\text{volume}}{\text{time}}$.

In engineering and technical fields, the system of units used for pressure is pounds per square inch; and the rate of fluid flow is measured in gallons per minute. With these units Equation 5 becomes:

$$P = \Delta p \left(\frac{\text{lb}}{\text{in}^2}\right) \times Q \left(\frac{\text{gal}}{\text{min}}\right) \times \frac{\text{ft}^3}{7.48 \text{ gal}} \times \frac{144 \text{ in}^2}{\text{ft}^2} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$P = \frac{\Delta p Q}{3.12} \, \frac{\text{ft} \cdot \text{lb}}{\text{s}} \text{ (if } \Delta P \text{ is in pounds/in}^2 \text{ and } Q \text{ in gal/min).}$$

$7.48 \text{ gal} = 1 \text{ ft}^3$

$1 \text{ ft}^2 = 144 \text{ in}^2$

$1 \text{ min} = 60 \text{ s}$

$1 \text{ hp} = 550 \, \frac{\text{ft} \cdot \text{lb}}{\text{s}}$

Power is normally expressed in horsepower (hp), as in Equation 6.

$$P = \frac{\Delta p Q}{3.12} \left(\frac{\cancel{\text{ft}} \cdot \text{lb}}{\cancel{\text{s}}}\right) \times \frac{1 \text{ hp}}{550 \left(\frac{\cancel{\text{ft}} \cdot \text{lb}}{\cancel{\text{s}}}\right)}$$

$$P = \frac{\Delta p Q}{1714} \qquad (6)$$

(where Δp is in lb/in^2, and Q is in gal/min).

Equation 6 may be used to calculate the power required to move fluid through any pressure differential. Equation 6 is used to solve a problem in Example A. This pressure differential may be caused by friction losses in the pipe; it may be a pressure differential required to operate a fluid motor; or it may be the pressure differential required to pump fluid to a new height. Often, this pressure is a result of an attempt to pump a fluid to a given height. The height of a liquid that causes a given pressure is called a pressure head; that is, as depth of a liquid increases, the pressure also increases. The depth of a liquid that causes a given pressure is also called a pressure head. The pressure, in pounds per square inch, corresponding to any pressure head is obtained by using Equation 7.

$$\Delta p = \rho g h \qquad (7)$$

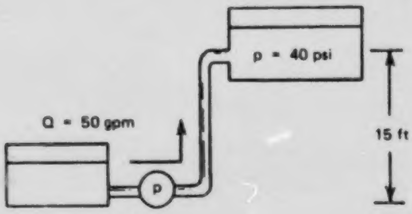
where

Δp = pressure.
ρg = weight density of fluid in lb ft^3.
h = height of fluid in ft.

EXAMPLE A: POWER IN A HYDRAULIC SYSTEM

Given: Water is being pumped into a tank 15 ft above the pump, at a rate of 50 gal/min, and at a constant tank pressure of 40 psi.

Find: hp required.



Solution: Total pressure across the pump is not 40 psi, but is greater by the amount of elevation of the tank. The pressure equivalent to the elevation is as follows:

$$p_e = 0.433\ \Delta h = (0.433)\ 15 = 6.5 \text{ psi}$$

and

$$\Delta p = 6.5 \text{ psi} + 40 \text{ psi} = 46.5 \text{ psi}$$

then

$$P = \frac{50\ (46.5)}{1714} = 1.36 \text{ hp (by Equation 6).}$$

Note: 1.36 hp is the theoretical horsepower required to move the water; and input, or motor, horsepower is greater because of losses in both the pump and the motor.

$\rho g = 62.4 \frac{\text{lb}}{\text{ft}^3}$

If the depth of fluid is expressed in feet, and if the density is the weight density of water (62.4 pounds per cubic foot), this equation reduces to Equation 8 as follows:

$$\Delta p = 62.4 \frac{\text{lb}}{\text{ft}^3} \Delta h \ (\cancel{\text{ft}}) \times \frac{1 \ \cancel{\text{ft}}^2}{144 \ \text{in}^2} = \frac{62.4 \Delta h}{144}$$

$$\Delta p = 0.433 \ \Delta h. \qquad (8)$$

(Δh in feet.)

If the liquid is not water, Equation 8 must be miltiplied by the specific gravity S_g of the liquid to get the required pressure, i.e.,

$$\Delta p = 0.433 \ S_g \Delta h. \qquad (9)$$

Equation 9 placed into Equation 6 gives Equation 10.

$$P = \frac{Q \ .433 \ S_g \Delta h}{1714} = \frac{S_g Q \Delta h}{3959} \qquad (10)$$

(Q in gal/min, Δh in feet.)

Equation 10 allows the calculation of horsepower required to pump fluid to any given height or pressure head.

The modern technical world depends heavily upon application of fluid power. Perhaps the most obvious uses are in modern automobiles that are equipped with hydraulic brakes, with automatic hydraulic transmissions and, very frequently, with power brakes, power steering and other fluid-power devices. Most construction and agricultural equipment now uses fluid power for operation or control, particularly of remotely-located components. Industrial equipment uses fluid power extensively for both operation and control, and the availability of fluid power has been one of the most important factors in the development of automation.

Some of the more spectacular uses of fluid power are exemplified in aircraft and missiles. A single aircraft similar to the Boeing 707 contains miles of hydraulic tubing plus hundred of hydraulic devices to power such components as flaps, control services and landing gear, in addition to the complex systems within each jet engine. Hydraulic systems are widely used to control the engines of missiles. Spacecraft attitude-correction devices are wholly fluid power units.

Fluid power is not the best answer for all power requirements. Each application must be evaluated individually. Fluid power has disadvantages as well as advantages when compared to other power and control systems; these strong and weak points can be summerized as follows:

Disadvantages:

- Hydraulic fluids are messy. Regardless of the type of fluid or the effectiveness of the maintenance and housekeeping procedures, some fluid always seeps out, and this seepage invariably attracts and holds dust and dirt.
- All fluid power systems are susceptible to damage by dirt or contamination. Good design and workmanship on the system will minimize the problem, and good maintenance is an obvious necessity. Under no circumstances, however, can a fluid power system withstand abuse or neglect as well as mechanical or electrical systems.
- Conductor failure, due to either overpress or to mechanical stress is always possible. With a hydraulic system, the possibility is great for physical injury from highspeed particles or for product contamination from escaping fluid.
- The fire or explosion hazard due to leaking hydraulic oils is a constant problem. This problem can be eliminated where fire-resistant liquids, such as glycol-water mixtures or phosphate esters, can be used.

Advantages:

- Hydraulic fluid will remove heat generated during power application. Heat is frequently a problem with electric motors, which must be large and well-ventilated to prevent disastrous heat buildup.
- A hydraulic device is mechanically rigid from the standpoint of the load, and it is possible to position accurately and to hold objects with no external mechanical lock. Positioning is not possible with either electrical or pneumatic devices, due to their slip or compressibility. In addition, hydraulic devices will continue to hold after the power source has been shut off, so long as the fluid pressure in the device is not released.
- Fluid power devices are highly responsive because of their weight-to-power ratio and the incompressibility of liquids.
- Fluid power devices are much easier to install than mechanical power-transmission equipment. They occupy a minimum of space and are easily controlled from a remote location.
- Fluid power drives easily are reversible and may be operated at either constant or variable torque in either direction.
- Fluid power control is compatible with any other control mode.
- Force multiplication for either power or control may by obtained with a minimum power loss.
- The use of fluid power may permit simplifications of machine design.

- Fluid power control may be used in hazardous areas where electrical controls cannot be used.

Fluid power is not always best for all requirements, but it should always be considered because of its obvious advantages under certain circumstances

SUMMARY

Hydraulic power is produced by a pressurized, incompressible liquid, and is used in a wide variety of applications to power and to control machines. Hydraulic power often is used when other power systems would be impractical or relatively expensive. Fluid is pressurized by hydraulic pumps and is piped to hydraulic cylinders or motors for conversion into mechanical energy and power.

1

DETERMINATION OF HORSEPOWER IN A FLUID SYSTEM

MATERIALS

Two large reservoirs of water.
Water pump.
Appropriate plumbing as indicated in Figure 3.
Scale for weighing the fluid.
500 ml graduate.
Two liquids (water and light oil) with known specific gravities. (Both fluids must be compatible with the type of pump being used.)

PROCEDURE

1. Connect pump and plumbing to large reservoir, as shown in Figure 3.
2. Measure height Δh of plumbing output above pump and record in Data Table. Place a large, empty reservoir beneath plumbing outlet.
3. With the experiment set up as shown in Figure 3, turn on pump and record time required to pump one gallon into the upper tank. Record this time and volume in Data Table.
4. Record specific gravity of the fluid used. Using all data recorded in Data Table, calculate horsepower required to produce this flow rate to this height. Record horsepower in Data Table.

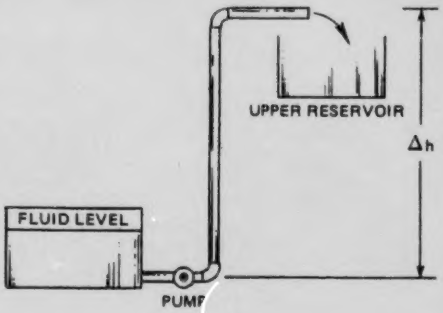


FIG 3

5. Compare calculated horsepower to that given by pump specification. (746 W = 1 hp.)
6. Repeat steps 1-5 for a system that has twice the height used previously.
7. Change fluids and repeat both experiments, recording all data in Data Table.
8. Why does the calculated horsepower differ from the specified?

DATA TABLE						
Fluid	Specific Gravity (S_g)	Volume (gal)	Time (s)	Δh (ft)	Horsepower	
					Calculated	Specified

SELF CHECK

1. Define the term pressure head.
2. Explain the major difference between pneumatic and hydraulic systems.
3. A tank contains a fluid with a specific gravity of 0.85. What is the pressure at a depth of 25 feet?
4. Water is to be pumped to a height of 15 feet at a rate of 50 gallons per minute. What is the pump horsepower required?
5. List three disadvantages and five advantages of hydraulic power.
6. What is the closed-tank pressure if water is being pumped into the closed tank from 20 ft below the tank and the pump has an output horsepower of 2 hp? Water is pumped into the tank at a rate of 100 gallons/minute.
7. A pump motor draws a current of 5 Amperes and operates on 110 d.c. volts. The pump is used to move water at a rate of 5 m^3/hour into a tank 10 meters above the pump. The tank pressure is 2×10^5 N/m^2. What is the efficiency of the pump?
8. A pump with a given output power can pump water into a tank (tank pressure = 40 psi) 25 feet above the pump at a rate of 50 gallons/minute. The same pump operating at the same output power can pump another liquid into a tank under the same pressure also 25 feet above the pump at a rate of 55 gallons/minute. What is the specific gravity of the liquid?

6E1

ELECTRICAL POWER MEASUREMENT

"Electrical power" is a term used frequently today. As the cost of electrical power rises, the availability of this commodity at times is very precious to our homes and industrial production. Electric light bulbs are rated and sold according to a singular specification — their power in watts. Electric heaters and hair dryers, likewise, are specified according to the power, or watts, consumed.

Power is an instantaneous quantity — energy used per second. An electric bill is computed according to the power (or average power) used for a given length of time. Electricity is paid for in watthours or kilowatthours, and measurement of power consumption gives the efficiency of a device.

In this module, the concept of electric power will be reviewed, power usage will be defined, and equipment for measuring watts and watthours will be described. In the laboratory, the student will measure electrical power.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define power in watts and energy consumption in watthours for d.c. and a.c. supplies.
- Given a diagram or actual wattmeter, describe the function of the watt scale, current coils, voltage coil, source, load and resistor.
- In a short paragraph each, describe how an a.c. and d.c. kilowatthour meter works.
- Given the appropriate equipment, (1) read the register kilowatts on a kilowatthour meter and (2) measure electrical power.

- Calculate cost of the electrical power required to operate appliances, given necessary information.

DISCUSSION

Electrical power is measured in watts, and its consumption is measured in kilowatthours. Power in watts designates the rate at which energy is consumed. Watthours or kilowatthours measure total energy used and involve a time factor. Power in watts is equal to the product of the current in amperes times the potential difference in volts. Equation 1 shows these relations for power in d.c. circuits.

$$P = VI \tag{1}$$

where

P = power in watts.
V = voltage in volts.
I = current in amps.

Equation 2 gives these relationships for a.c. circuits.

$$P = VI\ PF \tag{2}$$

where

PF = power factor; a correction factor with a value of between 0 and 1.

The term PF is a necessary correction factor when the alternating current wave peaks at a different instant of time than the alternating voltage wave; that is, when current and voltage are not precisely in step.

The basic definition of d.c. watthours is listed in Equation 3.

$$W = VIh \tag{3}$$

where

W = watthours (d.c.).
h = number of hours in operation.

Watthours are determined for a.c. applications by Equation 4.

$$W = VI\ PF\ h \tag{4}$$

The number of watts consumed in an electrical circuit changes as various loads are switched in and out of the circuit. Total watthours are calculated by summing the products of watts and time elapsed for the individual time intervals.

Note that a kilowatt is 1000 watts and a kilowatthour is 1000 watthours.

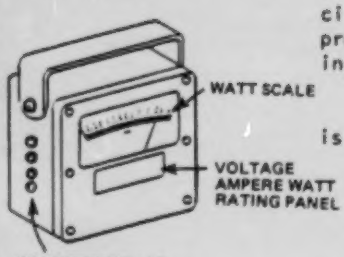


FIG. 1

BASIC WATTMETER

Figure 1 shows a typical four-terminal wattmeter. The four terminals are connected in the manner shown in Figure 2. CAUTION: When using the wattmeter, observe both voltage and ampere ratings on the meter. Neither should be exceeded individually. Furthermore, the watt rating itself should not be exceeded.

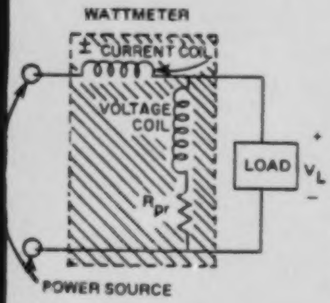


FIG. 2

Figure 3 illustrates the internal workings of a single-phase wattmeter. The voltage coil is basically the meter movement and is attached to the meter needle mounted on pivots in such a manner that it rotates clockwise against a return spring (not shown). As more voltage is applied, the voltage coil tends to rotate further against the spring. Current also passes through the two stationary current coils, as shown. These coils produce a magnetic field, in which the voltage coil turns. As current increases, the magnetic-field strength increases, and the voltage coil tends to rotate further against the spring.

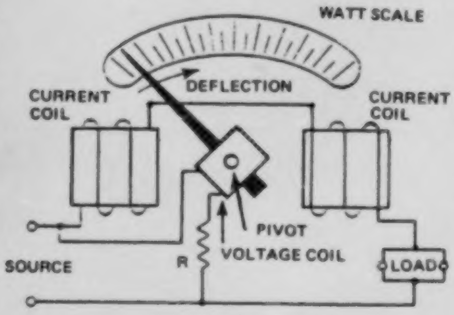


FIG. 3

As either voltage or current is increased, or as both are increased, there is a greater deflection of the needle, or a higher reading of the wattmeter. This deflection, like power itself, is proportional to the product of volts and amps; therefore, the meter scale can be calibrated to read watts directly. For a.c. circuits, if the volts and amps peak at different times, the deflection will be somewhat less than the value of volts x amps, and a power-factor correction is required, as indicated in Equation 2.

DETERMINING RESIDENTIAL POWER USAGE

The amount of energy consumed in a home can be determined with a wattmeter and a clock. A watt reading is taken each hour of one day and night. Each watthour reading is assumed to be the average value for all of the previous hour. The sum of all the readings gives some idea of watthour usage for the day. Hourly readings, however, do not give a true picture, since the actual power consumption can fluctuate every few minutes and may average higher or lower than the hourly reading.

Therefore, a kilowatthour meter that can measure total (cumulative) energy consumption is used. Its output is connected to a counter similar to the mileage counter (odometer) on an automobile. The meter is read monthly, and the previous month's reading is subtracted to obtain the kWh usage. This method is used by utility companies to determine energy used by customers.

DIRECT-CURRENT KILOWATTHOUR METER (KWH)

The direct-current (d.c.) kWh meter is seldom used, but it does illustrate some principles applicable to the a.c. kWh meter. The d.c. meter shown in Figure 4 is geared to a counter, which is calibrated so that each kWh fed through the meter to the load will cause meter rotation and indicate a count of 1 kilowatt. Voltage is applied to the armature coil "A." Current is applied to the field coils "F." In a manner similar to the wattmeter (Figure 3), the output rotation is directly proportional to watts (the product of volts and amps). When connected by gears to a counter, the net rotation indicates kWh.

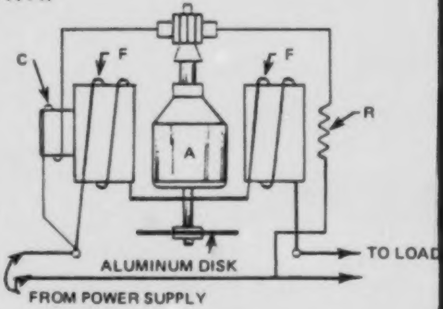


FIG. 4

ALTERNATING-CURRENT KILOWATTHOUR METER (KWH)

The alternating-current (a.c.) kWh meter also uses the motor principle. Figure 5 shows essential parts. Current passes through current coils, and voltage is applied across a separate coil. These coils produce magnetic flux lines that cause the armature disk to turn. The disk rotation is stabilized by the damping magnets. Rotation speed is proportional to watts in use and is geared to a counter to read kWh. A typical kWh meter found attached to houses is shown in Figure 6.

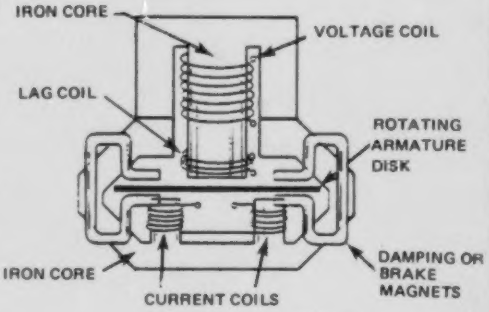


FIG. 5

Although residential electrical rate structure varies with time and location, a typical rate structure is shown in Table 1.

The rate structure in Table 1 will be used in Example A to demonstrate how an electric bill can be calculated. In the example, the total kilowatthour use is computed from an itemized knowledge of the power used per day for a 30-day month. In actual practice, the kWh meter (Figure 6) would be read to obtain the total monthly use as one figure. Thus, the kWh meter reading would indicate 6044 kWh used, if the meter had read 0000 at the beginning of the month.

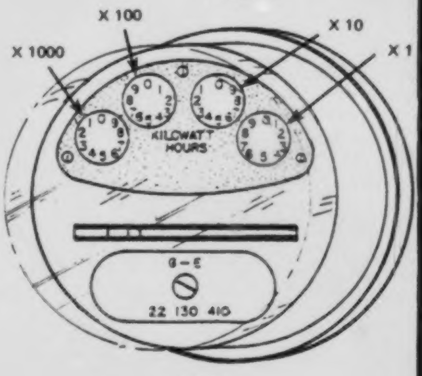


FIG. 6

TABLE 1: RESIDENTIAL ELECTRIC RATE STRUCTURE (JUNE 1977)

Fixed Charge	$2.75 plus	
First	200 kWh/mo	$0.0435/kWh
Next	1300 kWh/mo	$0.0314/kWh
Over	1500 kWh/mo	$0.0287/kWh
Plus electric adjustment factor x kWh use		
Plus 4% tax		
(Adjustment factor 0.00138)		

EXAMPLE A: CALCULATING AN ELECTRIC BILL

Given: Two 1-watt clocks run continuously. One 1.5-kW heaters run 11 hours per day. Four 100-watt bulbs run 9 hours per day.

Find: Monthly total electric bill using Table 1 data below. Total kWh is:

Item	No	kW	h/d	days	kWh
Clocks	2	0.001	24	30	1.44
Heater	1	1.5	11	30	495.00
Bulbs	4	0.1	9	30	108.00
					604.44 kWh
			Use factor →		0.83 kWh
			Total →		605.27 kWh

Solution: Correcting for use factor (adjustment factor, Table 1).

604.44 x 0.00138 = 0.83 kWh

Fixed cost	$ 2.75
First 200 kW x 0.0435	8.70
Next 405.27 x 0.0314	12.73
	24.18
+ 4% tax	.97
Total cost =	$25.15/month

SUMMARY

Electrical power is measured to determine kWh usage and cost. It also can be measured to identify those pieces of equipment or appliances that consume the most electricity; this procedure can be important in energy conservation if those high-consumption devices are operated only when necessary or at capacity. (Operating at full capacity means, for example, that an electric clothes dryer or oven is loaded to capacity to prevent repeated use later.) Power in electrical circuits usually is measured with a wattmeter, and in residential and commercial structures with a kilowatthour meter.

READING THE KWH METER

MATERIALS

Kilowatthour meter (use one attached to lab, house or building).

PROCEDURE

The dial of the kWh meter is fairly easy to read, even if the pointer is slightly off. The reading in Figure 7 is obtained as follows:

1. Begin with the left dial. Since the pointer is between 2 and 3, the first number is 2, indicating somewhere between 2 and 3 thousand kilowatts.
2. Move right to the next dial (labeled "hundreds" in Figure 7). The pointer lies close to, but not yet at, 2. Thus, the next figure is 1. So far, we have 2100 kilowatts.

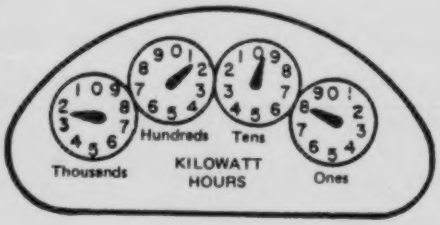


FIG. 7

3. Continuing on, the "tens" dial reads 9 and the "ones" dial reads 8.
4. Thus, the total reading is 2198 kilowatthours.

   Notice that for a small period of time, revolutions of the disk can be counted. A typical value for one revolution of the disk is 0.0035 kWh per revolution. Check actual value for particular meter used by counting revolutions required to move the small indicator dial (ones) one kWh and by dividing one kWh by number of revolutions of the disk.

5. Discuss how such tests of power could help to identify power waste and how energy could be conserved.

MEASURING KWH USED BY APPLIANCES

MATERIALS

Kilowatthour meter (use one attached to lab, house or building).
Electric appliances, such as a refrigerator, radio, TV, that can be left running for 8 to 12 hours or more.

PROCEDURE

This experiment will show how many kWh of electricity a home, lab, or building is using and the affect produced on electrical consumption when major appliances are turned off.

1. Locate the kWh meter that serves area to be measured.

2. Using information obtained in laboratory experiment 1, obtain and record a meter reading for a one-hour period.

3. At the end of the test period, switch off TV, radio, and any other electric appliances, except a minimum number of lights, and obtain and record a meter reading for a one-hour period.

4. Compare two readings and explain differences. Note: A longer test period gives a better average consumption, but use the same period for both tests.

    (a) Power consumed during first time period = ______ kWh.

    (b) Power consumed during second time period = _____ kWh.

    Power saved = (b) - (a) = _____ kWh.

    Multiply this figure by number of time periods selected in 30 days to determine total monthly power savings.

CALCULATION OF AN ELECTRIC BILL

MATERIALS

Not applicable.

PROCEDURE

Calculate the cost savings for one month using the typical rate structure shown in Table 1 or the current rate structure in the local community.

SELF CHECK

1. Define power in d.c. watts and in a.c. watts.
2. Define energy in d.c. watthours and in a.c. watthours.
3. Describe working parts of a wattmeter.
4. In general terms, explain how a d.c. kWh meter works.
5. In general terms, explain how an a.c. kWh meter works.
6. Use the schedule in Table 1 to calculate the monthly electric bill for a residence that uses 2000 kWh of electrical energy.
7. How much money does the resident in problem 6 save if the monthly electrical power consumption is reduced by 786 kWh?
8. A wattmeter has a linear scale. If a current of 3 amps causes full scale deflection, what current is necessary to cause half of full scale deflection? Assume that the voltage is the same in each case.
9. What is the power factor for an a.c. circuit if the input power to a motor which operates on 110 V and draws 2 Amperes is 200 watts?

10. What current does a d.c. motor that is 50% efficient and has an output of 400 watthours in 90 minutes draw if the motor operates on 110 d.c. volts?

EFFICIENCY OF ELECTRIC MOTORS AND GENERATORS

Because electrical energy can be transmitted and controlled simply, reliably and efficiently, other forms of energy often are converted to and from electrical energy. An electromechanical device operating as a generator converts mechanical energy to electrical energy. The same device operated as a motor converts electrical energy to mechanical energy. Because of losses within a motor, its output power cannot equal the input power.

This module discusses the efficiency of energy conversion in motors and generators and explains the factors contributing to energy losses. In the laboratory, the student will determine the efficiency of an electric motor.

OBJECTIVES

Upon completion of this module, the student should be able to:

- State the three classifications of loss in an electric motor or generator, and give two examples of each.
- Explain the origin of hysteresis and eddy currents and how they contribute to losses in an electric motor.
- Explain the purpose of laminated armature cores.
- Explain what happens to the portion of the input energy of a motor or generator that is not converted to output energy.
- Given the input voltage and current of a direct-current electric motor, its output torque and rotational rate, determine its efficiency.

- Given the appropriate equipment, determine the efficiency of an electric motor.

DISCUSSION

Electric motors and generators (Figure 1) are two applications of the same device. When this device is employed to convert mechanical energy to electrical energy, it is called a "generator." When it converts electrical energy to mechanical energy, it is called a "motor." In both cases, some energy losses occur. As a result, the output energy always is less than the input energy. Because these losses are the same for motors and generators, a discussion of loss factors in either device also applies to the other. This module uses the motor as an example.

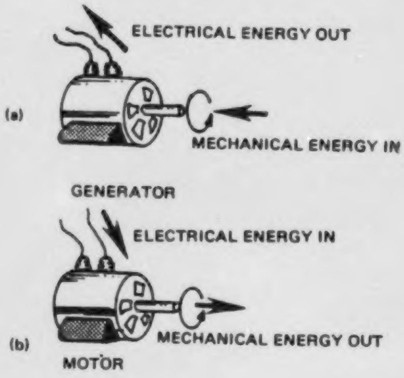


FIG. 1

When the conversion of energy in a motor or generator takes place at a uniform rate (that is, when the energy received by the device per unit of time and the energy delivered in the same unit of time are both constant), power is converted from one form to another. The efficiency of the device can be calculated from Equation 1 if the input and output power are known.

$$\eta = \frac{P_{out}}{P_{in}}(100) \qquad (1)$$

where

P_{out} = output power.
P_{in} = input power.
100 = multiplier to obtain %.

Example A shows how to use this equation in solving a problem.

EXAMPLE A: EFFICIENCY OF AN ELECTRIC MOTOR

Given: An electric motor produces an output power of 1/2 horsepower when the input power is 434W.

Find: Efficiency of motor.

Solution: Convert output power to watts: 1 hp = 746W.

$$P_{out} = (1/2\ \text{hp})(746\text{W/hp})$$

$$P_{out} = 373\ \text{W}$$

$$\eta = \frac{P_{out}}{P_{in}}(100)$$

$$\eta = \frac{373W}{434W}(100)$$

$$\eta = 86\%$$

The remainder of the energy (14% of 61 J/S in Example A) must be accounted for in some way. This energy appears as heat energy in the motor. If input energy is converted to heat at a high rate, the motor can be damaged and "burn out."

POWER LOSSES IN MOTORS

There are three classifications of losses in an electric motor:

- Rotational losses are those that are caused by the rotation of the armature.
- Copper losses are the result of electric current flowing through the conductors of the motor.
- Stray load losses are additional losses that cannot be accounted for in the two categories above, and often are difficult to analyze.

Rotational losses may be subdivided into the following classes:

- Bearing friction.
- Brush friction.
- Wind friction (windage).
- Hysteresis.
- Eddy currents.

The first three rotational losses are mechanical, caused by rotation of the armature. They always can be reduced by good mechanical design and lubrication. The last two are electrical in nature, but are classified as rotational losses because they also result from, and are dependent upon, the rate of rotation of the armature.

Hysteresis occurs when a material is magnetized. Materials that can be magnetized, such as the iron in an armature core, are composed of many tiny magnetic domains, each with its own field of orientation, as in Figure 2. In iron that is unmagnetized, the direction of the magnetic fields of the domains are random. When an external magnetic field is applied, the domain fields line up to produce a net magnetic field in the material.

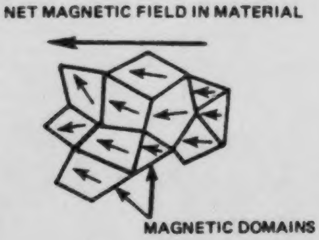


FIG. 2

In a motor armature, the direction of the applied field changes as the armature turns. This causes the direction of the magnetic fields of the domain to change. The directions of the domain fields always lag behind the direction of the applied field. This lag is called "hysteresis." The energy necessary to change the direction of fields of magnetic domains eventually appears as heat in the armature. Hysteresis losses always are present in electric motors.

Eddy currents are the result of the motion of conducting parts of the armature through the magnetic fields in the motor. When the armature rotates in the magnetic field, the outer edges of the armature core move through the field at a greater velocity than the center portions. This difference in velocity causes electrical potential differences to appear within the core, and creates the eddy currents shown in Figure 3. These eddy currents heat the material of the core by Joule heating (I^2R).

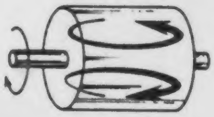


FIG. 3

Most of the eddy current flows parallel to the axis of rotation. It never can be eliminated completely, but it can be reduced greatly by the use of a laminated core composed of thin plates of conductor coated with varnish. The smaller eddy currents of such a core are shown in Figure 4.

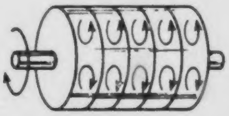


FIG. 4

Copper losses are the result of currents flowing through the resistance presented by electrical conductors within the motor. These losses are dependent upon the current and resistance (I^2R), and are independent of armature rotation. There are three sources of copper loss:

- Resistance of the armature windings.
- Resistance of field windings.
- Resistance of brush contacts.

Copper losses may be reduced by using larger-diameter wire to reduce the resistance of the windings. Thus, motors with high efficiencies are larger and more expensive than lower efficiency motors.

Stray load losses are attributed to the lack of a uniform division of current through various paths in the windings, to short-circuit currents in coils undergoing commutation, to radiated electromagnetic energy and to distortion of magnetic flux lines because of armature reaction. In most cases, the magnitude or exact causes of stray load losses are impossible to determine; but in modern motors, they are small. A value of 1% usually is assumed for stray load losses in large motors, and they are often neglected completely in efficiency calculations for small motors.

The overall efficiency of electric motors varies with the type and size of the motor and with the power that the motor delivers. Peak efficiencies in the range of 80-90% are common. The efficiency of a particular motor varies as the load varies. A typical graph

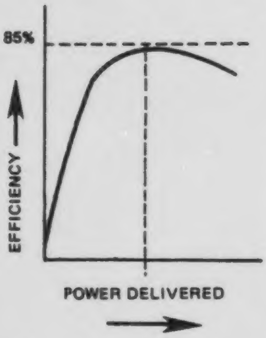


FIG. 5

for efficiency versus output power of an electric motor is shown in Figure 5.

MEASURING MOTOR EFFICIENCY

While determining the exact sources and magnitudes of losses in an electric motor may be complicated, experimental determinations of efficiency may be made with relative ease by the measurement of the electrical input power and the mechanical output power. For direct current motors, the input power may be determined by the measurement of the input voltage and current and multiplying to obtain power. In alternating-current motors, the power factor also must be considered. The task of measuring the input power is simplified by the use of an a.c. wattmeter that measures electrical power directly in watts.

The mechanical output power is the product of the angular velocity of the motor and the torque delivered. The rotational rate may be measured with a tachometer or with a stroboscope. Torque is measured by a dynamometer or with a prony brake. Mechanical power then is determined by Equation 2.

$$P_{out} = \tau\omega \qquad (2)$$

where

P_{out} = mechanical output power.

τ = torque.

ω = angular velocity.

Example B shows the use of this equation in calculating the efficiency of an electric motor.

EXAMPLE B: EFFICIENCY CALCULATION

Given: A d.c. electric motor has an input of 18 amps at 24 volts. It rotates at 1500 rpm and produces a force of 26.5 oz on a prony brake with a lever arm of one foot.

Find: Efficiency of motor.

Solution: Input Power: $P_{in} = IV$

$$= (18\text{A})(24\text{V})$$

$$P_{in} = 432\ \text{W}$$

520

Power Out:

Torque: $\tau = F\ell$

$$= (26.5\ \text{oz})\left(\frac{1\ \text{lb}}{16\ \text{oz}}\right)(1\ \text{ft})$$

$$\tau = 1.66\ \text{ft}\cdot\text{lb}$$

Angular velocity: $\omega = (1500\ \text{rev/min})\left(\frac{2\pi\text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$

$$\omega = 157\ \text{rad/s}$$

$$P_{out} = \tau\omega$$

$$= (1.66\ \text{ft}\cdot\text{lb})(157\ \text{rad/s})$$

$$= 260.6\ \text{ft}\cdot\text{lb/s})$$

$$= (260.6\ \text{ft}\cdot\text{lb/s})\left(\frac{1\ \text{hp}}{550\ \text{ft}\cdot\text{lb/s}}\right)$$

$$= 0.474\ \text{hp}$$

$$= (0.474\ \text{hp})\left(\frac{746\text{W}}{\text{hp}}\right)$$

$$P_{out} = 353.6\text{W}$$

Efficiency: $\eta = \frac{P_{out}}{P_{in}}(100)$

$$= \frac{353.6\text{W}}{432\ \text{W}}(100)$$

$$\eta = 81.9\%$$

SUMMARY

Electrical efficiency (in %) of an electric motor is calculated by the division of input power into output power, times 100. Output power cannot equal input power because of power losses within the motor. % efficiency is the percent of power remaining after losses. Mechanical efficiency is the product of motor torque and angular velocity.

ELECTRIC MOTOR EFFICIENCY

MATERIALS

110V shunt a.c. motor, 1/4 or 1/2 horsepower, fitted with appropriate pulley.
0-1000W a.c. wattmeter.
Prony brake.
0-48 oz spring scale.
Stroboscope or tachometer.
Power cord.

PROCEDURE

1. Set up the motor and prony brake as shown in Figure 6.

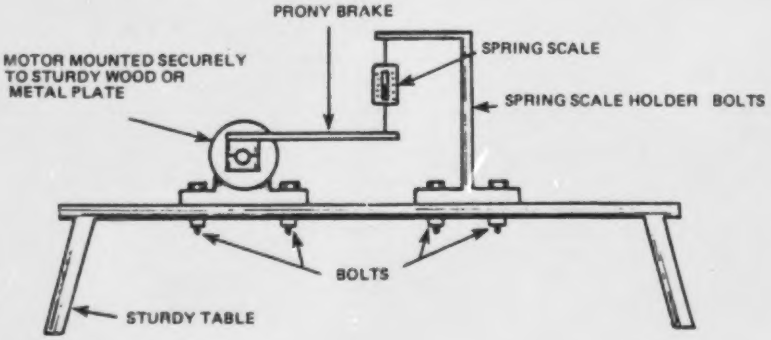

PRONY BRAKE
MOTOR MOUNTED SECURELY TO STURDY WOOD OR METAL PLATE
SPRING SCALE
SPRING SCALE HOLDER BOLTS
BOLTS
STURDY TABLE

FIG. 6

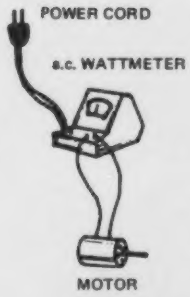

POWER CORD
a.c. WATTMETER
MOTOR

FIG. 7

2. Connect the motor electrically as shown in Figure 7.

3. Tighten prony brake wing nuts just enough to hold brake shoe in light contact with motor pulley.

4. Turn on motor, and adjust prony brake wing nuts to produce spring scale reading of 4 oz.

5. Measure rotational rate of motor with stroboscope or tachometer.

6. Measure input power with a.c. wattmeter.

7. Record the following in the Data Table.

   Input power (W).
   Rotational rate (rpm).
   Spring scale reading (oz).
   Lever arm of prony brake (in).

8. Adjust prony brake wing nuts for 8 oz, and repeat measurements.

9. Continue process at 4-oz intervals until input power reaches 1.5 times rated motor output power.

   Rated power: 1/2 hp = 373W
   1/4 hp = 186.5W

   Caution: Do not run motor above rated power for long periods. If motor stalls, turn off electrical power immediately.

10. Turn off motor, and disconnect prony brake from pulley.

11. Turn on motor. Measure and record motor speed and input power with no load.

12. Perform necessary calculations and complete Data Table. (Refer to Example B.)

13. Use Figure 8 to plot graph of motor efficiency versus output power. Select appropriate horizontal (power) scale for data.

14. At what output power is motor most efficient? Why?

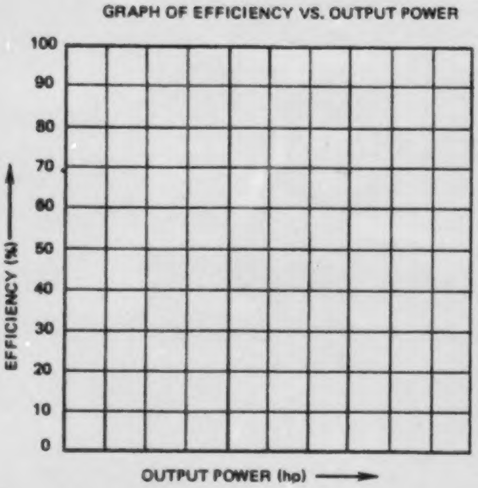


FIG. 8

DATA TABLE

SPRING SCALE READING		LEVER ARM	ROTATIONAL RATE		OUTPUT POWER			INPUT POWER	EFFICIENCY
(oz)	lb	in ft	rpm	rad/s	ft·lb/s	hp	W	W	η %
Desired	Actual								
0 (no load)									
4									
8									
12									
16									
20									
24									
28									
32									
36									
40									

SELF CHECK

1. Give two examples of each of the following losses:
   a. Rotation loss.
   b. Copper loss.
   c. Stray load loss.
2. Explain how hysteresis causes loss in an electric motor.
3. Explain the origin of eddy currents.
4. How are eddy currents reduced in armature cores?
5. What happens to input power not converted to output power?
6. A d.c. motor requires 12 A and 12 V. The output torque is 0.86 ft·lb, and the rotation rate is 1200 rpm. Determine the efficiency of the motor.

7. A generator has a mechanical input of 250 hp. It is 88% efficient and has an output voltage of 125 V. How much current does it produce?

8. Assume that the only losses in a 90% efficient motor are copper (I^2R) losses. If the motor has an effective resistance of 2 Ω and an output power of 100 watts, what effective current is responsible for the I^2R losses?

9. What is the angular speed of a motor which is 75% efficient, develops a torque of 5 N·m and draws a current of 12 Amperes while operating on 24 volts?

10. What is the operating voltage of a motor which is 90% efficient, draws 18 Amperes and has a power output of 600 watts?

6T1

HEAT EXCHANGERS

In thermal systems, power is the rate at which heat, or thermal energy, is transferred. A heat exchanger is a device that moves thermal energy from one part of a thermal system to another. Examples of heat exchangers are automobile cooling systems, refrigerators, air conditioners, transistor heat sinks, cooling towers, and even the human blood-circulatory system.

This module discusses the operational principles of several types of heat exchangers, especially of the circulating water-forced air heat exchanger system. In the laboratory, the student will determine the operating characteristics of such a system.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe the operation of each of the following exchangers:

  Solid-to-air heat exchanger.
  Water-to-air heat exchanger.
  Water-to-water heat exchanger.
  Refrigeration system.

- Given the water flow rate and the input and output temperature of a flowing water heat exchanger, calculate the power expended.
- Given the heat load, water flow rate, and output temperature of water flowing through a heat exchanger, calculate the water inlet temperature.

- Given the inlet and outlet temperatures of water flowing through a heat exchanger and its power, calculate the water flow rate.

- Given the appropriate equipment, set up a circulating water-forced air heat exchanger system; determine its operating characteristics; and discuss power differences between those read and those calculated.

DISCUSSION

Heat always flows from regions of higher temperature to those of lower temperature. The function of the heat exchanger is to increase the rate at which heat can flow from one region to another. Consider four types of heat exchangers.

The solid-to-air heat exchanger (Figure 1) consists of metal shaped to have a large surface area. Metal is chosen because a solid is an excellent thermal conductor. The high temperature component to be cooled is connected directly to the heat exchanger. Heat flows into the conducting metal, raising its temperature. Forced cooler air, flowing past the hot metal surface, carries the heat away. This type of heat exchanger, often called a "heat sink," frequently is employed in the electronics industry and in air-cooled engines.

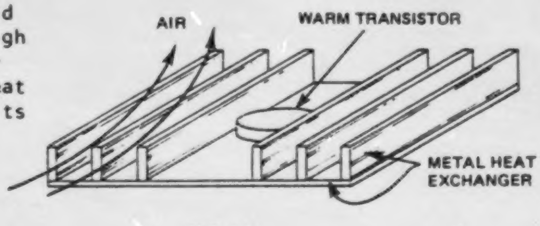


FIG 1

The liquid-to-air heat exchanger consists of a recirculating liquid system that moves heat from the high-temperature component or heat load to a large surface-area radiator. The automobile cooling system, a common liquid-to-air heat exchanger, is shown in Figure 2. Forced air from the fan cools the radiator and the hot liquid it contains. The cool liquid then returns to the heat source, the hot engine, to draw away more heat from the pistons. Because a liquid is used, this system is more efficient than the solid-to-air type, providing lower temperature operation. Liquid-to-air heat exchangers are used to cool internal combustion engines. Water is used as the heat transfer liquid, because it has a high heat capacity, is cheap, is available, and is relatively noncorrosive.

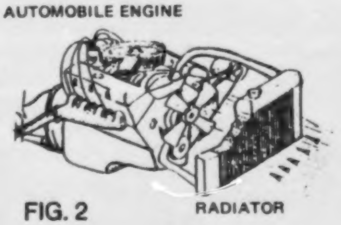


FIG. 2

Liquid-to-liquid heat exchangers are even more efficient, because the "radiator" of this system is immersed in a flowing liquid, such as water. This type of heat exchanger, shown in Figure 3, often is found on electronic equipment requiring deionized water cooling. Figure 3 shows a fluid circulating between a high-temperature laser and the heat exchanger. The fluid is heated at the laser; is piped to the heat exchanger, where it circulates through coils immersed in running water; and is returned to the laser. At the heat exchanger, the hot fluid transfers heat energy to the flowing water.

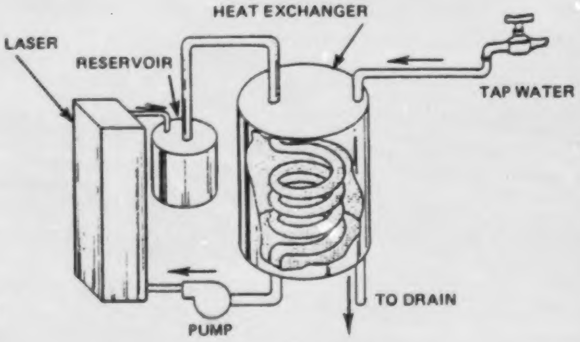


FIG. 3

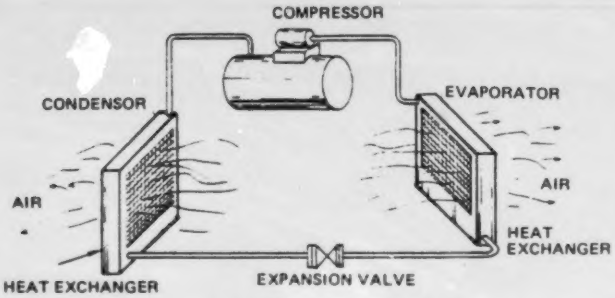


FIG. 4

Refrigeration systems contain two heat exchangers (Figure 4). The heat load of the system is located at the evaporator. A spray of cooled liquid refrigerant enters the evaporator via the expansion valve. The pressure in the evaporator is relatively low (about 30 psi). The liquid absorbs thermal energy from the walls of the evaporator, thus providing the cooling or refrigeration of the unit. This heat energy causes the liquid refrigerant to vaporize. The vaporized liquid flows to the compressor, where its pressure is increased to about 150 psi. The gas then moves to the condenser, where it is cooled and reliquified, giving up heat to the outside air. The gas condenses back to a liquid as the heat energy is transferred to the outside air. The liquid then flows back to the expansion valve to repeat the process. Thus, in the refrigeration system, one heat exchanger is used to transfer heat from the unit to the circulating refrigerant, and the second heat exchanger is used to transfer heat from the circulating refrigerant to the outside air.

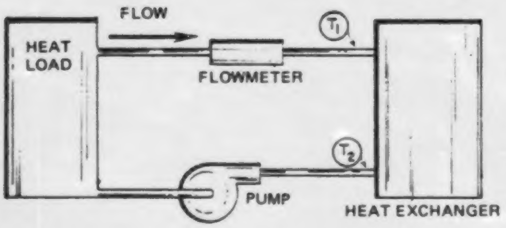


FIG. 5

Figure 5 shows a heat exchanger in a system that uses a recirculating liquid coolant. The heat exchanger in the system may be either water-to-air or water-to-water. When the output temperatures, T_1 and T_2, of the heat exchanger and the flow rate of the coolant are known, the power rating of the heat exchanger is calculated using Equation 1.

$$P = (T_1 - T_2)\left(\frac{\Delta V}{t}\right)(c) \qquad (1)$$

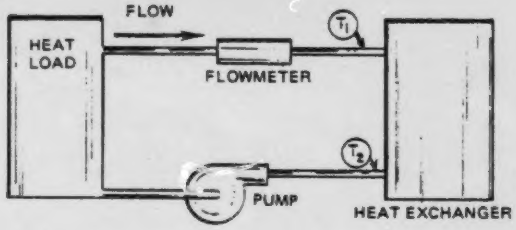
where

P = power of heat exchanger.
T_1 = input temperature.
T_2 = output temperature.
ΔV = volume of liquid flow.
t = time in seconds.
c = specific heat of coolant.

One calorie of heat energy will raise the temperature of one gram of water (1 cubic centimeter) by 1 Celsius degree; thus, the heat capacity of water is 1 cal/g °C. If the heat exchanger changes the temperature of 1 g of water by 1 C° in 1 second, it has a power of 1 cal/s. Equation 1 shows that the power of a heat exchanger is found by multiplying the specific heat of the fluid times the change in temperature times the volume of water flowing through the heat exchanger each second. Examples A and B show how to use Equation 1 to solve problems.

EXAMPLE A: DETERMINING THE POWER OF A HEAT EXCHANGER

Given: Heat exchanger shown:



$T_1 = 80°C$
$T_2 = 50°C$
$\frac{\Delta V}{t} = 10.2\ \ell/\text{min}$
$c = 1\ \text{cal/g °C}$ (water)

Find: Power of heat exchanger.

Solution: $P = (T_1 - T_2)\frac{\Delta Vc}{t}$
$P = (80°C - 50°C)(10.2\ \ell/\text{min})(1\ \text{cal/g °C})$

Convert liters per minute to grams per minute.

$$10.2\ \frac{\cancel{\ell}}{\text{min}} \times \frac{1000\ \cancel{\text{cm}^3}}{1\ \cancel{\ell}} \times \frac{1\ \text{g}}{1\ \cancel{\text{cm}^3}} = 10.2 \times 10^3\ \text{g/min}$$

$P = (30°C)(10.2 \times 10^3\ \text{g/min})(1\ \text{min}/60\ \text{s})(1\ \text{cal/g °C})$
$= (30°C)(170\ \text{g/s})(1\ \text{cal/g °C})$
$= (5100\ \text{cal/s})(4.1861\ \text{J/cal})$ (converting calories
$= 2.135 \times 10^4\ \text{J/s}$ to Joules)
$P = 21.35\ \text{kW}$

The power of the heat exchanger is 21.35 kilowatts.

In a recirculating liquid system, the power of the heat exchanger is dependent upon the change in temperature and the flow rate of the coolant. If the input temperature of such a system is increased with a constant coolant flow rate, the output temperature will also increase. In order to drop the output temperature under an increased heat load (higher inlet temperature), the coolant (water) must be moved through the exchanger at a slower rate to allow more time for heat to transfer.

EXAMPLE B: DETERMINING RATE OF FLOW IN A HEAT EXCHANGER

Given: A water-to-air heat exchanger has a power of 8 kW and an input temperature of 38°C.

Find: Water flow rate necessary to achieve an output temperature of 26°C.

Solution:

$$P = (T_1 - T_2)\,\frac{\Delta V c}{t}$$

Solve for $\frac{\Delta V}{t}$

$$\begin{aligned}
\frac{\Delta V}{t} &= \frac{P}{c\ (T_1 - T_2)} \\
&= \frac{(8000\ \cancel{J}/s)(1\ \text{calorie}/4.186\ J)}{(1\ \text{cal/g °C})(38°C - 26°C)} \\
&= \frac{1911\ \text{cal/s}}{12\ \text{cal/g}} \\
&= (159\ \cancel{g}/s)(1\ cm^3/\cancel{g}) \\
&= (159\ \cancel{cm^3}/\cancel{s})\left(\frac{1\ \ell}{1000\ \cancel{cm^3}}\right)\left(\frac{60\ \cancel{s}}{1\ min}\right) \\
\frac{\Delta V}{t} &= 9.54\ \ell/min
\end{aligned}$$

Recall:
1 cal = 4.186 J
1 cm^3 water = 1 g
1 ℓ = 1000 cm^3
60 s = 1 min

Thus, 9.54 liters must flow through the heat exchanger every minute if the temperature of the water is to drop from 38°C at the inlet to 26°C at the outlet.

SUMMARY

The heat exchanger moves (exchanges) thermal energy from one part of a thermal system to another. The most common heat exchangers are the following: (1) solid-to-air (heat sink), (2) liquid to air (by use of a radiator), and (3) liquid-to-liquid (by use of a radiator cooled by liquid). Air conditioners employ two heat exchangers. Some heat exchangers now are used efficiently to cool homes and businesses during hot weather and to warm them during cold weather; they are commonly called "Heat Pumps."

Large energy savings are possible with industrial heat exchangers, as evidenced by the following observation by Mark Halper:

> An Energy User News sampling of manufacturers shows that new product ideas are emphasizing heat exchangers that can function in the severe heat (up to 3,000 degrees F) of an industrial process. Both GTE Sylvania and Hague International are updating their ideas on the use of ceramic materials to recover heat from industrial furnace exhaust air, thereby cutting fuel bills up to 80 percent.

OPERATING CHARACTERISTICS OF HEAT EXCHANGER

MATERIALS

Automobile radiator or cooling coils from an air conditioner.
Electric fan.
Two thermometers with connectors (0° to 100°C).
Small ice chest or two-liter graduated beaker.
Electrical heating element.
Variable autotransformer.
Wattmeter (0 - 1500 W).
Water pump.
Connecting hoses.
Pinch or hose clamp.
Water flowmeter (0 - 5 liters/min).
Graph paper.
Friction tape.

PROCEDURE

1. Arrange equipment as shown in Figure 6. Check to be sure that system is water tight. Ice chest should be above the level of pump and radiator. Turn on pump to begin circulating water through system. When pump, connecting hoses and radiator are completely filled, ice chest should be about half full of water.

2. Verify proper operation of autotransformer and wattmeter.

3. Adjust hose clamp or valve to give a water flow rate of approximately 4 ℓ/min.

4. Turn on cooling fan.

5. Set autotransformer to deliver a power of 200 W.

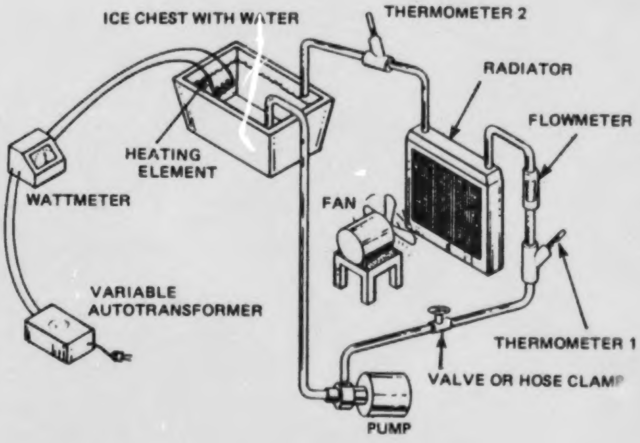

ICE CHEST WITH WATER
THERMOMETER 2
RADIATOR
FLOWMETER
HEATING ELEMENT
WATTMETER
FAN
VARIABLE AUTOTRANSFORMER
THERMOMETER 1
VALVE OR HOSE CLAMP
PUMP

FIG. 6

6. Wait for temperature at both thermometers to stabilize. Then record inlet and outlet temperatures in Data Table.

7. Set autotransformer to deliver 400 watts, and repeat step 6. Repeat step 6 at 600 W, 800 W and 1000 W. Record.

8. Adjust flow rate downward to 3 ℓ/min, and repeat steps 6 and 7. Repeat again at 2 ℓ/min and 1 ℓ/min.

9. Complete results portion of the Data Table, and draw four graphs of change in temperature versus measured power of heat exchanger on the same sheet of graph paper. Label each curve to indicate flow rate. (The Data Table follows sample calculations.)(See step 10 below.)

SAMPLE CALCULATION

Use the data from each line of the Data Table, and calculate the following.

Change in temperature: $\Delta T = (T_1 - T_2)$

$\Delta T =$ _______ C°.

Power of heat exchanger: $P = (\Delta T)(\frac{\Delta V}{t})(c)$
(from Equation 1)

P = _______ C° x _______ ℓ/min x (1 cal/g °C) x (1000 g/1ℓ)

= _______ cal/min x (4.186 J/1 cal)(1 min/60 s)

= _______ J/s

P = _______ watts.

10. Why is there a difference between the power read on the wattmeter and the power of the heat exchanger?

DATA TABLE

	Data				Results
Flow Rate (/min)	Power From Wattmeter (W)	T_1 (°C)	T_2 (°C)	$(T_1 - T_2) = \Delta T$	Power of Heat Exchanger (Watts)
V_1 = 4ℓ/min	P = 200 W	T_1=	T_2=	ΔT=	P_1=
	P = 400 W				
	P = 600 W				
	P = 800 W				

	P = 1000 W				
V_2 = 3ℓ/min	P = 200 W				
	P = 400 W				
	P = 600 W				
	P = 800 W				
	P = 1000 W				
V_3 = 2ℓ/min	P = 200 W				
	P = 400 W				
	P = 600 W				
	P = 800 W				
	P = 1000 W				
V_4 = 1ℓ/min	P = 200 W				
	P = 400 W				
	P = 600 W				
	P = 800 W				
	P = 1000 W				

SELF CHECK

1. Describe the basic operation of each of the following heat exchangers.

   Solid-to-air.
   Water-to-air.
   Water-to-water.
   Refrigeration system.

2. A recirculating heat exchanger using water as a coolant has an input temperature of 52°C, an output temperature of 31°C, and a flow rate of 7.62 ℓ/min. What is its power in watts?

3. With a heat load of 6500 W and a water flow rate of 6.3 ℓ/min, a heat exchanger has an output temperature of 27.6°C. What is its input temperature?

4. What is the water flow rate if the input temperature is 25°C, the output temperature is 24°C and the power is 1000 W?

5. A heat exchanger uses a coolant other than water. If power of the heat exchanger is 1.2 kW and input and output temperatures are 70°C and 35°C respectively, what is the specific heat of the coolant? The volume flow rate is 0.1 m^3/minute. Density of coolant is 10 kg/m^3.

6. A heat exchanger draws a current of 18 Amperes and operates on 110 V d.c. What is the efficiency of the heat exchanger if the coolant used is water, the input temperature is 80°C, the output temperature is 45 C° and the volume flow rate is 0.5 m^3/minute?

POTENTIAL AND KINETIC ENERGY

APPLICATION MODULES

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* Identifies concept support modules that introduce new physics.

7M3

ROTATIONAL KINETIC ENERGY OF A FLYWHEEL

A flywheel is a device used to store mechanical energy in the form of rotational motion of a mass. This rotational kinetic energy can be converted to other forms for use at a later time.

In this module, the student will examine characteristics of flywheels as energy-storage devices. In the laboratory, the student will construct a simple flywheel and will determine its efficiency as an energy-storage device.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Given mass and diameter of a uniform disk, calculate its moment of inertia.
- Calculate kinetic energy of a planetary body in a circular orbit, given its mass, angular velocity and radius of orbit.
- Calculate kinetic energy stored in a flywheel, given its moment of intertia and angular velocity.
- Calculate angular velocity of a flywheel, given its kinetic energy and moment of inertia.
- Describe energy losses that occur in a simple flywheel system.
- Given the appropriate equipment, construct a simple flywheel and determine its stored energy and angular velocity.

DISCUSSION

Figure 1 shows a particle of mass m in a circular orbit of radius r. Velocity of the particle is v, and angular velocity is $\omega = v/r$, where r is the radius.

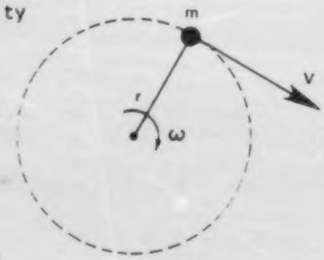


FIG. 1

In Concept Module 7-0, "Potential and Kinetic Energy," kinetic energy of such a mass was shown to be the following:

$$E_k = \frac{1}{2} mv^2 = \frac{1}{2} mr^2\omega^2$$

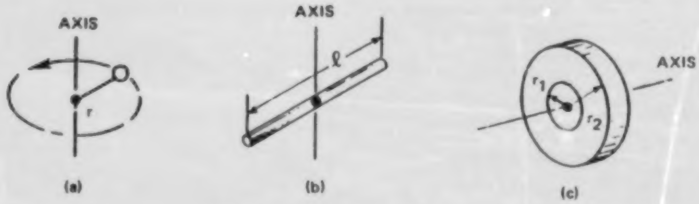
From Figure 2, the moment of inertia of a particle in a circular orbit is

$$I = mr^2.$$

Thus, kinetic energy of the orbital mass is given by Equation 1.

$$E_k = \frac{1}{2} I\omega^2 \qquad (1)$$

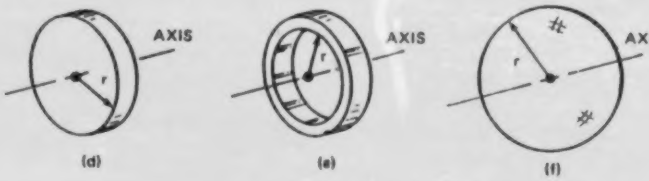
Equation 1 gives the kinetic energy of any rotating mass, as long as the correct expression is substituted for I, moment of inertia. In every case, the proper moment of inertia is that about the axis around which the body is rotating. Examples A and B show use of Equation 1 in solving problems.



Particle in circular orbit. $I = mr^2$

Slender rod, axis through center. $I = \frac{1}{12} m\ell^2$

Annular (ring) cyclinder. $I = \frac{1}{2} m\ (r_1{}^2 + r_2{}^2)$



Solid cylinder. $I = \frac{1}{2} mr^2$

Thin-walled hollow cylinder (wheel). $I = mr^2$

Solid sphere. $I = \frac{2}{5} mr^2$

FIG. 2

EXAMPLE A: ROTATIONAL K E OF A THIN-WALLED HOLLOW CYLINDER

Given: A thin-walled cylinder with a mass of 15 kg and a radius of 18 cm spinning at 2000 rpm.

Find: Its rotational kinetic energy.

Solution: $I = mr^2$ (from Figure 2)

$$= (15\text{ kg})(0.18\text{ m})^2$$

$$I = 0.486\text{ kg} \times \text{m}^2$$

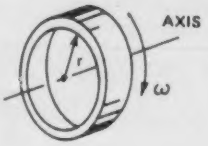
$$E_k = \frac{1}{2} I\omega^2$$

$$= (0.5)(0.486\text{ kg} \times \text{m}^2)(209\ \frac{\text{rad}}{\text{s}})^2$$

$$E_k = 1.06 \times 10^4\text{ J}$$

$$\omega = 2000\frac{\text{r}}{\text{min}}\ (\frac{2\pi\text{ rad}}{\text{r}})(\frac{1\text{ min}}{60\text{ s}})$$

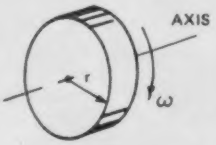
$$\omega = 209\ \frac{\text{rad}}{\text{s}}$$



EXAMPLE B: ROTATIONAL RATE OF A SOLID CYLINDER

Given: A solid cylinder with a mass of 25 kg and a radius of 25 cm used to store kinetic energy.

Find: What its rotational rate must be in rpm to store 100 kJ.



Solution:

$$I = \frac{1}{2}\ mr^2\text{(from Figure 2)}$$

$$= (0.5)(25\text{ kg})(0.25\text{ m})^2$$

$$I = 0.781\text{ kg} \times \text{m}^2$$

$$\text{rpm} = (506\ \frac{\text{rad}}{\text{s}})(\frac{1\text{ r}}{2\pi\text{ rad}})(\frac{60\text{ s}}{1\text{ min}})$$

$$\text{rpm} = 4832\text{ rpm}$$

$$E_k = \frac{1}{2}\ I\omega^2$$

$$\omega = \sqrt{\frac{2\ E_k}{I}}$$

$$= \sqrt{\frac{2\ (10^5\text{ J})}{0.781\text{ kg} \times \text{m}^2}}$$

$$\omega = 506\text{ rad/s}$$

APPLICATION OF FLYWHEEL

In many mechanical devices, flywheels are used to smooth the motion of reciprocating parts. Figure 3 shows the flywheel and one piston of an 8-cylinder internal-combustion engine. In this system part of the kinetic energy of the flywheel is used to force the piston upward to compress the air-fuel mixture in the cylinder. When the mixture burns, some of the energy produced is stored in the flywheel to be used in the next compression stroke.

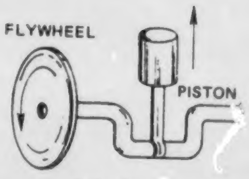

FLYWHEEL
PISTON

FIG. 3

The friction motors of toy cars are excellent examples of pure kinetic energy drives. The axle is connected to a flywheel by a series of gears. When the toy is pushed forward, the flywheel is caused to spin at a high rate. When the toy is released, the stored kinetic energy is transmitted to the axle by the same gear system, to maintain the forward motion.

In recent years, several investigations have considered storage of large amounts of rotational kinetic energy in flywheels as a solution to practical transportation problems. Flywheels weighing as much as 3000 lb, and smaller wheels with rotational rates of as high as 12,000 rpm, have been installed in vehicles. In some systems, direct mechanical coupling of the flywheel to the drive wheels has been employed; but most use the electrical conversion system shown in Figure 4. In this system, the flywheel drives a generator that supplies electrical power to a motor that powers the vehicle.

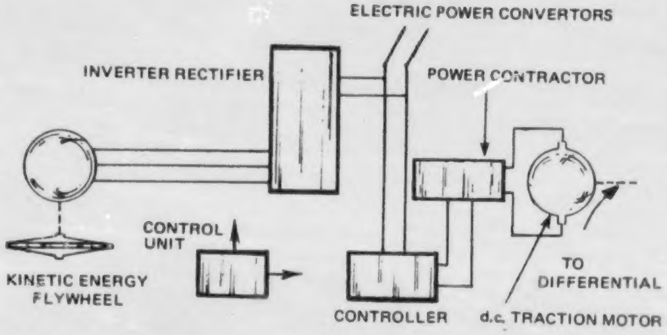

ELECTRIC POWER CONVERTORS
INVERTER RECTIFIER
POWER CONTRACTOR
CONTROL UNIT
KINETIC ENERGY FLYWHEEL
CONTROLLER
TO DIFFERENTIAL
d.c. TRACTION MOTOR

FIG. 4

Figure 5 shows a bus equipped with a kinetic-energy drive. This drive stores 25 MJ, and gives the bus a range of several miles. A similar system can run a small automobile for 7 to 9 miles at speeds of 30 miles per hour. Combinations of

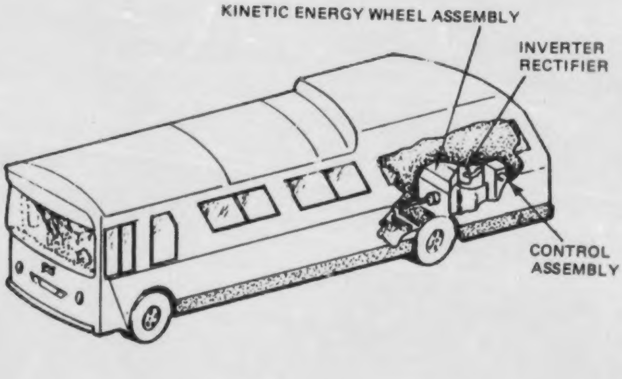

KINETIC ENERGY WHEEL ASSEMBLY
INVERTER RECTIFIER
CONTROL ASSEMBLY

FIG 5

kinetic energy and electrical drives offer both lower energy utilization and lower pollution for urban transportation.

SUMMARY

A turning flywheel possesses kinetic energy in the form of the mechanical, rotational energy of its mass. Kinetic energy stored in flywheels is used to smooth the motion of reciprocating parts (as, for example, in engines) and to provide energy for drive systems in some vehicles.

ROTATIONAL KINETIC ENERGY

MATERIALS

Wheel and axle. The wheel is an 18-inch circular disk of 1/2-inch aluminum. It is mounted on an axle consisting of 1-inch diameter wooden dowel.
Two 1-inch bearings.
Tack.
Nylon twine.
Metric weights.
Meter stick.
Metric balance.

PROCEDURE

1. Weigh wheel and axle; record appropriate mass (kilograms) in Data Table. Record radius of wheel; calculate its moment of inertia. (Note: This calculation is based upon a solid, uniform disk, and ignores the small change due to presence of wooden axle at center of disk.)

2. Mount wheel as shown in Figure 6. Be sure that wheel turns freely. Fasten string securely to axle by pressing a tack through end to prevent slipping. Cut other end of string at a length that will allow weight to hang without touching table top. Attach 200 g to free end of string. Measure, and record distance (h_1) from bottom of weight to table top in Data Table.

3. Turn wheel to wind string around axle, raising weight almost to axle. Measure, and record distance (h_2) from bottom of weight to table top in Data Table.

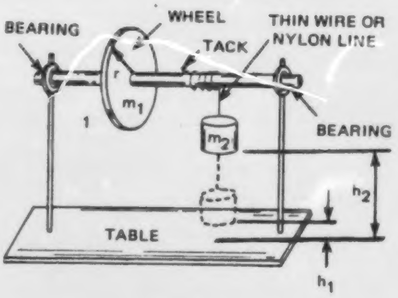


FIG. 6

4. Release wheel, and allow weight to drop, storing kinetic energy in flywheel. The stored energy then will raise weight above table as string is rewound on axle. Hold wheel when

it comes to a stop. Measure, and record new height (h_3) of weight in Data Table.

5. Complete calculations in Data Table.
6. Repeat with a 500-g weight.
7. What happened to energy lost in this flywheel system?

DATA TABLE

Trial 1

Mass of wheel and axle: m_1 = ______ kg

Radius of wheel: r = ______ m

Moment of inertia: $I = \frac{1}{2} mr^2$

$= 0.5 \times (\_\_\_\_\text{ kg})(\_\_\_\_\text{m})^2$

$I = \_\_\_\_\text{ kg} \times \text{m}^2$

Suspended mass = m_2 = _____ kg

Minimum height of suspended weight: h_1 = _____ m

Height of weight before release: h_2 = _____ m

Height of weight after return: h_3 = _____ m

Distance weight fell: $d_1 = h_2 - h_1 = (\_\_\_\_\text{ m}) - (\_\_\_\_\text{ m})$

$= \_\_\_\_\text{ m}$

Distance weight rose: $d_2 = h_3 - h_1 = (\_\_\_\_\text{ m}) - (\_\_\_\_\text{ m})$

$= \_\_\_\_\text{ m}$

Potential energy of weight before release:

$E_{p_1} = m_2 d_1 g = \_\_\_\_\text{ kg} \times \_\_\_\_\text{ m} \times 9.8\text{ m/s}^2 = \_\_\_\_\text{ J}$

Potential energy of weight after return:

$E_{p_2} = m_2 d_2 g = \_\_\_\_\text{ kg} \times \_\_\_\_\text{ m} \times 9.8\text{ m/s}^2 = \_\_\_\_\text{ J}$

% Efficiency of energy storage:

$$\eta = \frac{E_{p_1} - E_{p_2}}{E_{p_1}} 100 = \frac{(\_\_\_\_\text{ J}) - (\_\_\_\_\text{ J})}{(\_\_\_\_\text{ J})} 100$$

$= \_\_\_\_\ \%$

Assume that half of the energy loss occurs during conversion of potential energy of weight to kinetic energy of flywheel. Maximum kinetic energy of the flywheel then is as follows:

Stored energy: $E = \dfrac{E_{p_1} + E_{p_2}}{2} = 0.5 \times (\_\_\_\_\_\ \mathrm{J} + \_\_\_\_\_\ \mathrm{J})$

$= \_\_\_\_\_\ \mathrm{J}$

Maximum angular velocity of flywheel: $E = \frac{1}{2} I\omega^2$

$$\omega = \sqrt{\frac{2\ E}{I}}$$

$$= \sqrt{\frac{2 \times (\_\_\_\_\_\ \mathrm{J})}{(\_\_\_\_\_\ \mathrm{kg} \times \mathrm{m}^2)}}$$

$\omega = \_\_\_\_\_$ rad/s

Trial 2

$I = \_\_\_\_\_$ kg x m^2

$m_2 = \_\_\_\_\_$ kg

$d_1 = \_\_\_\_\_$ m

$d_2 = \_\_\_\_\_$ m

$E_{p_1} = \_\_\_\_\_$ J

$E_{p_2} = \_\_\_\_\_$ J

η (% eff) = _____ %

Stored Energy: $E = \_\_\_\_\_$ J

$\omega_{max} = \_\_\_\_\_$ rad/s

SELF CHECK

1. A uniform disk of mass 4.7 kg has a diameter of 35 cm. Calculate its moment of inertia.

2. A body of mass 2.2 kg moves in a circular orbit with a radius of 0.85 m and angular velocity of 50 rad/s. What is its kinetic energy?

3. A flywheel consists of a uniform disk with a radius of 30 cm and a mass of 400 kg. If it spins at 5000 rpm, what is the stored kinetic energy?

4. Assume that a flywheel has a kinetic energy of 10 kJ and a moment of inertia of 0.032 kg x m^2. Calculate its angular velocity.

5. At what rate must the above flywheel spin to store 2 MJ?

7M4

CONSERVATION OF ENERGY IN COLLISIONS

Moving objects possess both momentum and kinetic energy. When two such objects collide, momentum always is conserved, but kinetic energy after the collision usually is less than kinetic energy before the collision. Since total energy always is conserved, the kinetic energy lost is converted to other forms of energy. This module discusses kinetic energy in collisions and the final form of the kinetic energy lost.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Explain in a short paragraph the difference between conservation of linear momentum and conservation of energy in a collision.
- Explain in a short paragraph how the law of conservation of energy applies to a totally elastic collision and to an inelastic collision.
- Given the masses and velocities of two objects before a totally inelastic collision, determine their velocity after the collision and their total kinetic energy before and after the collision.
- Given the mass of a projectile, the mass of a ballistic pendulum, and the height of swing of the pendulum, determine the initial velocity of the projectile and the amount of kinetic energy converted to heat in the collision.
- Given the appropriate equipment, use a ballistic pendulum to determine the velocity of a projectile.

CONSERVATION OF MOMENTUM IN COLLISIONS

In Concept Module 4-0, "Momentum," linear momentum was defined by Equation 1:

$$p = mv \tag{1}$$

where

p = linear momentum.
m = mass of object.
v = velocity of object.

Since velocity is a vector quantity, linear momentum also must be a vector quantity. Module 5-0 demonstrated that the total momentum of an isolated system does not change. In other words, the total momentum before a collision equals the total momentum after the collision. The conservation of momentum in a collision is described by Equation 2.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \tag{2}$$

where

m_1 = mass of object one.
m_2 = mass of object two.
v_1 = initial velocity of object one.
v_2 = initial velocity of object two.
v_1' = final velocity of object one.
v_2' = final velocity of object two.

This equation indicates that linear momentum always is conserved in collisions.

KINETIC ENERGY IN COLLISIONS

Kinetic energy is the energy of motion. In Concept Module 7-0, "Potential and Kinetic Energy," the kinetic energy of a body was defined by Equation 3.

$$E_K = \frac{1}{2}mv^2 \tag{3}$$

where

E_K = kinetic energy.
m = mass.
v = velocity.

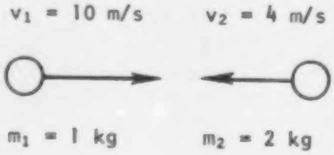
Unlike momentum, kinetic energy is a scalar quantity.

When two completely elastic bodies collide, kinetic energy is conserved, and the total kinetic energy after the collision is equal to the total kinetic energy before the collision. The only completely elastic collisions known are those between atomic particles. Some other collisions, such as those between billiard balls, are almost completely elastic; but even in those cases, some of the kinetic energy is changed into other forms of energy.

Totally inelastic collisions are those in which the two colliding bodies stick together. In this type of collision, a large fraction of the original kinetic energy is lost. Example A illustrates the loss of kinetic energy in a totally-inelastic collision.

EXAMPLE A: KINETIC ENERGY IN A TOTALLY INELASTIC COLLISION

Given: The two bodies here depicted collide head-on and stick together.



Find: Kinetic energy before and after the collision.

Solution: The kinetic energy before the collision is the sum of the individual kinetic energies.

$$E_K = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$= (0.5)(1\ \text{kg})(10\ \text{m/s})^2 + (0.5)(2\ \text{kg})(4\ \text{m/s})^2$$

$$= 50\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} + 16\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

$$= 66\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

$$E_K = 66\ \text{J}$$

Before the kinetic energy after the collision can be calculated, the final velocity v_f of the two bodies first must be determined. Substitution of $v_f = v_1' = v_2'$ into Equation 2 results in the following:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f$$

$$v_f = \frac{(m_1v_1 + m_2v_2)}{(m_1 + m_2)}$$

$$= \frac{(1\ \text{kg})(10\ \text{m/s}) + (2\ \text{kg})(-4\ \text{m/s})}{(1\ \text{kg} + 2\ \text{kg})}$$

(Right is chosen as positive.)

$$= \frac{(10\ \frac{\text{kg}\cdot\text{m}}{\text{s}} - 8\ \frac{\text{kg}\cdot\text{m}}{\text{s}})}{3\ \text{kg}}$$

$$v_f = 0.67\ \text{m/s}$$

$$E_K = \frac{1}{2}mv^2 \quad (\text{where } m = m_1 + m_2 = 3\ \text{kg})$$

$$= (0.5)(3\ \text{kg})(0.67\ \text{m/s})^2$$

$$= 0.67\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

$$E_K = 0.67\ \text{J}$$

Kinetic Energy Lost = 66 J - 0.67 J = 65.33 J

Since total energy in any system is constant according to the law of conservation of energy, this decrease in kinetic energy must be accompanied by an equivalent increase in other forms of energy. This equivalent increase of energy usually appears either in the form of temperature increases (increased kinetic energy of molecules) or in the form of deformations of the objects.

THE BALLISTIC PENDULUM

The ballistic pendulum is a device that can measure the velocity of a projectile. The projectile makes a totally-inelastic collision with a stationary body of much greater mass. The momentum of the system immediately after the collision is the same as the momentum of the projectile before the collision. This momentum can be determined if the kinetic energy after the collision is known.

Figure 1 depicts a bullet of mass m_1 striking a stationary pendulum of mass m_2. The velocity of the bullet before the collision is v_1, and the velocity of the pendulum and bullet after the collision is v_2. The center of mass of the pendulum swings upward through a vertical height h. The kinetic energy of the system immediately after the collision is as follows:

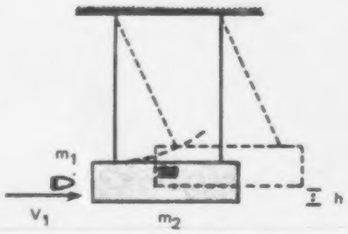


FIG. 1

$$E_K = \frac{1}{2} (m_1 + m_2) v_2^2$$

This value is equal to the potential energy of the system at the top of the pendulum swing:

$$E_P = (m_1 + m_2) gh$$

Thus,

$$\begin{aligned} E_K &= E_P \\ \frac{1}{2} (m_1 + m_2) v_2^2 &= (m_1 + m_2) gh \\ v_2^2 &= 2 gh \\ v_2 &= \sqrt{2 gh} \qquad (4) \end{aligned}$$

From the conservation of momentum —

$$\begin{aligned} m_1 v_1 &= (m_1 + m_2) v_2 \\ v_1 &= \frac{(m_1 + m_2)}{m_1} v_2 \qquad (5) \end{aligned}$$

Substitution of Equation 4 into Equation 5 yields Equation 6.

$$v_1 = \left(\frac{m_1 + m_2}{m_1}\right) \sqrt{2 gh} \qquad (6)$$

Thus, the initial velocity of the bullet can be determined from the masses and the height of the pendulum swing. The use of Equation 6 is illustrated in Example B.

EXAMPLE B: THE BALLISTIC PENDULUM

Given: A bullet having a mass of 20 g strikes a ballistic pendulum having a mass of 1.5 kg. The pendulum swings through a vertical height of 5 cm.

Find: Velocity of the bullet before the collision.

Solution: $v_1 = \frac{m_1 + m_2}{m_1} \sqrt{2 gh}$

$$v_1 = \frac{1500\ g + 20\ g}{20\ g}\sqrt{2\ (9.8\ m/s^2)(0.05\ m)}$$

$$= (76)\ \sqrt{0.98\ m^2/s^2}$$

$$= (76)(0.99\ m/s)$$

$$v_1 = 75.24\ m/s$$

SUMMARY

Kinetic energy is the energy of the motion of a mass and is equal to the product of one-half the mass and the square of its velocity. Although linear momentum (mass times velocity) is conserved in all collisions, kinetic energy is conserved only in totally-elastic collisions. In any other collision, a portion of the kinetic energy is converted into heat energy that appears as a temperature increase in the bodies in collision. Use of the ballistic pendulum employed in the laboratory of this module involves calculations of both kinetic energy and momentum in the determining of projectile velocity.

BALLISTIC PENDULUM

MATERIALS

Balance.
Meter stick.
Ballistic pendulum apparatus (Sargent-Welch, Bow-and-Arrow Form) consisting of a bow with attached arrow guide and table clamp, an arrow of known mass m_1, and a ballistic pendulum consisting of a container of modeling clay supported by a quadrifilar suspension system. (Refer to Figure 2.)

PROCEDURE

1. Determine mass of arrow and pendulum, and record in Data Table.
2. Set up equipment as in Figure 2.
3. Fire arrow into pendulum; measure and record height of swing. Be sure to measure height of swing between identical points on the pendulum, while at the bottom and top of the swing.
4. Perform the calculations indicated to complete the Data Table.
5. Write a short paragraph explaining how total energy is conserved in this experiment.

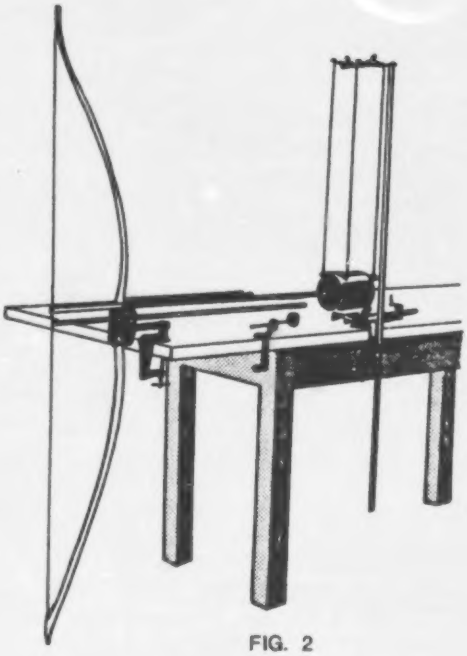


FIG. 2

DATA TABLE

Mass of arrow: m_1 = ________

Mass of pendulum: m_2 = ________

Height of swing: h = ________

Initial velocity of arrow (from Equation 6): v_1 = ________

Initial kinetic energy of arrow: E_{K_1} = ________

Initial momentum of arrow: P_1 = ________

Velocity after collision (from Equation 5): v_2 = ________

Kinetic energy after collision: E_{K_2} = ________

Momentum after collision: P_2 = ________

Kinetic energy changed to heat energy in collision:

$E = E_{K_1} - E_{K_2}$ = ________

SELF CHECK

1. Explain the difference between the conservation of linear momentum in collisions and the conservation of energy in collisions.

2. What happens to the kinetic energy in each of the following collisions?

   a. Totally-elastic.
   b. Totally-inelastic.
   c. Partially-elastic.

3. A 1000-kg automobile traveling to the right at 10 m/s has a totally inelastic collision with a 1250-kg automobile traveling to the left at 8 m/s. How much kinetic energy is converted to heat in the collision?

4. A bullet having a mass of 9 g is fired into a ballistic pendulum having a mass of 2 kg. The pendulum swings through a vertical distance of 3.2 cm. Determine the following:

a. Initial velocity of the bullet.
b. Initial kinetic energy of the bullet.
c. Velocity of the pendulum after the collision.
d. Kinetic energy converted to heat in the collision.

7F2

VENTURI METERS

A venturi meter is a device used in industry for flow rate measurements. It illustrates the interchange of kinetic and potential energy that occurs in fluid flow. The energy interchange obeys the fluid conservation-of-energy equation, Bernoulli's theorem. Although Bernoulli's theorem is strictly correct only for incompressible, nonviscous liquids, it often is applied to all liquids with sufficient accuracy for many engineering purposes.

This module presents the application of Bernoulli's equation to a venturi meter and the derivation of an equation for the flow velocity in a horizontal pipe. A discussion of potential and kinetic energy changes for fluid flowing through a horizontal pipe is included. In the laboratory, the student will measure velocity of flow with a venturi meter.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe with the aid of diagrams and equations the application of Bernoulli's equation to fluid flow in a horizontal pipe in which the cross-sectional area changes along the flow direction.
- Calculate fluid velocity in a horizontal pipe using a velocity equation developed for a venturi flowmeter inserted into the pipe.
- Calculate pressure in the throat constriction of a venturi flowmeter.

- Given the appropriate equipment, measure the velocity of fluid flow with a venturi meter.

DISCUSSION

In Concept Module 7-0, Bernoulli's equation was explained as a conservation-of-energy law that applies to streamline flow of an incompressible fluid. Bernoulli's equation states that the net work done on, or by, a fluid must equal the change in kinetic and potential energy of the fluid mass. This energy relationship is given by Equation 1.

$$[p + \rho gh + \frac{1}{2}\rho v^2] = \text{constant} \qquad (1)$$

where

p = pressure of fluid at a given point.
ρ = density of fluid (constant for incompressible fluid).
g = acceleration due to gravity.
h = height of fluid mass at given point above a reference level.
v = velocity of fluid at given point in stream.

APPLICATION OF BERNOULLI'S EQUATION TO VENTURI METER

In this module, the changes in pressure and kinetic energy that occur at different cross sections in the venturi meter will be examined. This will permit determination of the flow velocity. To measure the flow rate of a liquid with a venturi flowmeter, a U-tube manometer can be attached to the bottom of the flowmeter, as illustrated in Figure 1.

The left-hand member of Equation 1 will be the same for any two points in 1 and 2 in Figure 1. Bernoulli's equation takes the form of Equation 2.

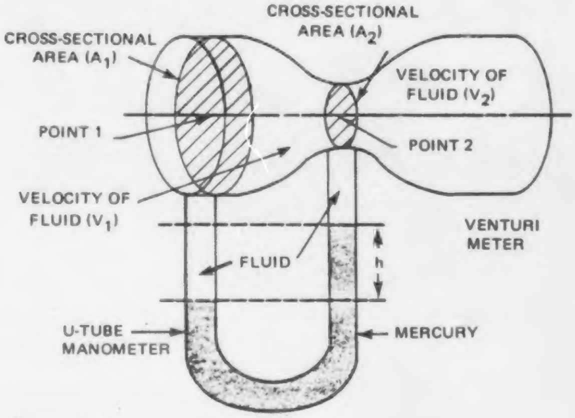

CROSS-SECTIONAL AREA (A$_1$)
CROSS-SECTIONAL AREA (A$_2$)
VELOCITY OF FLUID (V$_2$)
POINT 1
POINT 2
VELOCITY OF FLUID (V$_1$)
VENTURI METER
FLUID
h
U-TUBE MANOMETER
MERCURY

FIG. 1

$$p_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2 \qquad (2)$$

Because points 1 and 2 are the same elevation —

$$\rho gh_1 = \rho gh_2 .$$

Equation 2 therefore reduces to the form of Equation 3 or 4.

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 \qquad (3)$$

or

$$p_1 - p_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \qquad (4)$$

In Equation 4, the pressure difference between points 1 and 2 (left-hand side) equals the change in kinetic energy of fluid flow between points 1 and 2 (right-hand side).

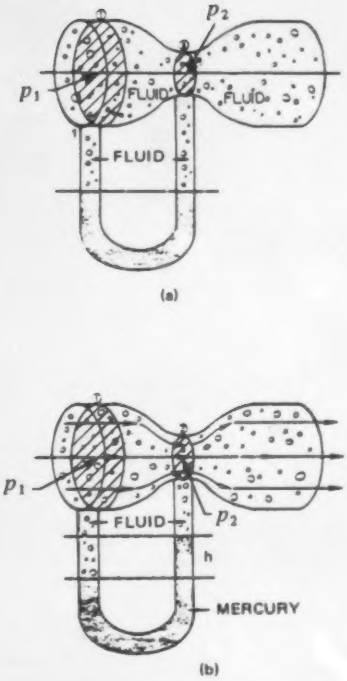


FIG. 2

Pressure change between positions 1 and 2 in Figure 1 can be measured with a U-tube manometer. The simplified diagrams in Figure 2 illustrate how a U-tube manometer registers the pressure drop. In Figure 2a, representing a condition of no-flow in the venturi meter, the mercury in the U-tube is at the same level in each arm. In the lower drawing, under flow conditions, the pressure at position 2 decreases relative to position 1. This drop is registered by a difference in the height of the mercury level in the two arms of the U-tube.

Pressure difference $p_1 - p_2$ is obtained from Equation 5.

$$p_1 - p_2 = (\rho_M - \rho_F)\,gh \qquad (5)$$

where

ρ_M = density of mercury (13.6 g/cm^3).
ρ_F = density of liquid flowing through venturi flowmeter.
h = height differential of mercury columns, cm.
g = acceleration due to gravity (980 cm/s^2).

For incompressible, streamline fluid flow, cross-sectional area and velocity are related in the manner shown by Equation 6.

$$A_1 v_1 = A_2 v_2 \qquad (6)$$

where

A_1 = cross-sectional area at position 1.
v_1 = flow velocity at position 1.
A_2 = cross-sectional area at position 2.
v_2 = flow velocity at position 2.

The essential parameters p_1, v_1, A_1; and p_2, v_2, A_2 are shown in Figure 3 for positions 1 and 2.

Because the venturi flowmeter is inserted into a fluid system in which the natural flow velocity is v_1, a single equation that yields v_1 in terms of known or measurable parameters is useful. When Equations 4, 5, and 6 are combined appropriately, Equation 7 is the result.

$$v_1 = \sqrt{2\left(\frac{\rho_M - \rho_F}{\rho_F}\right)\left(\frac{A_2^2}{A_1^2 - A_2^2}\right) gh} \qquad (7)$$

Flow velocity can be calculated from Equation 7 if the following values are known: density of mercury (ρ_M), density of the flowing liquid (ρ_F), cross-sectional area of the venturi flowmeter at the input (A_1) and at constriction (A_2), and the differential mercury height (h) in the U-tube manometer.

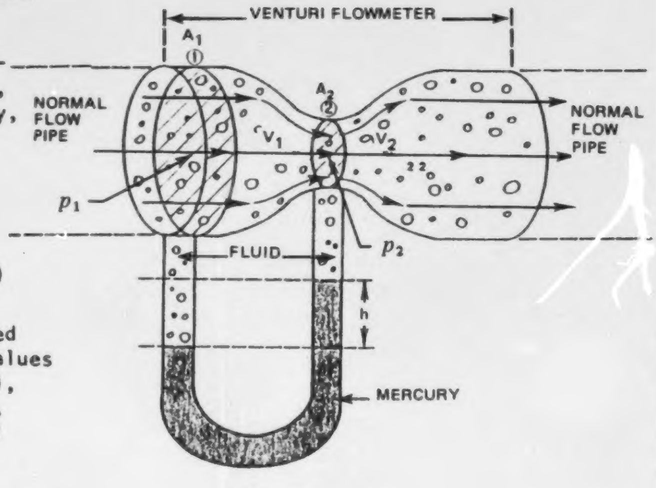


FIG. 3

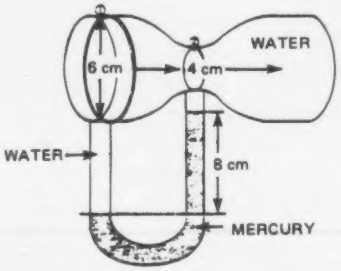
As the cross-sectional area is equal to $\pi d^2/4$ where d is diameter, Equation 7 can also be written in the form of Equation 8.

$$v_1 = \sqrt{2\left(\frac{\rho_M - \rho_F}{\rho_F}\right)\left(\frac{d_2^4}{d_1^4 - d_2^4}\right) gh} \qquad (8)$$

Example A illustrates use of Equation 8 in determining flow velocity in a flow pipe.

EXAMPLE A: FLOW MEASUREMENT WITH A VENTURI FLOWMETER

Given: Water flows through a pipe 6-cm in diameter into a venturi tube with a constriction of 4 cm at the throat. Mercury fills the manometer U-tube; and, as the steady flow proceeds, a height differential of 8 cm is established.



Find: Velocity of fluid in the pipe.

Solution: From Equation 8 —

$$v_1 = \sqrt{2\left(\frac{\rho_M - \rho_F}{\rho_F}\right)\left(\frac{d_2^4}{d_1^4 - d_2^4}\right) gh}$$

$$= \sqrt{2\left(\frac{13.6 - 1}{1}\right)\left(\frac{(4)^4}{(6)^4 - (4)^4}\right)(980)(8)}$$

$$v = 220\ \frac{\text{cm}}{\text{s}}$$

ENERGY LOSS IN A VENTURI TUBE

When fluid flows through a constriction in a real pipe or through a venturi flowmeter where resistance exists due to drag at the walls of the pipe, the constriction reduces total energy of the fluid. This reduction is analogous to the action of an electrical resistor that reduces electrical energy in a circuit and causes a voltage drop along the direction of current flow.

The experiment in this module uses a triple-arm manometer. The purpose of the third arm is to show that the fluid pressure recovered after flowing through the venturi flowmeter does not quite equal the pressure of the fluid at the entrance to the flowmeter. The third arm of the manometer therefore clearly shows the effect of frictional drag on pressure loss in real pipes. Any constriction or sudden bend in real hydraulic lines causes a similar pressure loss.

SUMMARY

The venturi tube is a short tube arranged with a constricted segment at the center to cause a pressure difference for the purpose of measuring the fluid flow in pipes. The fluid velocity is higher through the constriction than through either end of the tube. When fluid increases in velocity, it decreases in pressure, making possible a measurement of the rate of flow. Factors influencing measurement include the cross-sectional area of tube ends and restriction and the density of the fluid.

USE OF A VENTURI METER TO DETERMINE FLOW VELOCITY

MATERIALS

Glass venturi tube (18 cm long, 18 mm in diameter; tube narrows at center and has three outlet tubes at center; ends of tube are flared). (Suggested sources: Science Kit Inc., Central Scientific Co., Sargeant Welch Co.)
Two lengths of 3/4-inch automobile heater hose.
Triple-arm manometer, 30 cm-long to fit outlet tubes of venturi tube. (Suggested source: Sargeant Welch Co.)
Rubber tubing (1/4-inch inside diameter) for connections to outlet tubes of venturi flowmeter.
Metric scales for manometer arms.
Circular clamp for 3/4-inch hose.
Two large containers (5-gallon buckets or trash cans).
Two platform scales graduated in kilograms.
Stopwatch.
Index cards (any size).
Scissors.

PROCEDURE

1. Arrange venturi tube, closed triple-arm mercury manometer, and associated equipment as shown in Figure 4.

2. Measure inside diameters of entrance and exits of venturi; record in Data Table 1.

3. Measure diameter of venturi throat, or obtain from manufacturer's specifications. Record in Data Table 1.

4. Place exit hose in excess water con-

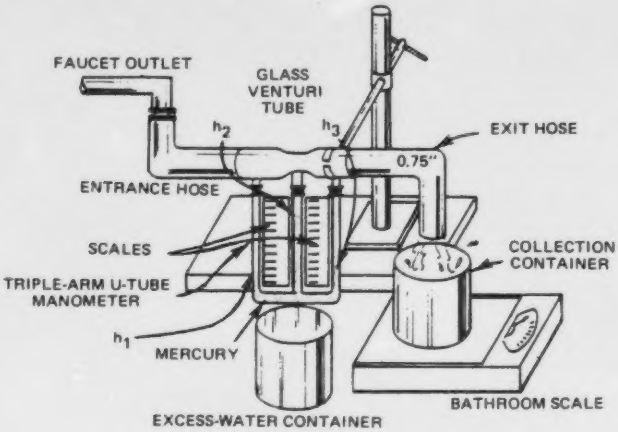

FAUCET OUTLET
GLASS VENTURI TUBE
h_2
h_3
EXIT HOSE
0.75"
ENTRANCE HOSE
SCALES
COLLECTION CONTAINER
TRIPLE-ARM U-TUBE MANOMETER
h_1
MERCURY
BATHROOM SCALE
EXCESS-WATER CONTAINER

FIG. 4

tainer; slowly turn on faucet, checking system for leaks. Shut off faucet; repair leaks if necessary.

5. Slowly turn faucet on; adjust water flow to obtain a 2-cm difference in mercury levels of entrance and throat manometer arms.

6. Record mercury meniscus scale readings for all three arms of manometer.

7. Simultaneously switch exit hose to collection container, and start timer. After a time interval somewhat in excess of 2 minutes, stop timer, while simultaneously switching exit hose to excess water container. (Anticipate this action to minimize mess that can occur.)

8. Record total mass readings of scales and time elapsed (from timer).

9. Empty collection containers and excess water container as needed.

10. Repeat Steps 5, 6, 7, and 8 for mercury level differences of 6 and 10 cm between entrance (h_1) and venturi throat (h_2). Record data and results from each trial.

11. Do calculations necessary to complete Data Table 2.

    a. Determine water added to collection container during each trial (kg at end of trial, minus kg at start of trial).
    b. Calculate mass flow rate and volume flow rate. Calculate velocity v of fluid from relation given below. (This equation comes essentially from Equation 6 where the product of Av is equal to a constant — the volume flow rate, Q_v.)

    $$v = \frac{Q_v}{A_2}$$

    Do this calculation for each trial in experiment. Record results in Data Table.
    c. For each trial, calculate velocity of fluid flow from Equation 8. Record results.
    d. For each trial, calculate a % difference between velocity in b and velocity in c. Record.
    e. For each trial, calculate pressure difference (Δp) between venturi entrance and throat from Equation 5. For measurements made in Figure [illegible], this becomes —

    $$\Delta p = (\rho_M - \rho_F)\ g\ (h_2 - h_1)$$

f. For each trial, calculate pressure difference (Δp_v) between venturi entrance and exit, using this equation:

$$\Delta p_v = (\rho_M - \rho_F)\ g\ (h_3 - h_1)$$

DATA TABLE 1. MEASUREMENTS

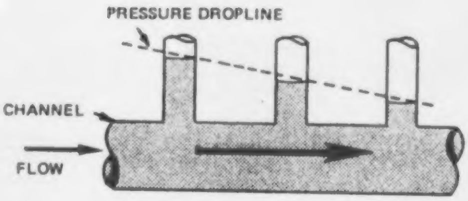
	Scale Readings of Mercury Meniscus Level			Scale Reading		
Trial	Entrance h_1 (cm)	Throat h_2 (cm)	Exit h_3 (cm)	Start of Trial (kg)	End of Trial (kg)	Water Added During Trial (kg)
1						
2						
3						
4						
5						

Inside diameters: Venturi entrance: _____ cm
Venturi throat: _____ cm
Venturi exit: _____ cm
Time water is collected: ______ s

DATA TABLE 2. RESULTS OF CALCULATIONS							
Trial	Mass Flow Rate (kg/s)	Volume Flow Rate (cm^3/s)	Measured by Flow Rate (cm/s)	Velocity of Flow Calculated Using Equation 8 (cm/s)	% Difference	Pressure Change Δp (kg/m^2)	Pressure Loss Across Tube (Δp_v) (kg/m^2)
1							
2							
3							
4							
5							

SELF CHECK

1. Explain why height of liquid in the standpipes of the adjoining illustration indicates that pressure drops along the flow pipe, although the flow pipe has a uniform cross section and the flowing liquid is incompressible.



2. By a dimensional analysis, show that Bernoulli's equation is dimensionally consistent. Show that the units of each term in Bernoulli's equation (Equation 1) are energy per unit volume.

3. Height difference registered in the two arms of a mercury manometer with water above the mercury is 17.5 cm when connected to a horizontal venturi meter. Diameters of the pipe and throat are 15 cm and 7 cm, respectively. Neglecting frictional losses, calculate the volume rate of discharge through the meter.

4. A horizontal pipe has a cross-sectional area of 4.0 in^2. In the center of the pipe, ther is a constriction having a cross-sectional area of 1.0 in^2. Gasoline (weight density 42 lb/ft^3) flows with a speed of 6.0 ft/s^2 in the larger section. $p = 10$ lb/in^2. Calculate (a) gasoline speed in the constriction and (b) pressure in the constriction.

5. What is the density of a fluid flowing through a venturi meter with throat and pipe areas of 20 cm^2 and 100 cm^2 respectively if the height difference in a U tube filled with mercury is 16 cm when the velocity of fluid flow through the pipe is 2 m/s?

6. Find an expression similar to Equation 7 that gives V_2 in terms of P_m, P_F, A_1, A_2, g and h.

7. If the height differential in a manometer connected to a venturi meter is 12 cm when water is flowing through the venturi meter what should the height differential be when a fluid with a density of 4 g/cm^3 flows through the meter at the same rate that the water was flowing?

7F3

PITOT TUBES

The pitot tube is a device designed for velocity measurement in a fluid. It finds extensive use on boats and aircraft as a simple, low-cost speed indicator. In measurement of air speeds and other fluid velocities in industrial applications, the pitot tube is used in an exploratory manner. It is used to check velocity of fluids passing through orifice plates, venturis, and other flow-measuring instruments, or to determine radial velocity distributions across a large pipe. Air or liquid impacting with the tube has kinetic energy that causes a measurable pressure differential.

This module will present the operating principles of the pitot tube, its advantages and its limitations in fluid velocity measurement, and its various features of construction and associated equipment. The pitot tube equation for velocity flow is derived from Bernoulli's equation.

In the laboratory, the student will measure the air speed of a moving automobile using a pitot tube.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Describe features of the construction of a pitot tube and environmental conditions in which it can be used.
- List merits and limitations of an industrial pitot tube given in this module.
- Use the pitot tube equation to calculate fluid speeds.
- Given the appropriate equipment, set up a pitot tube device in an actual experimental situation, and take measurements

necessary to calculate fluid flow velocity with the aid of the pitot-tube equation.

DISCUSSION

The pitot tube is a device that measures speed of fluid flow by measuring the impact pressure or force required to stop a small portion of the fluid. It is used for measurement of speed of a gas or of an incompressible liquid. A pitot tube of popular design is illustrated in Figure 1. It is constructed of two concentric tubes. The inner tube is open to the moving fluid, and is referred to as the "pitot tube." The outer tube, referred to as the "static tube," is shielded from the oncoming flow, although in its side it has small holes perpendicular to the flow of the fluid stream.

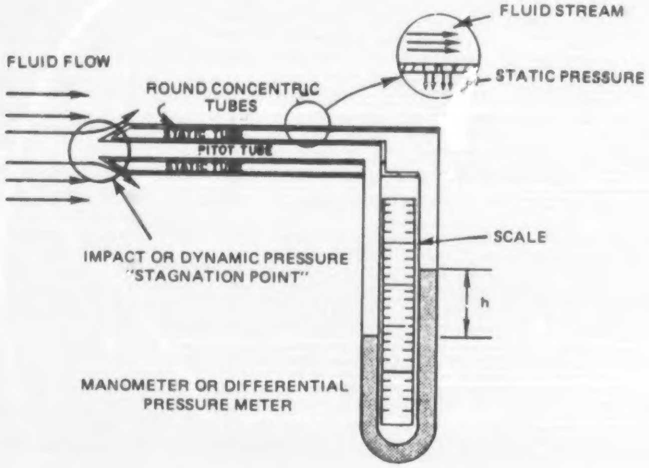

FLUID STREAM
FLUID FLOW
ROUND CONCENTRIC TUBES
STATIC PRESSURE
STATIC TUBE
PITOT TUBE
STATIC TUBE
SCALE
h
IMPACT OR DYNAMIC PRESSURE "STAGNATION POINT"
MANOMETER OR DIFFERENTIAL PRESSURE METER

FIG. 1

The small holes in the static tube provide a means by which static stream pressure is measured. The dynamic, or impact, pressure occurs at the mouth of the instrument. Velocity of the fluid at this impact region is zero, and the region is sometimes called the "stagnation point." Speed of the fluid stream is proportional to the difference in impact pressure and static pressure. Pitot tubes use a sensitive differential-pressure measuring device, such as the manometer shown in Figure 1, to measure this pressure difference.

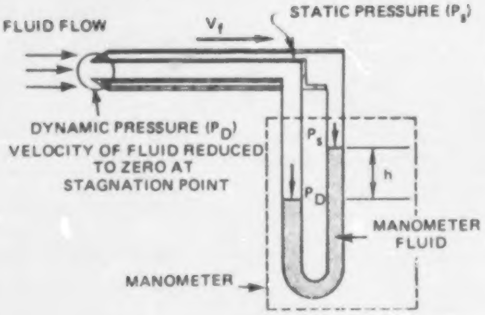

FLUID FLOW
V_f
STATIC PRESSURE (P_s)
DYNAMIC PRESSURE (P_D)
VELOCITY OF FLUID REDUCED TO ZERO AT STAGNATION POINT
P_s
P_D
h
MANOMETER FLUID
MANOMETER

FIG. 2

The equation that describes the function of a pitot tube can be derived by applying conditions and variables illustrated in Figure 2 to Berboulli's equation. The steps in this derivation provide a means of understanding the energy exchanges that occur in the pitot tube.

DERIVATION OF A PITOT TUBE EQUATION

Equation 1 consists of:

$$\left(p_S + \frac{1}{2}\rho_F v_F^2 + \rho_F g h_S\right)_{\text{static}}$$

$$= \left(p_D + \frac{1}{2}\rho_F v_D^2 + \rho_F g h_D\right)_{\text{dynamic}} \qquad (1)$$

where

p_S = static pressure.
ρ_F = density of fluid.
v_F = velocity of fluid.
g = acceleration due to gravity.
p_D = dynamic pressure.
v_D = dynamic velocity.
h_S = elevation of static point.
h_D = elevation of dynamic point.

At the stagnation point, velocity of the fluid V_D is zero. Elevations of the static and dynamic points are essentially equal, so the two terms cancel in Equation 1. When these conditions are applied to Equation 1, they yield the results shown in Equation 2.

$$p_S + \frac{1}{2}\rho_F v_F^2 = p_D$$

or

$$p_D - p_S = \frac{1}{2}\rho_F v_F^2 \qquad (2)$$

An examination of Equation 2 shows that the kinetic energy of the fluid impacting the central tube is absorbed and made evident in the form of a pressure differential.

Bernoulli's equation also can be applied to the manometric liquid, which is subject to the same pressure differential.

If h is difference in the height of the liquid in the manometric arm, and ρ_L is density of the manometric liquid, Bernoulli's equation would reduce to the familiar form of Equation 3.

$$p_D - p_S = \rho_L g h \qquad (3)$$

But Equation 2 shows that $p_D - p_S = 1/2\ \rho_F v_F^2$. Thus, because the left-hand members of Equations 2 and 3 are identical, the right-hand members of each are also identical. The equating of right-hand members leads to Equation 4.

$$\frac{1}{2}\, \rho_F v_D^2 = \rho_L g h \qquad (4)$$

The significant feature of Equation 4 is that the left-hand member is the kinetic energy loss by the fluid stream in the central tube and the right-hand member represents increase in gravitational potential energy stored in the manometer liquid. When Equation 4 is solved for velocity of fluid (v_D), Equation 5 results.

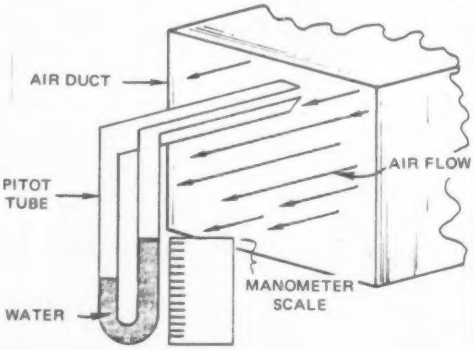
$$v_D = \sqrt{2\ g\ \left(\frac{\rho_L}{\rho_F}\right)\ h} \qquad (5)$$

Equation 5 is used in the laboratory experiment in this module and is referred to as the pitot-tube equation. Also, Example A is provided to show how this equation is applied to an air-velocity problem.

EXAMPLE A: AIR VELOCITY MEASUREMENT

Given: A pitot tube is used to measure the velocity of dry air at a point in a large duct. The manometer is filled with water, and the differential height read from the manometer scale is 1 centimeter.

Find: Air velocity in m/s.



Solution:

$$v_D = \sqrt{2\ g\ (\frac{\rho_L}{\rho_F})\ h}$$

Density of water (ρ_L): 1 g/cm³.

Density of air (ρ_F): 0.00129 g/cm³ (dry air).

Therefore,

$$v_D = \sqrt{2\ (980\ \frac{cm}{s^2})\ (\frac{1\ g/cm^3}{0.00129\ g/cm^3})\ (1\ cm)}$$

$$= \left(1233\ \frac{cm}{s}\right)\left(\frac{10^{-2}\ m}{1\ cm}\right)$$

$$v_D = 12.33\ m/s$$

PRACTICAL CONSIDERATIONS FOR MEASUREMENTS

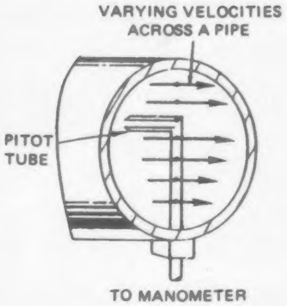


FIG. 3

Pitot tubes range from 1/8 inch to 1 inch in diameter. The main disadvantage is that the tube can be used to measure velocities only at one point at a time in the cross section of a pipe or duct, as shown in Figure 3. As a fixed instrument, the pitot tube would not provide an accurate measure of flow through the pipe. If a velocity distribution is desired, however, the pitot tube may be positioned at different radial locations across the pipe; and the data obtained may be used to produce a velocity profile for the fluid flow. In industry, the pitot tube is used primarily as an exploratory device and is rarely implanted permanently in a piping system. It also becomes clogged very easily with foreign matter in the fluid flow. No appreciable pressure loss is caused by insertion of a pitot tube into a fluid stream. It can be inserted into a small hole through the pipe or duct, without requiring much system modification.

Pressure drops created in the pitot tube are small and require sensitive differential-pressure meters or manometers. Frequently, a slanted manometer is used to increase sensitivity. Water is a light, inexpensive manometer fluid when air velocity measurements are made. Colored alcohol also can be used.

Precautions should be taken to ensure that the axis of the pitot tube always is aligned in the direction of the fluid flow. A component of the fluid stream striking on the static pressure ports will cause pressure inaccuracies to occur; however, the misalignment angle would have to be greater than 10 or 15 degrees to produce significant inaccuracies.

AVIATION APPLICATIONS

The traditional aviation pitot head shown in Figure 4 is a design variation of the basic pitot tube. Dynamic pressure is picked up through port A and is transferred through tube D to the cockpit Airspeed Indicator and to the Mach Number sensing units. Baffle plate B reduces water intake, and water trap C stops any remaining water and exits it through drain holes E and F.

Static pressure is picked up through holes G and H, and is carried through tube I to appropriate instruments. Electrical connection L provides current to electric heaters J and K to prevent ice formation. Many modern aircraft are equipped with pitot heads that pick up only the dynamic pressure. Static pressure is obtained through ports on the fuselage.

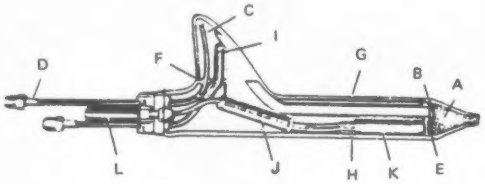


FIG. 4

True airspeed is determined only when these factors are added to the dynamic pressure: (1) the temperature of air in which aircraft is flying and (2) the pressure altitude at which the plane is flying above sea level. The all-important true air speed is calculated by use of Equation 6.

$$v_t = v_i \frac{T_h + kp_s}{T_s + kp_h} \qquad (6)$$

where

v_t = true airspeed.
v_i = indicated airspeed.
T_s = standard temperature (59°F or 15°C).
p_s = standard pressure (29.92 in. Hg or 760 mm Hg).
T_h = air temperature at flight altitude.
p_h = air pressure at flight altitude, in. or mm Hg.
k = constant value for changing temperatures to absolute values (459.4 for °F and 273 for °C).

SUMMARY

Fluid striking a pitot tube possesses kinetic energy that causes a measurable pressure differential. The tube transfers this pressurized fluid to a measuring device (manometer or other instrument), which, in turn, indicates the velocity either of the fluid flow or of the object (such as an aircraft) passing through the fluid.

DETERMINATION OF AIR SPEED WITH A PITOT TUBE

MATERIALS

Pitot tube differential gage with manometer and scale (available from Central Scientific Company).
Automobile with speedometer.
Meter stick.

PROCEDURE

1. Convert all velocities in column 1 of Data Table to centimeters per second; record in column 2 of Data Table.

2. Solve Equation 5 for h. Using this equation, calculate predicted manometer difference for each velocity indicated in column 1. Record results of calculations in column 3.

3. An automobile is required. Team with another class member; designate one as driver and other as technician. Each team member has certain duties to perform, as shown in Figure 5. Driver should be concerned with safe driving practices and occasional glances at car speedometer during experiment. Technician is responsible for positioning pitot tube some 2 or 3 ft from car, out of open window on passenger side. Technician must also align pitot tube in car's forward direction and maintain this alignment throughout experiment. Technician must read and record manometer differential heights at various speeds.

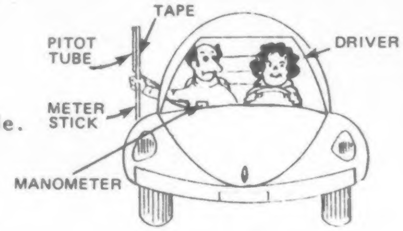


FIG. 5

4. Auto driver should choose a fairly straight and safe stretch of road and hold a constant speed of 10 miles per hour. When auto is moving at 10 miles per hour, driver signals technician, who reads measured manometer difference (h_M) and records both speedometer reading and h_M in Data Table.

5. Team should repeat step 4 for 30 and 50 miles per hour.

6. If possible, repeat steps 4 and 5 moving in opposite direction; record in Data Table.

7. Calculate measured air speed for each speedometer reading, using Equation 5; record in Data Table.

8. Compare measured air speed with speedometer readings.

9. Note weather conditions; record significant conditions causing error in air-speed calculations.

10. Explain the operation of the pitot tube used in this experiment.

DATA TABLE

Velocity Of Air Stream (mi/h) (Speedometer Reading)	Velocity Of Air Stream (cm/s)	Predicted Manometer Difference (h_p) (cm)	Measured Manometer Difference (h_M) (cm)	Measured Air Speed (mi/h)
10				
30				
50				

SELF CHECK

1. Describe, with the aid of diagrams, the construction and operation of a pitot tube and a manometer suitable for use with the pitot tube.

2. List merits and limitations of the industrial type of pitot tube.

3. A pitot tube used to measure air velocity in a wind tunnel is connected to a manometer that indicates a differential head of 0.20 in of water. Compute air velocity in ft/s if the density of air is 0.075 lb/ft^3.

4. To calibrate a pitot tube, compute the theoretically-expected air velocity for each head recorded.

head (h) - inches of water	3.04	6.92	9.63	14.00
velocity (v) - ft/s				

Why would this theoretical velocity not be achieved in actual practical measurements with this pitot tube?

5. The velocity of dry air is measured by a pitot tube to be 20 m/s when the differential height in the manometer is 2 cm. What is the velocity of the air when the height differential is 7 cm?

6. What is the density of the liquid used in the pitot tube of problem 5?

7. The airspeed indicated by an aviation pitot tube is 750 m/s. At flight altitude the air temperature is 8°C and the air pressure is 600 mm Hg. What is the true air speed?

7F5

GAS LAWS

Gases possess kinetic energy in the form of moving molecules that collide with one another and with objects in which they are contained, causing pressure that increases as the temperature increases. Many devices, such as the internal combustion engine, involve expansion or compression of gases. This module presents instructions that illustrate the solution of many gas problems, although chemical reaction and departure from ideal gas behavior are not included. In the laboratory, the student will measure, calculate, and plot gas pressure and temperature.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Explain the relationship between kinetic energy and molecular motion in a gas.
- Define the General Gas Law.
- Describe these three gas laws:

    Boyle's Law.
    Charles' Law.
    Gay-Lussac's Law.

- Define the concept of an ideal gas.

- Given the initial pressure and temperature of a given volume of air, calculate the pressure when the temperature increases to a given point.
- Given the appropriate equipment, measure, and plot gas pressure versus volume at a fixed temperature.
- Given the appropriate equipment, measure and plot gas pressure versus temperature for a fixed volume of gas.

DISCUSSION

All gases possess kinetic energy because of the motion of their molecules. Molecules of a solid vibrate and rotate about fixed positions; those of a liquid slide over and collide with one another and with the walls of the containers holding them. Gas molecules, however, are farther apart and move at greater speeds than those in a solid or a liquid. A gas is, therefore, greatly compressible, in spite of the constant molecular motion.

Such molecular motion within a gas is illustrated by Figure 1, where molecules are shown striking the container and its movable, air-tight top with such force that a weight upon the top is suspended, or balanced, there. The combined force of the top and the weight are compressing the gas until balanced by the upward force exerted by the gas.

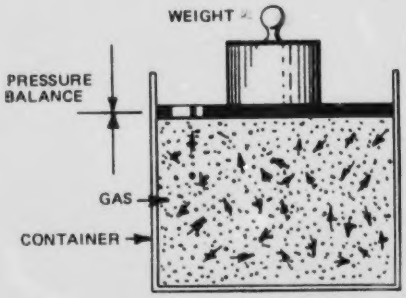


FIG. 1

Pressure is a force per unit area, and the force to contain gas is the force necessary to "reflect" gas atoms or molecules from the walls of the container and to reverse their momentum. Momentum is the product of the mass and velocity of the atom ($p = mv$), and velocity is a function of the temperature of the gas. A higher temperature causes a higher molecular velocity. The temperature of a gas is proportional to the average kinetic energy ($\frac{1}{2}mv^2$) of its molecules; thus, T is proportional to v^2. Consequently, when the temperature of a gas is raised, the pressure increases if the volume does not change, as shown by the tire in Figure 2. Two views of the same tire are shown: In Figure 2a the tire is cold and almost flat, while in Figure 2b the same tire, containing the same air, is fully inflated. An increase in temperature caused increased molecular speed and air pressure. An automobile actually rides upon the pressure created by millions of molecules within the tires (containers).

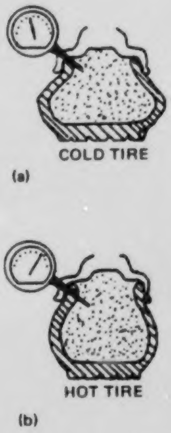


FIG. 2

THE GENERAL GAS LAW

The General Gas Law relates absolute pressure, volumes, and absolute temperatures of a fixed mass of gas, as stated in Equation 1.

$$\frac{p_1V}{T_1} = \frac{p_2V_2}{T_2} = \frac{p_3V_3}{T_3} = C \qquad (1)$$

where

p_1 = pressure, initial.
V_1 = volume, initial.
T_1 = absolute temperature, initial.
C = a constant.

subscripts 2 and 3 = later states.

For any condition, therefore, Equation 1 can be written as Equation 2.

$$\frac{pV}{T} = C \qquad (2)$$

Equation 2 assumes that there is no change in the amount of gas and no change of state (such as condensation).

THREE SPECIAL FORMS OF THE GENERAL LAW

Boyle's Law, Charles' Law, and Gay-Lussac's Law are special applications of the general gas law.

BOYLE'S LAW

Boyle's Law is designed for a constant temperature process; it is indicated by Equation 3.

$$pV = C \qquad (3)$$

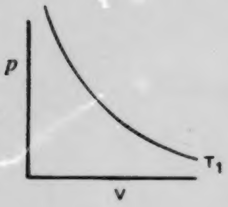


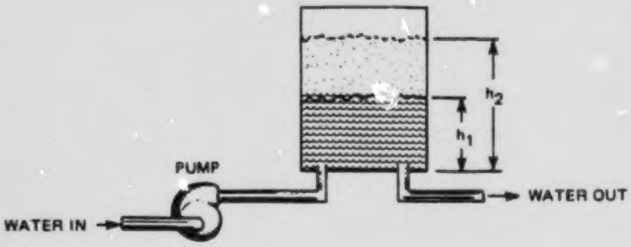
FIG. 3

As displayed in Figure 3, decreases in volume are accompanied by an increase in pressure, or a decrease in pressure results in an increase in volume, when the temperature is held constant. The relationship between pressure and volume first was stated as a law in 1662 by Robert Boyle, after whom it was named.

Equation 3 is utilized in solving the problem in Example A.

EXAMPLE A: SURGE TANK PRESSURE AND BOYLE'S LAW	
Given:	An 85-gallon surge tank is installed in a water system, as illustrated in the drawing below. The tank is 63" high and 20" in diameter. Pressure switches in the surge tank start the pump at a pressure of 20-psig (gage) and stop the pump when the pressure reaches

40-psig. The space above the water (called "ullage") is filled with air. The temperature of the air remains essentially constant during this process. The water is 32" high when the pressure is 20 psig.



Find: Water height when the pressure is 40 psig.

Solution: Symbols used in calculations:

psia = pounds per sq in absolute.
V_T = total tank volume.
V_L = volume of water at h_1.
V_H = volume of water at h_2.
V_1 = volume of air when water volume is V_L.
V_2 = volume of air when water volume is V_H.

$$V_T = \frac{\pi}{4} d^2 h_T = \frac{\pi}{4} (20 \text{ in})^2 (63 \text{ in})$$

$$V_T = 19792 \text{ in}^3$$

$$V_L = \frac{\pi}{4} d^2 h_1$$

$$V_L = \frac{\pi}{4} (20 \text{ in})^2 (32 \text{ in})$$

$$V_L = 10053 \text{ in}^3$$

$$V_1 = V_T - V_L$$

$$V_1 = 19792 - 10053 = 9739 \text{ in}^3$$

$$p_1 = 20 \text{ psig} + 14.7 \text{ psia} = 34.7 \text{ psia}$$

$$p_2 = 40 \text{ psig} + 14.7 \text{ psia} = 54.7 \text{ psia}$$

From Equation 3:

$$P_1 V_1 = P_2 V_2$$

or

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

Therefore,

$$V_2 = V_1 \frac{P_1}{P_2}$$

So,

$$V_2 = (9739 \text{ in}^3) \left(\frac{34.7 \text{ psia}}{54.7 \text{ psia}}\right)$$

$$V_2 = 6178 \text{ in}^3$$

And,

$$V_H = 19792 - 6178$$

$$V_H = 13614 \text{ in}^3$$

Then,

$$h_2 = \frac{4\ V_H}{\pi\ D^2}$$

$$h_2 = \frac{4\ (13614 \text{ in}^3)}{\pi\ (20 \text{ in})^2}$$

$$h_2 = 43.3 \text{ in}$$

CHARLES' LAW

Charles' Law is designed for a constant pressure process, and is stated in Equation 4.

$$\frac{V}{T} = C \qquad (4)$$

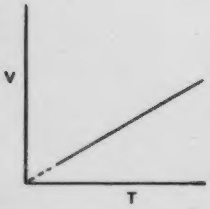


FIG. 4

Increases in temperature cause an increase in volume when the pressure is maintained at the same value. This relationship is named after its discoverer, Jacques Charles. Figure 4 indicates how pressure is plotted against volume and temperature using Charles' Law.

GAY-LUSSAC'S LAW

Gay-Lussac's Law is for a constant-volume process, as plotted in Figure 5 and as stated in mathematical terms by Equation 5.

$$\frac{p}{T} = C \qquad (5)$$

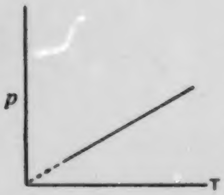


FIG. 5

Increases in temperature of a gas when the volume is held constant cause an increase in pressure.

Equation 5 is used to solve a problem in Example B.

EXAMPLE B: TIRE PRESSURE AND GAY-LUSSAC'S LAW

Given: An automobile tire with a volume of 1500 in^3 is pressurized to 25 psig at a temperature of 70°F.

Find: Pressure in the tire after the car has been driven several hours and air temperature in the tire has increased to 170°F.

Solution: p_1 = 25 psig + 14.7 psia = 39.7 psia
T_1 = 70°F + 460 = 530°R (converting to an absolute
T_2 = 170°F + 460 = 630°R temperature scale)

Assume that the volume of the tire remains essentially constant, Equation 5 is used as follows:

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

or

$$p_2 = \frac{P_1 T_2}{T_1}$$
$$= \frac{(39.7\text{ psia})(630°\text{R})}{(530°\text{R})}$$
$$p_2 = 47.2\text{ psia}$$

or

$$p_2 = 47.2\text{ psia} - 14.7\text{ psia}$$
$$p_2 = 32.5\text{ psig}$$

THE IDEAL LAW

The concept of an ideal, imaginary gas is useful because it permits its behavior to be predicted. Behavior of an actual gas is predictable from ideal gas laws also, to the extent that the gas conforms to the requirements of the ideal gas. At temperatures much higher than the boiling temperature, and at normal or low pressures, most real gases come very close to satisfying the conditions of an ideal gas. For most gases, these conditions are satisfied at 70°F and at normal atmospheric pressure. However, steam must be "superheated" to several hundred degrees before its behavior approximates that of an ideal gas.

Thomas Andrews investigated the pressure-volume relationships of CO_2 at Queen's College, Belfast, between 1861 and 1869. He observed, as indicated in Figure 6, that as he lowered the temperature, the smooth curves began developing irregularities that eventually led to a change in volume without a change in pressure. Andrews accurately concluded that the gas actually was changing to a liquid under these conditions. The highest temperature at which the gas can liquefy is the "critical temperature"; the pressure at the critical point (Figure 6) is the "critical pressure."

The "critical point" indicates where to expect the failure

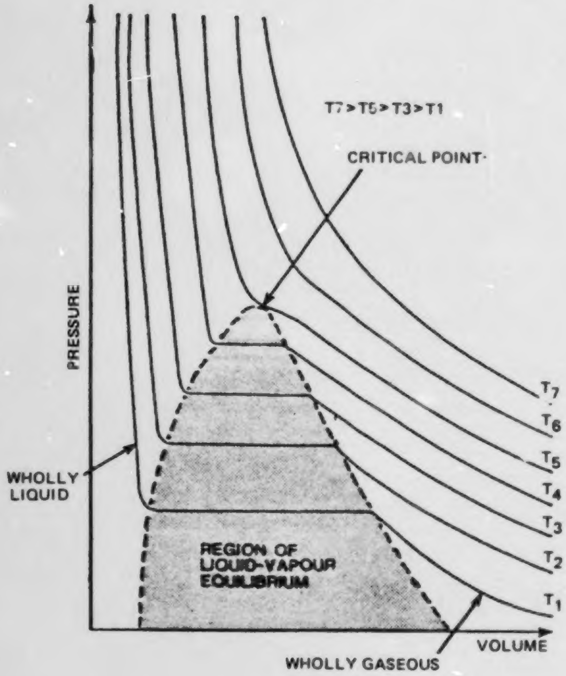


FIG. 6

of the ideal gas law. There is considerable deviation from the ideal gas law for temperatures and pressures near the critical point. Several more sophisticated equations have been developed to represent the behavior of a gas that does not obey the ideal gas law. These equations are empirical and only can approximate the behavior of gases under conditions far removed from their liquefaction points. Such equations are beyond the scope of this module.

Caution must be exercised in the application of the laws expressed in Equations 1 through 5 to assure that the required conditions are not violated. For example, since compression is so rapid in an internal combustion engine, there is no opportunity for appreciable heat transfer. As a result, the temperature of gas in the cylinder is not constant. Thermal processes of this type are called "adiabatic processes." Equation 3, designed for use with constant temperature, is not applicable. In such a case, gas temperature is unknown and is not computed easily from first principles. This problem can be handled by use of Equation 6, which allows for such rapid changes.

$$PV^k = C \qquad (6)$$

where

$k = 1.4$ for air.

Equation 6 and Equation 3 (Boyle's Law) are compared and used to solve a problem in Example C.

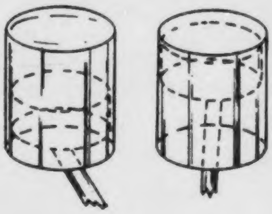
EXAMPLE C: INTERNAL COMBUSTION ENGINE PRESSURE

Given: An eight-cylinder, internal-combustion engine has a compression ratio of 10 and a displacement of 350 in^3. Assume that the cylinder is filled with air at a pressure of 0 psig (atmospheric pressure) and that no ignition occurs.

Find: Cylinder pressure when the piston is at top dead center, (a) assuming a constant temperature process (Equation 3) and (b) assuming no heat loss (Equation 6).

Solution: Since the engine has a compression ratio of 10, then $\frac{V_2}{V_1} = \frac{1}{10}$; that is, the gas volume after compression is $\frac{1}{10}$ the initial gas volume, assuming that no ignition has occurred.

(a) From Equation 3,



$$p_1V_1 = p_2V_2$$

or,

$$\frac{p_1}{p_2} = \frac{V_2}{V_1}$$

Therefore,

$$\frac{p_1}{p_2} = \frac{1}{10}$$

And,

$$p_2 = 10\ p_1$$

Since p_2 = 0 psig + 14.7 psia = 14.7 psia, then

$$p_2 = 10\ (14.7\ \text{psia}) = 147\ \text{psia}$$

or

$$p_2 = 147\ \text{psia} - 14.7\ \text{psia} = 132.3\ \text{psig}$$

(b) From Equation 6,

$$p_1V_1{}^k = p_2V_2{}^k$$

or

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^k$$

Therefore,

$$\frac{p_1}{p_2} = \left(\frac{1}{10}\right)^{1.4}$$

$$\frac{p_1}{p_2} = 0.0398$$

Then,

$$p_2 = \frac{p_1}{0.0398} = \frac{14.7\text{ psia}}{0.0398}$$

$$p_2 = 369.3\text{ psia}$$

or

$$p_2 = 369.3\text{ psia} - 14.7\text{ psia}$$

$$p_2 = 354.6\text{ psia}$$

A comparison of the two calculated pressures (132.3 psig and 354.6 psig) reveals a considerable difference, which could be an important factor, depending upon how these results are used. If the initial temperature (T_1) and the temperature following compression (T_2) had been measured, then Equation 1 could have been used.

DEMONSTRATION OF BOYLE'S LAW -PV-C

MATERIALS

Bicycle pump.
Manometer.

PROCEDURE

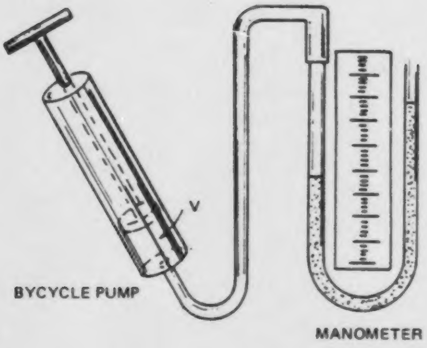


FIG. 7

1. Disassemble bicycle pump, and measure inside diameter and stroke.
2. Compute and plot volume of the cylinder as a function of the displacement of pump handle.
3. Reassemble pump.
4. Connect manometer to bicycle pump, in Figure 7.
5. Slowly push in on handle of pump, stopping at four measured distances. Measure pressure at each step. Record distances and pressure at each step in Data Table 1. CAUTION: Do not pull back on pump handle while manometer is connected.
6. Plot measured pressure points as a function of volume. Use graph in Data Table 1.
7. Why must pressure at each distance in Step 5 be measured at the same temperature?
8. Does the graph plotted in Data Table 1 agree with that effected from the equation $pV = C$?

DEMONSTRATION OF GAY-LUSSAC'S LAW - $P = CT$

MATERIALS

Bunsen burner or hot plate.
Insulated metal container having two holes in top.
Thermometer.
Two corks to fit holes in container.
Manometer.
Stand to hold container above burner.
Small-diameter flexible tubing and rigid tubing.

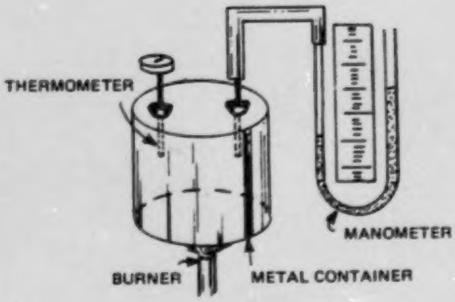


FIG. 8

PROCEDURE

1. Push thermometer through a cork, and insert cork into metal container. (Refer to Figure 8.)

2. Insert a piece of small-diameter, rigid tubing into a cork; install cork in metal container.

3. Place metal container on a stand over Bunsen burner.

4. Connect manometer to tubing in cork in metal container, using flexible tubing.

5. Light burner, and heat air inside container.

6. Measure pressure and temperature of gas, twice as it increases in temperature and twice as it cools. Record in Data Table 2.

7. Plot measured temperature as a function of pressure. Use graph in Data Table 2.

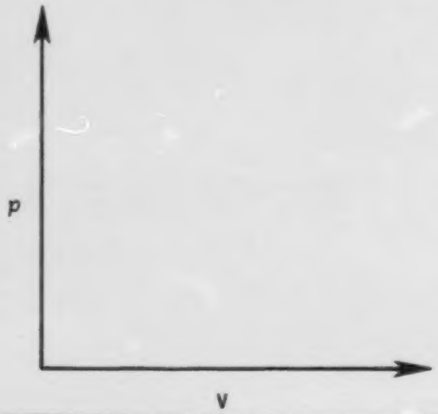
8. Why must the volume of gas in the container remain unchanged during these measurements?

9. Does the graph plotted in Data Table 2 agree with that predicted by the equation $\frac{P}{T} = C$?

DATA TABLE 1

	Cylinder Distance (Measured)	Volume (Calculated)	Pressure (Measured)
#1	________ in	________ in^3	________ psia
#2	________ in	________ in^3	________ psia
#3	________ in	________ in^3	________ psia
#4	________ in	________ in^3	________ psia

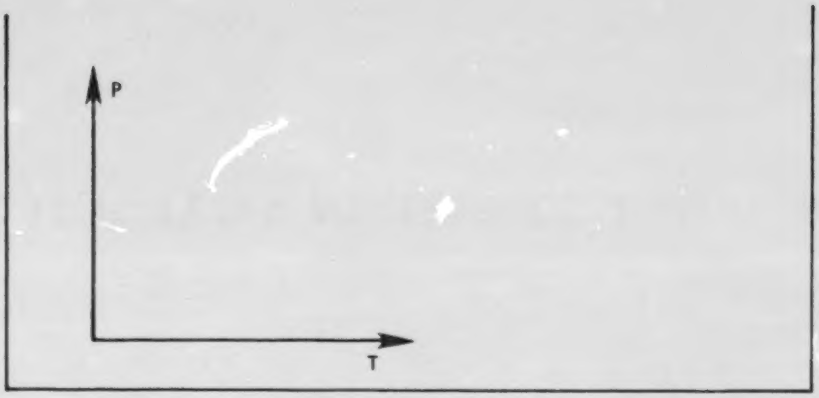
Plot p versus V for four points given above.



DATA TABLE 2

	Pressure	Temperature
(Increasing gas temperature)		
1st reading →	______ psia	________ °R
2nd reading →	______ psia	________ °R
(Decreasing gas temperature)		
3rd reading →	______ psia	________ °R
4th reading →	______ psia	________ °R

Plot four pair of values on plot shown. Use appropriate scales.



SELF CHECK

1. Explain the relationship between kinetic energy and molecular motion in a gas.

2. State the three properties of a gas related by the General Gas Law.

3. Describe the following gas laws — use words and equations. State what "variable" is held constant in each law.

   Boyle's Law.
   Charles' Law.
   Gay-Lussac's Law.

4. Why is the concept of an "ideal gas" important in studying the behavior of gases?

5. The tires on an automobile are pressurized to 28 psig at a temperature of -30°F. The car is driven across the country to a warmer climate, where the temperature is 110°F. Compute the air pressure in the tires, assuming that the volume of the tires remains essentially constant at 2000 in^3.

7E2

ENERGY STORED IN A CAPACITOR

A capacitor is a device consisting of two closely-spaced conductors separated by a layer of insulating material, used to store electrical potential energy. This energy-storage capability is employed in many electronic circuits, including pulsed power supplies for lasers and other high-intensity light sources.

This module investigates the energy stored in a capacitor. In the laboratory, the student will measure such stored energy.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate energy stored in a capacitor, given a known capacitance and voltage.
- Calculate voltage necessary to store a given energy in a capacitor of known capacitance.
- Determine the charge on each plate of a capacitor, given the size of the capacitor and the energy stored by it.
- Given an oscilloscope and other appropriate equipment, measure the energy stored in a capacitor.

DISCUSSION

A capacitor is a device used to store electrical potential energy. It usually consists of two large plates of conducting material separated by a layer of insulating material (dielectric). In one type of capacitor, the plates are made of metal foil, and the insulating layer is paper or plastic, allowing for the folding or rolling of the capacitor to attain a large plate area into a small package (Figure 1).

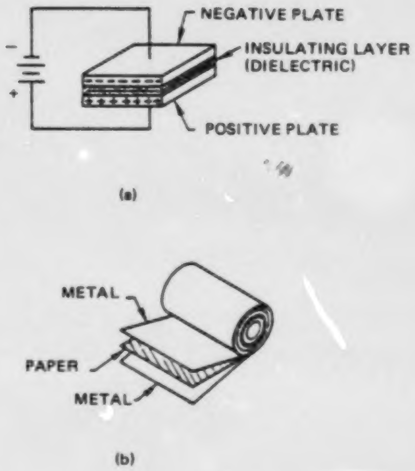

NEGATIVE PLATE
INSULATING LAYER (DIELECTRIC)
POSITIVE PLATE
-
+
(a)
METAL
PAPER
METAL
(b)

FIG. 1

Capacitance, as defined by Equation 1, is a measure of the ability of a capacitor to store charge.

$$C = \frac{Q}{V} \qquad (1)$$

where

C = capacitance.
Q = charge stored.
V = voltage (potential difference).

The unit of capacitance is the farad, equal to a coulomb of charge per volt of potential difference. Because a coulomb of charge is extraordinarily large, the farad is too large a unit of capacitance for practical applications. Capacitors are rated more frequently in microfarads (μF), where one microfarad equals 10^{-6} farads ($1\ \mu f = 10^{-6}$ F).

Figure 2 illustrates a capacitor in a simple charging circuit.

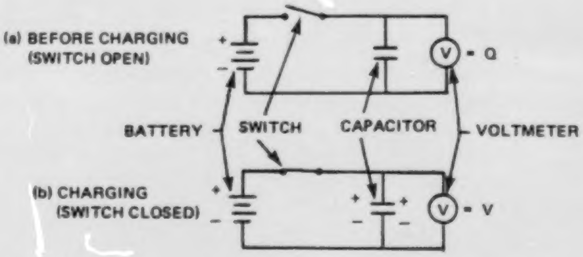

(a) BEFORE CHARGING (SWITCH OPEN)
(b) CHARGING (SWITCH CLOSED)
BATTERY
SWITCH
CAPACITOR
VOLTMETER
V = 0
V = V
+
-

FIG. 2

Assume that the capacitor is uncharged initially and that the switch is open (a). When the switch is closed (b), a current will flow to charge the capacitor. As more and more charge builds up on the capacitor, the transfer of additional charge becomes more and more difficult. Movement of charge stops when potential difference across the capacitor equals the potential difference of the battery. Work required to move the electrical charges is supplied by the battery and stored in the capacitor as electrical potential energy.

The average potential difference through which the charge is moved is given by Equation 2.

$$V_{av} = \frac{V_{final} + V_{initial}}{2} = \frac{V + 0}{2} = \frac{1}{2} V \qquad (2)$$

where

V_{av} = average voltage.

The total work done by the battery equals the energy stored in the capacitor; therefore, the energy stored is the product of the total charge transferred and the average voltage, as indicated by Equation 3.

$$\text{Work done} = \text{energy stored} = QV_{av} = Q\left(\frac{1}{2} V\right) = \frac{1}{2} QV \qquad (3)$$

Substitution of $Q = CV$ from Equation 1 yields the frequently used form of Equation 4 for energy stored in a capacitor.

$$E_p = \frac{1}{2} CV^2 \qquad (4)$$

where

E_p = energy stored in a capacitor.

When C is in farads and V in volts, E_p will be in joules.

Examples A and B demonstrate the use of Equation 4 in solving a problem.

EXAMPLE A: ENERGY STORED IN A CAPACITOR

Given: A 10-μF capacitor is charged with a voltage of 600 V.

Find: Amount of energy stored in the capacitor.

Solution:

$$E_p = \frac{1}{2} CV^2$$

$$= \frac{1}{2} (10 \times 10^{-6}\ F)(600\ V)^2 \qquad (\text{Recall: } 1\ \mu F = 10^{-6}\ F.)$$

$$= (5 \times 10^{-6}\ F)(3.6 \times 10^5\ V^2)$$

$$= 1.8\ FV^2 = 1.8 \left(\frac{\text{coulomb}}{\cancel{\text{volt}}}\right)(\text{volt})^{\cancel{2}}$$

(Note: A volt is a joule/coulomb.)

$$= 1.8 (\cancel{\text{coulomb}})\left(\frac{\text{joule}}{\cancel{\text{coulomb}}}\right) =$$

$$E_p = 1.8 \text{ joule}$$

EXAMPLE 8: VOLTAGE TO CHARGE A CAPACITOR

Given: 100 joules of energy must be stored in a 25-μF capacitor.

Find: The necessary voltage.

Solution:

$$E_p = \frac{1}{2} CV^2$$

$$V^2 = \frac{2\,E_p}{C}$$

$$V = \sqrt{\frac{2\,E_p}{C}}$$

$$= \sqrt{\frac{2(100\text{ J})}{25 \times 10^{-6}\text{ F}}}$$

$$= \sqrt{8 \times 10^6 \frac{J}{F}}$$

$$= 2.83 \times 10^3 \sqrt{\frac{J}{F}}$$

$$V = 2830\text{ volts}$$

Note:

$$\sqrt{\frac{J}{F}} = \sqrt{J \times \frac{1}{F}}$$

$$= \sqrt{\text{joule} \times \left(\frac{\text{volt}}{\text{coulomb}}\right)}$$

$$= \sqrt{\left(\frac{\text{joule}}{\text{coulomb}}\right) \times \text{volt}}$$

$$= \sqrt{\text{volt}^2} = \text{volt}$$

SUMMARY

Capacitors are devices that store electrical potential energy. They consist of large plates of conducting material separated by layers of dielectric (insulating material). A popular type of capacitor is constructed of foil plates separated by paper or plastic, rolled into a convenient, small package. There are also large industrial capacitors, such as those employed on electrical power transmission equipment. Energy stored in capacitors is utilized in many circuits, such as pulsed power supplies and high-intensity light sources. They are used as "starters" on some electric motors and on fluorescent lights, where they deliver their stored energy in sudden bursts.

ENERGY STORED BY CAPACITORS

MATERIALS

Oscilloscope with a triggered sweep.
d.c. power supply (0-10 V, 50 W typical).
Three capacitors (approximately 500 μF, 800 μF and 1000 μF).
Assorted resistors (approximately 1 kΩ and 2 kΩ).
Clip leads.
Single-pole, double-throw switch.
Graph paper (10 x 10).

PROCEDURE

1. Select a resistor, R, and a capacitor, C, that will give an RC time constant (R x C) of about one second. Note: (ohm)(farad) = $\left(\frac{\cancel{\text{volt}} \times \text{second}}{\cancel{\text{coulomb}}}\right)\left(\frac{\cancel{\text{coulomb}}}{\cancel{\text{volt}}}\right)$ = second.
2. Connect equipment as shown in Figure 3.
3. Set oscilloscope controls in such a manner that an input of zero volts will result in a trace at lower edge of scope face and that d.c. voltage of power supply will give a trace near upper edge. Set trigger mode of oscilloscope to "line" and to "automatic." Start with time setting of 1 s/cm. This value may be changed later if necessary.
4. Throw switch to position A, as shown in Figure 3; observe scope. In five seconds (if RC was chosen to equal one second),

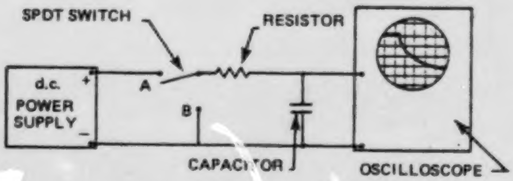


FIG. 3

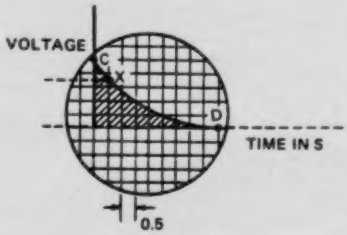


FIG. 4

capacitor should be almost fully charged. Throw switch to position B. Dot on scope should trace out a curve similar to that in Figure 4.

5. Point C in Figure 4 represents voltage across capacitor at the instant that switch is thrown to B position. This point is the beginning of capacitor discharge. Point D represents zero voltage across capacitor and is the point at which all charge has left the capacitor plates (about 5 seconds).

6. On the oscilloscope face, the horizontal portion of each square represents a certain time; and the vertical portion represents a certain voltage. These dimensions are indicated by the setting of oscilloscope knobs. For example, a knob setting of 0.5 s/cm means that horizontally each square represents 0.5 seconds. A knob setting of 2 volt/cm means that vertically each square represents 2 volts.

7. Graphing the oscilloscope trace.
   Throw switch to position A; after about 5 seconds, throw switch back to position B. Notice how many blocks the dot drops vertically as it moves one block horizontally. For example, if it drops one and a half blocks vertically, as in Figure 4, the point is 3-1/2 blocks above baseline; therefore, at 0.5 seconds, voltage across the capacitor is 7 volts (for a knob setting of 2 volt/cm). Enter voltage and time in Data Table 1. Also make a graph of voltage versus time. Repeat this procedure until enough points are chosen to make a good, smooth curve resembling Figure 4.

   When horizontal axis is calibrated in time and vertical axis is calibrated in power, total energy stored in the capacitor and later dissipated by the resistor is represented by area under the trace curve (shaded area in Figure 4). To calibrate vertical axis as power, remember that power can be calculated from this equation:

$$P = \frac{V^2}{R}$$

   where
   P = power.
   V = voltage.
   R = resistance.

   Power level at point C is determined from this equation if V represents the full voltage supplied.

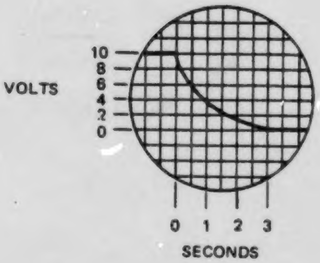
8. Make several trials; determine approximate energy stored in capacitor by estimating area under the trace curve. (Refer to Example C.) Record results in Data Table 2.

9. Repeat process with two other capacitors.
10. Calculate energy stored in each capacitor; use $E_p = \frac{1}{2} CV^2$.
11. Compare calculated results with estimated energy stored.
12. Discuss the importance of capacitors as energy-storage devices.

EXAMPLE C: ESTIMATING ENERGY STORED IN A CAPACITOR

Given: Oscilloscope trace similar to figure on right.

Find: Energy stored in capacitor.



Solution: Assume the following values:

$C = 500\ \mu F$
$R = 2k\Omega$
$V = 10\ V$

Note:
$RC = 1\ s$

Consider the first time interval of one second.

Instantaneous voltage at start = 10 volts.
Instantaneous voltage at end = 3.7 volts.

$$\text{Average voltage over first second interval} = \frac{10 + 3.7}{2} = 6.85 \text{ volts}$$

$$\text{Average power over first second interval} = \frac{(6.85)^2}{2000} = 0.0235 \text{ watts}$$

Decrease in potential energy of capacitor over first second interval = $\overline{P}\Delta t$ = (0.0235 joules/s x 1 s) = 0.0235 joules.

Time (seconds)	Instantaneous Voltage, V (volts)	Average Voltage, $\overline{V}_{av}$ (volts)	Average Power, $\overline{P}_{av} = \frac{\overline{V}^2}{R}$ (watts)	Energy, $\overline{P}\Delta t$ (joules)
0	10	--		
1	3.7	6.85	0.0235	0.0235
2	1.4	2.55	0.0033	0.0033
3	0.5	0.95	0.0005	0.0005

Therefore, total energy stored in the capacitor can be estimated to be 0.0235 + 0.0033 + 0.0005 joules = 0.027 J = 27×10^{-3} J.

Use assumed values in Equation 4:

$$E_p = \frac{1}{2} CV^2$$

$$= \frac{1}{2} (0.5 \times 10^{-3}\ F)(100\ V^2)$$

$$E_p = 25 \times 10^{-3}\ J$$

This solution compares favorably with the estimated value.

DATA TABLE 1

Voltage	Time

DATA TABLE 2

Capacitance	Resistance	Voltage	Estimated Energy (Area under Curve, Example C)	Calculated Energy ($E_p = \frac{1}{2}CV^2$)	% Difference
500 µF					
800 µF					
1000 µF					

SELF CHECK

1. A 2000-μF capacitor is charged to 240 V. How much energy is stored?

2. A 375-μF capacitor is charged to 4000 V. How much energy is stored?

3. A 400-μF capacitor is to be used to store one kilojoule. What will the voltage be?

4. A 0.25-μF capacitor is to be used to store 72 J. What will the voltage be?

5. 200 J is stored in a 60-μF capacitor. What is charge on each plate of the capacitor?

7E3

ENERGY STORED IN AN INDUCTOR

Energy supplied to an electric circuit is dissipated by its resistance and is stored in its electric and magnetic fields. Electrical potential energy is stored in the electric field between the conductors of a capacitor. Magnetic potential energy is stored in the magnetic field of an inductor.

The concept of inductance is the subject of this module. Air-core and iron-core inductors are contrasted. The reaction of the inductor to changing currents is discussed in terms of energy stored in its magnetic field. In the laboratory, the student will measure the current through an inductor and will calculate the energy stored in its magnetic field.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Define "inductance."
- Calculate the inductance and magnetic-field strength of an air-core inductor.
- State the difference in inductance and magnetic field strength of a coil with and without an iron core.
- Describe how a variable inductor works.
- Calculate the energy stored in the magnetic field of an inductor, given the inductance and current.
- Sketch graphs of voltage, current, and energy stored as functions of time in an inductor through which the current is increasing and decreasing.

- Sketch a schematic of the ignition system of an automobile and describe how it produces a spark; include the functions of the coil, breaker points, condenser, and rotor.

- Given the appropriate equipment, measure the current through an inductor of known inductance; and calculate the energy stored in its magnetic field.

DISCUSSION

A varying magnetic field induces an electromotive force, or emf, in a conducting loop within the field (Figure 1). The emf is in such a direction as to oppose, by the magnetic action of a corresponding induced current, the changes in the magnetic field linking the loop (Lenz's Law); however, the induced emf occurs whether or not there is current in the loop. If the magnetic field is itself due to a current in the loop, then the induced emf will act in such a way as to resist changes in the magnetic field or changes in the loop current responsible for the magnetic-field changes. A current loop, thus, tends to preserve the status-quo through a kind of built-in inertia. This resistance to changing current is the inductance of the loop.

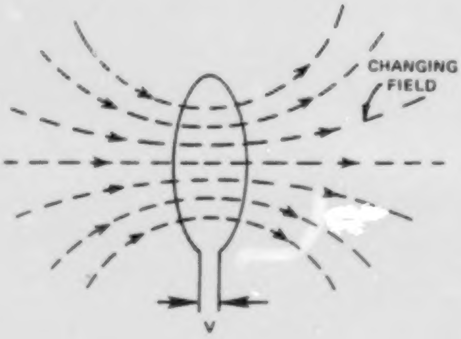


FIG. 1

If the loop is a closed circuit, an induced current accompanies the induced emf. It circulates in such a manner as to oppose the changes in the applied current. When the applied current and associated magnetic field are constant, the induced emf and current disappear.

Mathematically, inductance is defined as follows by Equation 1.

$$L = \frac{-V}{(\frac{\Delta I}{\Delta t})} \qquad (1)$$

where
L = inductance in Henries.
V = induced emf.
ΔI = average change in current.
Δt = time interval in which current change occurs.

The minus sign appears because the emf V opposes the change in current ΔI. The unit of inductance in the International System of units is the Henry, abbreviated H. From Equation 1, the Henry is defined as follows:

$$1 \text{ Henry} = 1 \text{ volt}/(1 \text{ amp}/1\text{s}) = 1 \text{ (volt-s/amp)}$$

or $$1 \text{ Henry} = 1\ \Omega\text{-s (since } 1\ \Omega = 1 \text{ volt/amp)}$$

Inductance in a circuit is represented in the schematic form of a coil, as illustrated in Figure 2. The common inductor is a coil of wire. The inductance of an air-core coil is given by Equation 2.

$$L = \mu_0 N^2 \frac{A}{\ell} \qquad (2)$$

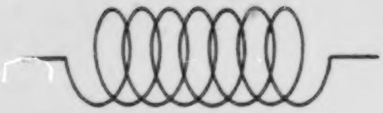


FIG. 2

where

L = inductance of the coil.
μ_0 = constant ($4\pi \times 10^{-7}$ H/m), called the "permeability of free space."
N = number of loops or turns of the coil.
A = cross-sectional area.
ℓ = length.

The mangetic field strength of an air core inductor is —

$$B = \mu_0 \frac{NI}{\ell} \qquad (3)$$

where
B = magnetic field strength, in Teslas.
I = current.

The unit of B is the Tesla, abbreviated T. The magnetic field is parallel to the axis of the coil, as indicated by Figure 3.

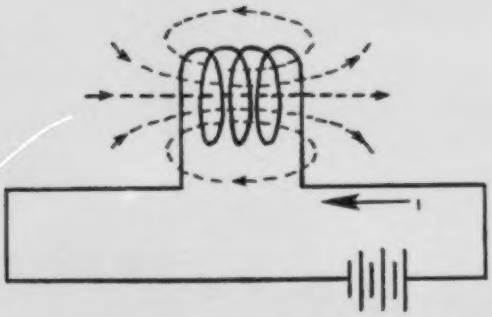


FIG. 3

The inductance and magnetic field strength of the coil are enhanced if the windings are wrapped on an iron core (Figure 4a). The inductance and magnetic field strength now are given by Equations 4 and 5.

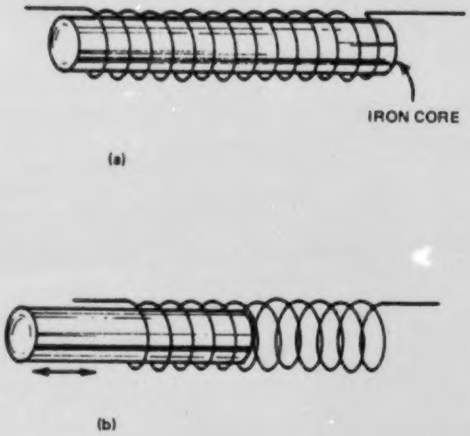
$$L = \mu_i N^2 \frac{A}{\ell} \qquad (4)$$

and $$B = \mu_i \frac{NI}{\ell} \qquad (5)$$

where
μ_i = permeability of iron.

The permeability of free space μ_0 has been replaced by the permeability of iron μ_i. The permeability of iron is not constant, but depends upon the current I through the coil and the history of the iron sample. The permeability of a material, therefore, is a measure of how easily and to what extent it can be magnetized. The permeability of iron is quite large. Once magnetized, iron will preserve a strong magnetic field, unless environmental factors destroy it. Because of this preservation, it is said to form a "permanent" magnet. However, its permeability changes as the external magnetic field changes; this phenomenon is known as "hysteresis."

A variable inductor is depicted in Figure 4b. As the iron rod slides into the coil, the effective permeability is due more to iron than to air. The inductance, therefore, increases as more of the rod slides inside the coil. Example A shows how the inductance and magnetic-field strength of an inductor are calculated.



(a)

(b)

FIG. 4

EXAMPLE A: AIR CORE INDUCTOR

Given: An air-core inductor has $N = 1000$ turns, $\ell = 2$ cm, and a radius $r = 2$ cm. The current $I = 1.0$ A.

Find:
(a) The inductance L of the coil.
(b) The magnetic-field strength B at the center of the coil.

Solution: (a)

$$A = \pi r^2$$
$$= \pi (0.02\text{ m})^2$$
$$A = 0.00125\text{ m}^2$$
$$L = \mu_0 \frac{N^2 A}{\ell}$$
$$= \frac{(4\pi \times 10^{-7}\text{ H/m})(1000)^2(.00125\text{ m}^2)}{(0.02\text{ m})}$$
$$L = 78.5\text{ mH}$$

(b)

$$B = \mu_0 \frac{NI}{\ell}$$
$$= \frac{(4\pi \times 10^{-7}\text{ H/m})(1000)(1.0\text{ A})}{(0.02\text{ m})}$$
$$B = 0.0628\text{ T}$$

ENERGY STORED IN AN INDUCTOR

As long as the current through the inductor is changing, there is an emf developed along the coil; therefore, power (recall: $P = VI$) or energy is supplied to the inductor from the circuit. If the current becomes constant, there is no induced emf and, therefore, zero power input. The energy supplied to the inductor is used to establish its magnetic field. This energy remains stored as magnetic potential energy as long as the current remains constant. If the circuit is opened, the magnetic field collapses; and the energy is returned to the circuit. The release of energy often is sufficient to create an arc across the switch that opens the circuit.

When the current in a circuit increases from zero to I, the energy supplied to the inductor can be calculated from Equation 6.

$$E = \frac{1}{2} LI^2 \qquad (6)$$

where

E = energy.

When a voltage V is applied suddenly to an inductor, as when switch S1 is closed in the circuit of Figure 5, the current in the circuit increases only gradually, because of the coil's resistance to change in the current. Thus, time is required for the energy supplied to the inductor from the battery to be stored in the magnetic field. Voltage across the inductor leads the current, but the time lapse between voltage and current typically changes only a few milliseconds. Graphs of voltage, current, and energy as functions of time are shown in figure 6.

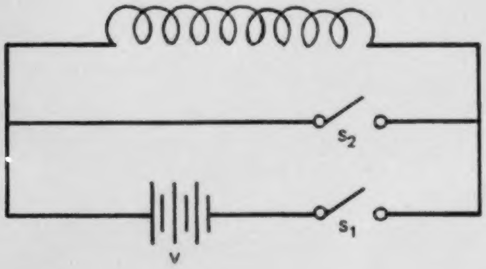


FIG. 5

Likewise, if the applied voltage drops suddenly to zero, as when switch S2 of Figure 5 is closed, time is required for the energy stored in the magnetic field of the inductor to be released to the circuit. The current decays exponentially to zero. The current change lags the voltage change. Graphs of voltage, current, and energy as functions of time during current decay are shown in Figure 7. Calculation of the energy stored in an inductor is illustrated in Example B.

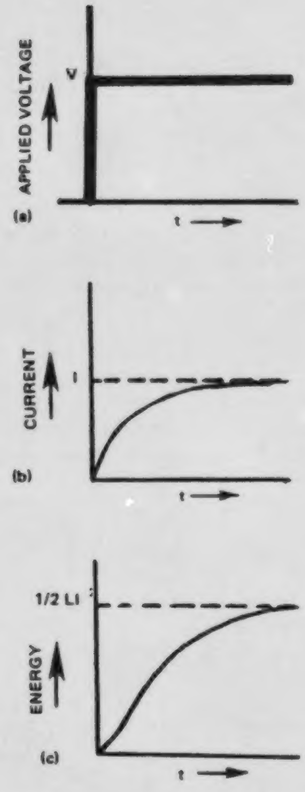


FIG. 6

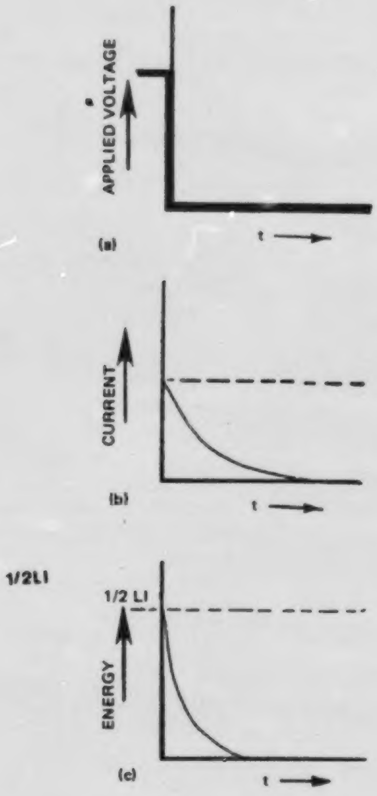


FIG. 7

EXAMPLE 8: ENERGY STORED IN AN INDUCTOR

Given: The current I through an air-core inductor of inductance L = 1.0 H is 2.0 A.

Find: The energy stored in the magnetic field of the inductor.

Solution:
$$E = \tfrac{1}{2} LI^2$$
$$= \tfrac{1}{2}(1.0\ \text{H})(2.0\ \text{A})^2$$
$$E = 2.0 \text{ joules}$$

THE IGNITION SYSTEM OF AN AUTOMOBILE

Figure 8 is a schematic diagram of an automobile ignition system. It consists of two separate circuits. The primary circuit is composed of the battery, ignition switch, primary winding of the ignition coil, breaker points, and condenser. The secondary circuit contains the secondary winding of the ignition coil, rotor, and spark plugs. The breaker points, condenser, and rotor are contained in the distributor.

The primary winding typically has several hundred turns of wire, while the secondary one has more than 10,000; therefore, the inductance of the secondary is very large compared to that of the primary. When the current through the primary changes, a large emf is induced in the secondary.

The process that results in the spark begins with the ignition switch and breaker points closed, establishing a constant current through the primary circuit. The breaker points are opened by a revolving cam. The rapidly-decreasing current in the primary induces a large emf in the secondary. The rotor is a rotary switch that connects the secondary to one of the spark plugs. The emf in the secondary is discharged across the gap in the spark plug, igniting the fuel in the cylinder. The process requires precise timing, since the breaker points are open for less than five thousandths of a second for each spark; and the spark must occur at the top of the compression cycle in the cylinder. The condenser is a capacitor that prevents sparking across the breaker points when they are open.

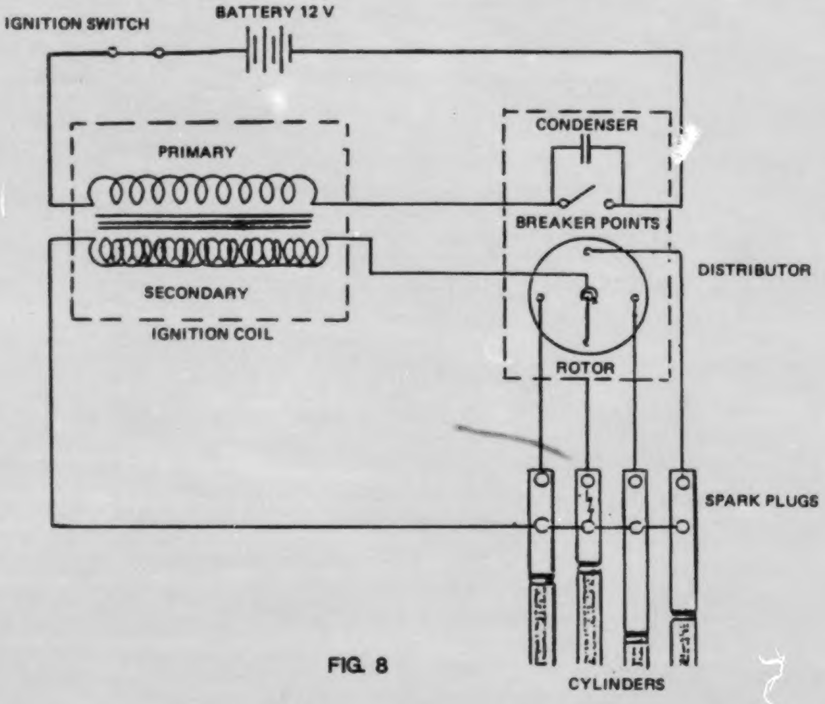


FIG. 8

SUMMARY

The inductor is a device that resists changes in the current passing through it. The energy stored in the magnetic field of an inductor is derived by the following equation:

$$E = \frac{1}{2} LI^2$$

where
E = energy.
L = inductance.
I = current.

An iron-core inductor has a larger inductance and, therefore, is capable of storing more magnetic energy than the same coil with an air core. The large induced emf of a high-inductance coil produces the spark that ignites the fuel in the cylinder of an automobile engine.

DETERMINATION OF ENERGY STORED IN AN INDUCTOR

MATERIALS

Automobile ignition coil.
d.c. power supply (0-12 V).
Switch.
2µf capacitor.
Ammeter.
Spark plug.

PROCEDURE

1. Wire the schematic shown in Figure 9. Close switch, and adjust power supply for an output voltage of 12 volts.

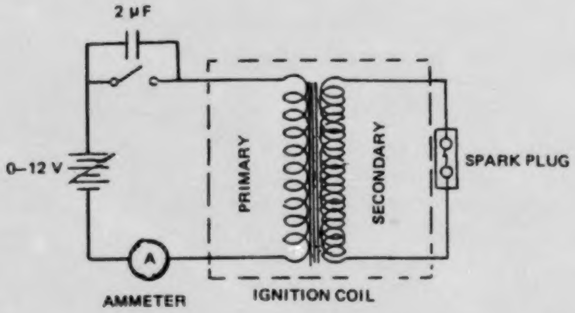


FIG. 9

2. In the Data Table, record the current registered on the ammeter.

3. Calculate energy stored in magnetic field of primary, using L = 4H and measured current; and record in results portion of Data Table.

4. The following procedure must be done with extreme caution because voltage developed across secondary is typically 10,000 to 20,000 volts. Open switch, and observe spark across spark plug.

5. Repeat steps 2, 3, and 4 for output voltages of 10, 8, 6, and 4 volts. Record values.

DATA TABLE		
		RESULTS
Output Voltage (volts)	Primary Current (amperes)	Energy $E = \frac{1}{2} LI^2$ (joules)
12		
10		
8		
6		
4		

SELF CHECK

1. Define "inductance."

2. An air core inductor has $N = 3500$ turns, $\ell = 7.0$ cm, and $r = 5.0$ cm. Calculate the following values:
   a. Inductance of the coil.
   b. Magnetic-field strength for a current of 0.5 A.
   c. Energy stored in the magnetic field of the coil for a current of 0.5 A.

3. State the differences in inductance and magnetic field strength of a coil with and without an iron core.

4. Describe how a variable inductor works.

5. Sketch graphs showing the time dependence of voltage, current, and energy for an inductor when the current increases and when it decreases.

6. Sketch a schematic of an automobile ignition system, and describe how it produces a spark. Include the functions of the coil, breaker points, condenser, rotor, and spark plugs.

7. How many turns are ncessary to produce an inductance of at least 50 mH in an air core inductor which has a 1 cm radius and is 3 cm long?

8. If the number of turns in an air core inductor is doubled without changing the length of the coil by what factor is the inductance of the coil changed?

9. Derive an expression relating the magnetic field strength at the center of an air core inductor to the inductance of the coil.

10. What current flows through a 10 mH inductor if 5 joules of energy is stored in the magnetic field of the inductor?

7E4

SERIES AND PARALLEL CAPACITORS

Often, circuits in which capacitors are arranged in combinations must be designed or analyzed. When the combinations are simple series or parallel networks, the analysis is particularly simple.

In this module, the student will become familiar with the properties of circuits consisting of capacitors connected in series or parallel. The concept of the potential energy of equivalent capacitance will be introduced, and practical applications will be discussed briefly. In the laboratory, the student will verify these properties and will apply them to measure an unknown capacitance.

OBJECTIVES

Upon completion of this module, the student should be able to:

- Calculate the equivalent capacitance of a series or parallel network of capacitors.
- Given two of the following three quantities, calculate the third:

    Equivalent capacitance.
    Source voltage.
    Charge.

- Given two of these three quantities, calculate the third:

    Equivalent capacitance.
    Source voltage.
    Energy.

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- List four properties of equivalent circuits for —

  Parallel-connected capacitors.
  Series-connected capacitors.

- Given the appropriate equipment, measure the voltage across capacitors connected in series or parallel, and calculate the charge and energy stored by each.

- Given the appropriate equipment, determine the capacitance of an unknown capacitor.

DISCUSSION

CAPACITORS IN PARALLEL

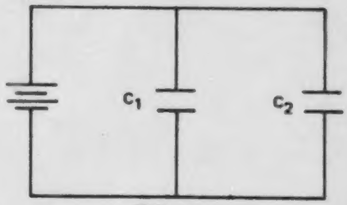


FIG. 1

Figure 1 is a schematic of two capacitors, C_1 and C_2, connected in parallel across a d.c. voltage source V. Because the capacitors are in parallel, the voltage across each must be the same as, and equal to, V. Hence it is a simple matter to calculate the magnitude of the charge carried on the plates of each capacitor. Equation 1 from Module 7E2 is as follows:

$$C = \frac{Q}{V} \qquad (1)$$

where
C = capacitance.
Q = charge stored.
V = voltage (potential difference).

When both sides of Equation 1 are multiplied by V, the result is Equation 2.

$$Q = CV \qquad (2)$$

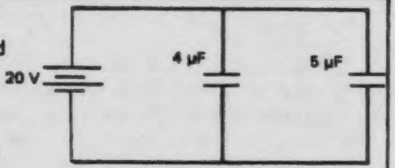
According to Equation 2, the charge Q_1 on capacitor C_1 in Figure 1 is equal to C_1V, and the charge Q_2 on C_2 is C_2V. In a similar manner, the energy stored by each capacitor can be calculated using Equation 4 from Module 7E2:

$$E_p = \frac{1}{2} CV^2 \qquad (3)$$

where
E_p = potential energy stored in a capacitor.

Equation 3 indicates that the energy E_1 stored by C_1 is equal to 1/2 C_1V^2, and the energy E_2 stored by C_2 is 1/2 C_2V^2. Example A illustrates how Equations 2 and 3 can be applied to solve a problem.

EXAMPLE A: CHARGE AND ENERGY STORED BY CAPACITORS IN PARALLEL



Given: A 4-µF and a 5-µF capacitor are connected in parallel across a 20-V d.c. source.

Find:
(a) The charge stored by each capacitor.
(b) The energy stored by each capacitor.

Solution: (a) $Q = CV$ (Equation 2)

4-µF capacitor:

$$Q = (4 \times 10^{-6}\ \mathrm{F})(20\ \mathrm{V})$$
$$= (80 \times 10^{-6}\ \mathrm{F})(\mathrm{V})$$
$$Q = 80 \times 10^{-6}\ \mathrm{C}$$

5-µF capacitor:

$$Q = (5 \times 10^{-6}\ \mathrm{F})(20\ \mathrm{V})$$
$$= (100 \times 10^{-6}\ \mathrm{F})(\mathrm{V})$$
$$Q = 100 \times 10^{-6}\ \mathrm{C}$$

Note: $1\ \mathrm{FV} = 1\ \frac{\mathrm{C}}{\mathrm{V}}\ \mathrm{V}$

$1\ \mathrm{FV} = 1\ \mathrm{C}$ (1 coulomb)

(b) $E_p = \frac{1}{2}CV^2$ (Equation 3)

4-µF capacitor:

$$E_p = (\tfrac{1}{2})(4 \times 10^{-6}\ \mathrm{F})(20\ \mathrm{V})^2$$
$$= (800 \times 10^{-6}\ \mathrm{F})(\mathrm{V}^2)$$
$$E_p = 800 \times 10^{-6}\ \mathrm{J}$$

5-µF capacitor:

$$E_p = (\tfrac{1}{2})(5 \times 10^{-6}\ \mathrm{F})(20\ \mathrm{V})^2$$
$$= (1000 \times 10^{-6}\ \mathrm{F})(\mathrm{V}^2)$$
$$E_p = 1000 \times 10^{-6}\ \mathrm{J}$$

Note: $1\ \mathrm{FV}^2 = 1\ \frac{\mathrm{C}}{\mathrm{V}}\ \mathrm{V}^2$

$= 1\ \mathrm{CV}$

$= 1\ \mathrm{C}\ (\frac{\mathrm{J}}{\mathrm{C}})$

$1\ \mathrm{FV}^2 = 1\ \mathrm{J}$ (1 joule)

A useful concept often used in analyzing combinations of capacitors in a circuit is "equivalent capacitance." The equivalent capacitance of a group of capacitors in a circuit is defined as "the capacitance of the single capacitor that can replace the group without changing the characteristics of the circuit." A circuit in which two or more capacitors are replaced by their equivalent capacitance is called an "equivalent circuit." For the case in which the original group of capacitors are connected in parallel, the equivalent capacitance C_p easily is found.

Figure 2a depicts the circuit previously shown and indicates the charge and energy associated with each capacitor. Since the characteristics of the equivalent circuit must be identical to those of the original circuit, the total potential energy E_p stored by C_p must be equal to the sum of the energies stored by C_1 and C_2. Similarly, the total charge Q_T on C_p must equal to the sum of the charges on C_1 and C_2 (Figure 2b). Either of these criteria can be used to determine the equivalent capacitance. Using the latter:

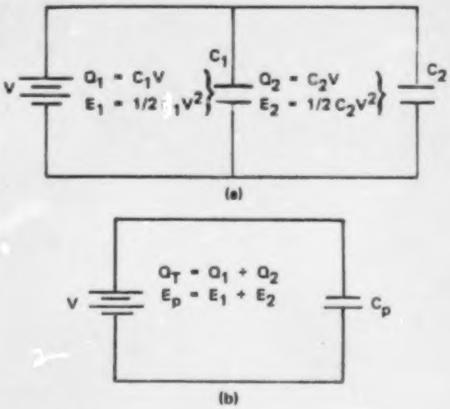


FIG. 2

$$Q_T = Q_1 + Q_2$$

$$C_pV = C_1V + C_2V \quad \text{(substitution from Equation 2)}$$

$$C_p = C_1 + C_2 \quad \text{(division of both sides by V)}$$

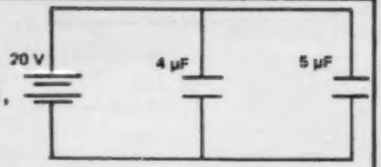
As seen above, the equivalent capacitance of C_1 and C_2 connected in parallel is exactly the sum of their individual capacitance. In general, for any number of capacitors connected in parallel, the equivalent capacitance is simply the sum of the individual capacitances, as indicated by Equation 4.

$$C_p = C_1 + C_2 + C_3 + \ldots \text{ etc.} \qquad (4)$$

In addition, the total charge and energy stored by the circuit can be calculated directly from Equations 2 and 3, respectively, by using the value of C_p for the capacitance.

EXAMPLE B: EQUIVALENT CAPACITANCE OF A PARALLEL NETWORK

Given: A 4-μF and a 5-μF capacitor are connected in parallel across a 20-V d.c. voltage source, as in Example A.



Find:
(a) Their equivalent capacitance.
(b) The total charge stored.
(c) The total energy stored.

Solution: (a)

$$
\begin{aligned}
C_p &= C_1 + C_2 \\
&= 4\mu F + 5\mu F \\
C_p &= 9\mu F
\end{aligned}
$$

(b)

$$
\begin{aligned}
Q_T &= C_pV \\
&= 9 \times 10^{-6}\ F\ (20\ V) \\
&= 180 \times 10^{-6}\ C \\
Q_T &= 180\mu C
\end{aligned}
$$

Note that this is the sum of Q_1 and Q_2 from Example A.

(c)

$$
\begin{aligned}
E_p &= \frac{1}{2}\,C_pV^2 \\
&= \frac{1}{2}\,(9 \times 10^{-6}\ F)(20\ V)^2 \\
E_p &= 1800 \times 10^{-6}\ J
\end{aligned}
$$

Note that this value is the sum of E_1 and E_2 from Example A.

The circuit behaves exactly as if a single 9-μF capacitor were connected across the voltage source.

In summary, the equivalent circuit of a group of capacitors connected in parallel exhibits the following properties:

1. The equivalent capacitance is equal to the sum of the individual capacitances:

$$C_p = C_1 + C_2 + C_3 + \ldots\ \text{etc.} \tag{4}$$

2. The voltage across each individual capacitor is the same as, and equal to, the total voltage V_T across the equivalent capacitor:

$$V_T = V_1 = V_2 = V_3 = \ldots\ \text{etc.} \qquad (5)$$

3. The total charge Q_T associated with the equivalent capacitor is equal to the sum of the charges of the individual capacitors:

$$Q_T = Q_1 + Q_2 + Q_3 + \ldots\ \text{etc.} \qquad (6)$$

4. The total potential energy E_p stored by the equivalent capacitor is equal to the sum of the energies stored by each individual capacitor:

$$E_p = E_1 + E_2 + E_3 + \ldots\ \text{etc.} \qquad (7)$$

The principal application of capacitors connected in parallel is in the building of large capacitance networks from smaller capacitors in parallel. This procedure results in high capacitance values at less expenditure than is required for a single large capacitor.

CAPACITORS IN SERIES

Figure 3a displays capacitors C_1 and C_2 connected in series across potential difference V. In general, the voltages V_1 and V_2 across each capacitor will be different — unlike the case for capacitors in parallel; however, the charge that accumulates on the plates of each capacitor is the same.

Figure 3b shows the circuit as the capacitors are beginning to charge. The process can be visualized step-by-step as follows: A negative charge (an electron) leaves the negative terminal of the battery and travels to the nearest plate of C_2, imparting to it a slight, negative net charge (Figure 3b). Because the charge on one plate of a capacitor induces an equal and opposite charge on the opposite plate, an electron leaves the opposite plate of C_2 and travels to the nearest plate of C_1 (Figure 3c). Since the plates of each capacitor at first are electrically neutral, a positive charge of equal magnitude is left behind on C_2. Upon arriving at C_1, the electron repels another electron from the opposite plate; this electron then moves to the positive terminal of the battery (Figure 3d). The process continues until the capacitors are charged completely, each carrying equal charges Q (Figure 3e). Notice that although each capacitor carries a charge

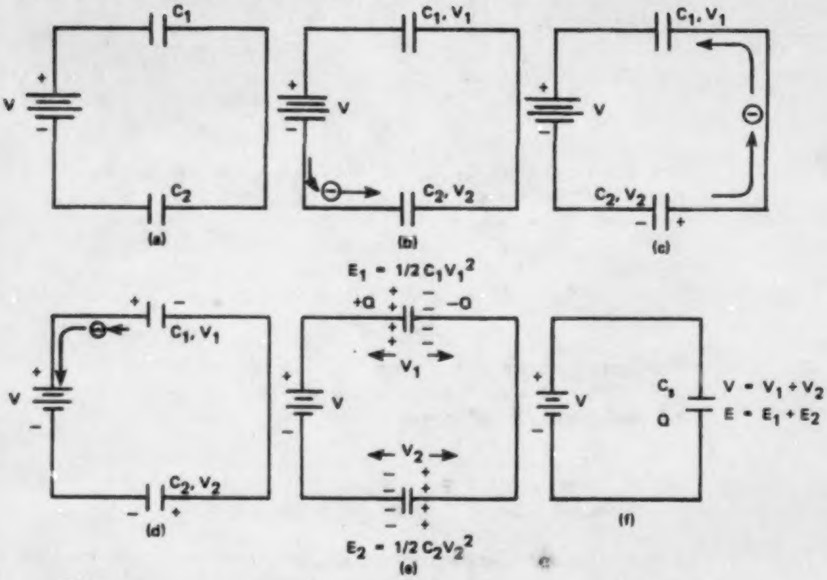


FIG. 3

equal to Q, the battery has only supplied a total charge Q_T to the entire circuit. The net effect, then, is that of a single capacitor carrying a charge Q. Thus, the charge Q_T carried by the equivalent capacitor should be the same as the charge on each of the individual capacitors.

In order that the equivalent circuit can behave in the same manner as the series circuit, two more criteria must be met: (1) The energy stored by the equivalent circuit must be equal to the sum of the energies stored in the individual capacitors. (2) The total voltage across the equivalent circuit must be equal to the sum of the voltages across the individual capacitors. Either of these criteria (shown in the equivalent circuit of Figure 3f) can be used to determine the value of the series equivalent capacitance, C_s. Equation 8 derives the total voltage.

$$V_T = V_1 + V_2 \tag{8}$$

Rearranged, Equation 1 gives $V = Q/C$. Substitution of the appropriate value of V and C in this result transforms Equation 8 into —

$$\frac{Q}{C_s} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

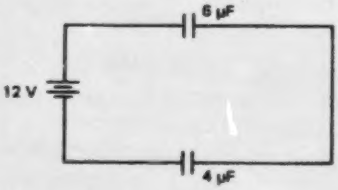
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{(Both sides divided by Q)}$$

Thus, the reciprocal of the equivalent capacitance of the two capacitors connected in series is equal to the sum of the reciprocals of the individual capacitances. This relationship is true in general, no matter how many capacitors are connected in series; therefore —

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \text{ etc.} \qquad (9)$$

Once C_s is known, the charge and energy stored by the equivalent circuit can be calculated from Equations 2 and 3 respectively, using the value of C_s for the capacitance. Example C demonstrates the solution of a typical problem involving capacitors in series.

EXAMPLE C: SERIES CAPACITOR CIRCUIT



Given: A 6-µF capacitor and a 4-µF capacitor are connected in series across a 12-V d.c. voltage source.

Find:
(a) The equivalent capacitance of the circuit.
(b) The total charge supplied by the battery.
(c) The total energy stored in the circuit.
(d) The voltage across each capacitor.
(e) The energy stored in each capacitor.

Solution: (a)

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{6\mu F} + \frac{1}{4\mu F}$$

$$= \frac{2}{12\mu F} + \frac{3}{12\mu F} \quad \text{(Expressed in terms of common denominators)}$$

$$= \frac{5}{12\mu F} \quad \text{(The fractions are added.)}$$

$$\frac{1}{C_s} = \frac{12\mu F}{5} \quad \text{(The reciprocal of both sides is taken.)}$$

$$C_s = 2.4\mu F$$

(b)

$$Q_T = C_s V$$

$$= 2.4 \times 10^{-6} \text{ F } (12 \text{ V})$$

$$Q_T = 28.8 \times 10^{-6} \text{ C}$$

(c) $E_p = \frac{1}{2} C_s V^2$

$= \frac{1}{2} (2.4 \times 10^{-6}\ \text{F})(12\ \text{V})^2$

$E_p = 172.8 \times 10^{-6}\ \text{J}$

(d) 6-μF capacitor:

$V = \frac{Q}{C}$

$= \frac{28.8 \times 10^{-6}\ \text{C}}{6 \times 10^{-6}\ \text{C}}$

$V = 4.8\ \text{V}$

4-μF capacitor:

$V = \frac{28.8 \times 10^{-6}\ \text{C}}{4 \times 10^{-6}\ \text{F}}$

$V = 7.2\ \text{V}$

Notice that the sum of the voltages across the individual capacitors is equal to the voltage across the equivalent capacitor (source voltage):

$$4.8\ \text{V} + 7.2\ \text{V} = 12\ \text{V}$$

(e) $E_p = \frac{1}{2} CV^2$

6-μF capacitor:

$E_p = \frac{1}{2} (6 \times 10^{-6}\ \text{F})(4.8\ \text{V})^2$

$E_p = 69.12 \times 10^{-6}\ \text{J}$

4-μF capacitor:

$E_p = \frac{1}{2} (4 \times 10^{-6}\ \text{F})(7.2\ \text{V})^2$

$E_p = 103.68 \times 10^{-6}\ \text{J}$

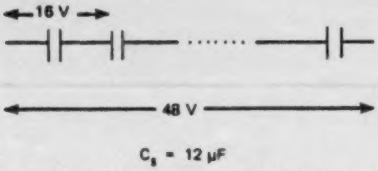
Notice that the sum of the energies stored by each capacitor is equal to the energy stored by the equivalent capacitor:

$$69.12 \times 10^{-6}\ \text{J} + 103.68 \times 10^{-6}\ \text{J} = 172.8 \times 10^{-6}\ \text{J}$$

Example D illustrates another application of capacitors in series.

EXAMPLE D: ENHANCED VOLTAGE RATING OF CAPACITORS IN SERIES

Given: A technician must construct a 12-µF equivalent capacitance network that can be placed across a 48-V d.c. voltage source. An unlimited selection of capacitors with maximum voltage ratings of 16 V each is available.



Find: The minimum number and capacitance of identical capacitors from which the network can be constructed.

Solution: Each individual capacitor can support a maximum voltage of 16 V; therefore, a series combination is required. The minimum number of capacitors needed is given by —

$$\text{number of capacitors} = \frac{\text{total voltage}}{16\text{ V}} = \frac{48\text{ V}}{16\text{ V}} = 3$$

Find the capacitance of each capacitor:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{(for three capacitors)}$$

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \quad \text{(Since } C_1 = C_2 = C_3 = C \text{ for identical capacitors)}$$

$$\frac{1}{12\mu F} = \frac{3}{C}$$

$$\frac{C}{12\mu F} = 3 \quad \text{(Both sides multiplied by C)}$$

$$C = 3(12\mu F)$$

$$C = 36\mu F \quad \text{(Both sides multiplied by } 12\mu F\text{)}$$

Thus, three 36-µF capacitors rated at 16 V each, connected in series, are equivalent to a single 12-µF, 48-V capacitor.

The equivalent circuit of a number of capacitors connected in series has the following properties:

1. The reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\text{ etc.} \qquad (9)$$

2. The charge associated with each individual capacitor is the same and equal to the charge on the equivalent capacitor:

$$Q_T = Q_1 = Q_2 = Q_3 = \ldots\ \text{etc.} \qquad (10)$$

3. The voltage across the equivalent capacitor is equal to the sum of the voltages across the individual capacitors:

$$V_T = V_1 + V_2 + V_3 + \ldots\ \text{etc.} \qquad (11)$$

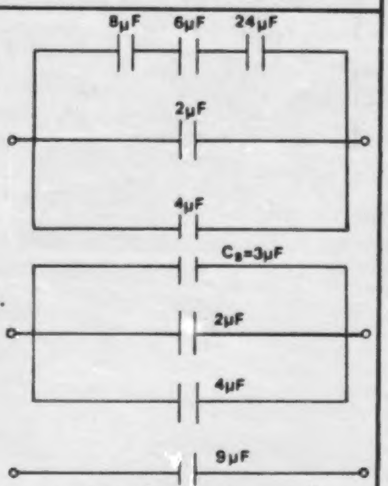
4. The potential energy E_p stored by the equivalent capacitor is equal to the sum of the energies by the individual capacitors:

$$E_p = E_1 + E_2 + E_3 + \ldots\ \text{etc.} \qquad (12)$$

CAPACITORS IN SERIES/PARALLEL NETWORKS

A circuit that contains both series and parallel networks of capacitors often must be analyzed. This is easily done using the rules already developed, as illustrated in Example E.

EXAMPLE E: EQUIVALENT CAPACITANCE OF A SERIES–PARALLEL NETWORK



Given: Five capacitors are connected as in adjoining sketch.

Find: Their equivalent capacitance.

Solution: Start with the simplest segment, in this case the upper branch. Here there are 3 capacitors in series. Find the equivalent capacitance of this series segment:

$$\frac{1}{C_s} = \frac{1}{8\mu F} + \frac{1}{6\mu F} + \frac{1}{24\mu F}$$

$$= \frac{3}{24\mu F} + \frac{4}{24\mu F} + \frac{1}{24\mu F} \quad \text{(common denominator is used)}$$

$$\frac{1}{C_s} = \frac{8}{24\mu F} = \frac{1}{3\mu F}$$ (fraction is added and reduced)

$C_s = 3\mu F$ (reciprocal of both sides is taken)

Draw an equivalent circuit with the original segment replaced by its equivalent capacitance. The three capacitors now are in parallel. Find the equivalent capacitance of this parallel arrangement:

$$C_p = 3\mu F + 2\mu F + 4\mu F$$

$$C_p = 9\mu F$$

The equivalent capacitance of the entire network is 9μF.

SUMMARY

Capacitors store and release charge and potential energy. "Equivalent capacitance" is "the capacitance of a single capacitor that can replace a group of capacitors without changing the characteristics of the circuit." Often, an equivalent circuit is more practical and economical than one containing a single large capacitor. The equivalent capacitance in parallel circuits is equal to the sum of the individual capacitances; and the total charge and total energy in the equivalent capacitor is equal to the sum of that in the individual capacitors, although the voltage across each individual capacitor is the same. In a series circuit, however, the reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of the individual capacitances; and the voltage and energy in the equivalent capacitance is equal to the sum of that in the individual ones. The charge in each capacitor is the same and equal to that on the equivalent capacitor.

SERIES AND PARALLEL CAPACITORS

MATERIALS

Power supply
Perf-board with solderless spring clips
Wire leads
Assortment of nonelectrolytic capacitors
Digital multimeter

PROCEDURE

1. Turn on the VTVM and allow it to warm up.
2. Connect three capacitors in parallel, as illustrated in Figure 4. Label Figure 4 with the capacitance values you have used.

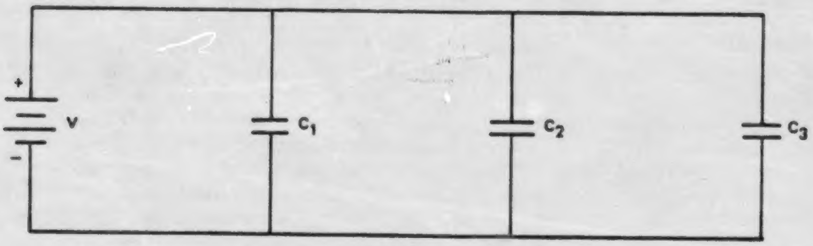


FIG. 4

3. Set the power supply voltage to about 3 V. Measure voltages across each capacitor and the voltage source. Record in Data Table 1.
4. Calculate the charge stored by each capacitor from Equation 2.
5. Calculate the energy stored by each capacitor from Equation 3.
6. Calculate the equivalent capacitance from Equation 4.
7. Calculate the total charge stored from the values of C_p and V using Equation 2.
8. Calculate the total energy stored from the values of C_p and V using Equation 3.

9. Verify Equations 5, 6, and 7 for this circuit.

10. Reconnect the capacitors in series, as shown in Figure 5. Label Figure 5 with the appropriate values of capacitance.

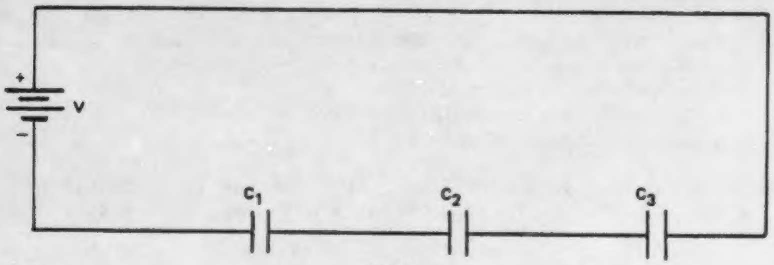


FIG. 5

11. Repeat steps 3-5 for this circuit. Record results in Data Table 2.

12. Calculate the equivalent capacitance C_s from Equation 9.

13. Calculate the charge Q and the energy E of the equivalent capacitor from values of C_s and V; use Equations 2 and 3, respectively.

14. Verify Equations 10, 11, and 12 for this circuit.

15. How does this experiment demonstrate the principles given in the Discussion section?

DETERMINATION OF UNKNOWN CAPACITANCE

MATERIALS

Same as for first experiment.

PROCEDURE

1. Connect a capacitor of unknown capacitance in series with a known capacitor across the power supply as shown in Figure 6. Record capacitance of the known capacitor in Data Table 3.

2. Set the power supply voltage to approximately 3 volts. Measure the voltage across each capacitor with the VTVM, and record these values in Data Table 3.

3. Use Equation 2 to calculate the charge Q on the known capacitor from its capacitance and the voltage across it. Since C_1 and C_2 are in series, C_2 must carry an equal amount of charge Q.

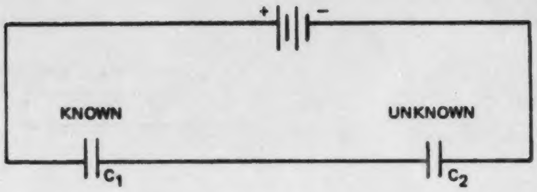


FIG. 8

4. From the values of Q and V, calculate the unknown capacitance. Use Equation 1. Verify your results with your instructor.

DATA TABLE 1

Source: $C_1 =$ ____ μF	$C_1 =$ ____ μF	$C_3 =$ ____ μF
$V =$ ____ V $V_1 =$ ____ V	$V_2 =$ ____ V	$V_3 =$ ____ V
$Q_1 = C_1V_1 =$ ____ C	$Q_2 = C_2V_2 =$ ____ C	$Q_3 = C_3V_3 =$ ____ C
$E_1 = \frac{1}{2} C_1V_1^2 =$ ____ J	$E_2 = \frac{1}{2} C_2V_2^2 =$ ____ J	$E_3 = \frac{1}{2} C_3V_3^2 =$ ____ J
$C_p = C_1 + C_2 + C_3 =$ ____ F	$Q_T = C_pV =$ ____ C	$E_p = \frac{1}{2} C_pV^2 =$ ____ J

DATA TABLE 2

Source: $C_1 =$ ____ μF	$C_2 =$ ____ μF	$C_3 =$ ____ μF
$V =$ ____ V $V_1 =$ ____ V	$V_2 =$ ____ V	$V_3 =$ ____ V
$Q_1 = C_1V_1 =$ ____ C	$Q_2 = C_2V_2 =$ ____ C	$Q_3 = C_3V_3 =$ ____ C
$E_1 = \frac{1}{2} C_1V_1^2 =$ ____ J	$E_2 = \frac{1}{2} C_2V_2^2 =$ ____ J	$E_3 = \frac{1}{2} C_3V_3^2 =$ ____ J
$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \rightarrow C_s =$ ____ μF		
$Q_T = C_sV =$ ________	$E_p = \frac{1}{2} C_sV^2 =$ ________	

DATA TABLE 3	
C_1 = _____ μF V_1 = _____ V	V_2 = _____ V
$Q_T = C_1V_1$ = _____ C	$C_2 = \frac{Q}{V_2}$ = _____ F = _____ μF

SELF CHECK

1. Calculate the equivalent capacitance of each of the circuits in Figure 7.

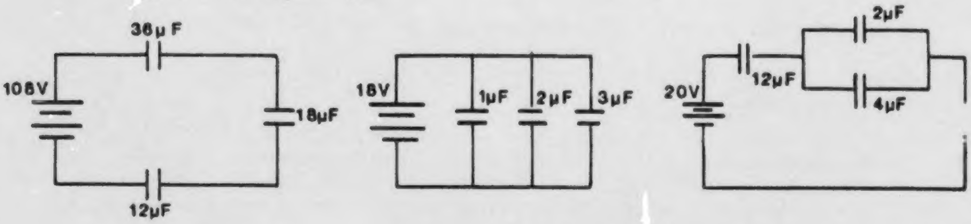


FIG. 7

2. Calculate the total charge and energy stored in each of the circuits of Figure 7.

3. Calculate the voltage across each capacitor in Figure 7a.

4. Calculate the charge and energy stored for each capacitor in Figure 7b.

5. List four properties of the equivalent circuit for each of the following:

   Parallel-connected capacitors.
   Series-connected capacitors.

6. What capacitance should be placed in parallel with a 6 μf capacitor in order to store 5.0×10^{-4} joules of energy in a circuit which contains only the two capacitors and a 12 volt battery?

7T2

ELECTRICAL EQUIVALENT OF HEAT

In electrical systems a portion of the electrical energy is invariably converted to kinetic energy of atoms in the form of heat. In systems designed to convert electrical energy to mechanical energy, it is desirable to reduce to a minimum the heat energy produced in this process. In other electrical systems, however, heat may be the desired result and all of the electrical energy is transformed to heat energy. In either case, it is important to understand the relationship between electrical energy and heat energy.

In this module, the process by which electrical energy is converted to heat energy will be investigated. Experiments will be used to measure the electrical equivalent of heat, that is, the energy in electrical units equivalent to a given quantity of energy in heat units.

OBJECTIVES

To complete this module, the student should be able to:

- Calculate electrical energy expended, given a quantity of charge Q and potential difference v through which that charge is moved.
- Calculate electrical energy expended, given current I, voltage V and duration of current flow t.
- Calculate heat energy gained by a material, given its mass m, initial and final temperatures T_1 and T_2 and specific heat capacity c.
- Explain in a short paragraph how electrical energy is con-

verted to heat energy in an electrical conductor.

- Provided with necessary equipment, find energy produced, corresponding increase in heat energy of a liquid and electrical equivalent of heat of a system in which electrical energy is converted to heat energy.

DISCUSSION

Potential or voltage of a source of electrical energy is a measure of its ability to do work in moving an electric charge. One joule of energy is required to move a charge of one coulomb through a potential difference of one volt. Thus, electrical energy expended (work done) by a source of voltage in moving a given amount of charge is given as:

Electrical energy = charge x voltage.

$$\text{In symbols:} \quad E_e = QV. \qquad (1)$$

Example A illustrates the use of Equation 1 in solving a problem.

EXAMPLE A: ELECTRICAL ENERGY CALCULATION

Given: A battery with a potential of 12 volts used to move 3 coulombs of charge through a circuit.

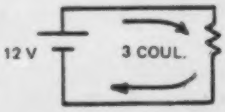
Find; How much electrical energy is used.

Solution:

$E_e = QV$

$= (3 \text{ coulombs})(12 \text{ volts})$

$E_e = 36 \text{ C} \times \text{V} = 36 \text{ joules}.$



All electrical energy expended by the source must appear in some other form. At least a portion of this energy is converted to heat. Figure 1 shows how electrical potential energy is converted to heat energy in an electrical conductor.

An electron at point A is accelerated by the electric field supplied by the source. The electron is accelerated toward the positive terminal, its electrical potential energy is converted

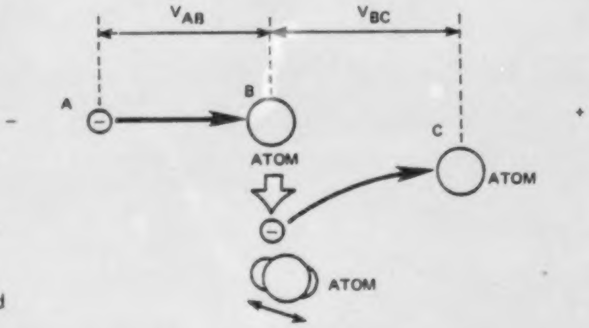


FIG. 1

to kinetic energy. In moving from point A to point B, kinetic energy of the electron is increased by the product of the electron charge and the voltage V_{AB}. At point B, the electron collides with an atom and part of its kinetic energy is transferred to the atom. The part transferred to the atom appears as vibrational energy in the material carrying the current. The electron then continues to gain kinetic energy as it accelerates toward point C, where the process is repeated. Vibrational energy imparted to the atoms of the conductor appears as heat energy, and its temperature rises. This process is called Joule heating after physicist James P. Joule, who first described it.

In a purely resistive load, all of the electrical energy is changed to heat energy. The power of an electric heating element or resistor is the rate at which electrical energy is converted to heat within the element. Since current is given by:

$$I = \frac{Q}{t}$$

and energy is $E = QV$,

power is $P = \frac{E}{t} = \frac{QV}{t}$

or, finally, $P = IV$,

total energy delivered by a pure resistive load is given by Equation 2.

Energy = power x time.

Energy = voltage x current x time.

In symbols: $E_e = VIt$. (2)

Example B illustrates how to use Equations 1 and 2 in solving a problem.

Recall from Concept Module 3-0, "Work," Equation 3 for the heat energy necessary to cause a temperature rise in a material.

Heat energy = mass x specific heat x change in temperature.

In symbols: $E_h = m \times c \times \Delta t$. (3)

EXAMPLE B: ENERGY AND TOTAL CHARGE CALCULATION

Given: A 220-ohm resistor connected to a 12-volt battery as shown. The switch is closed for 30 seconds.

Find: a) Energy converted to heat within the resistor and b) total charge that flows through the circuit.

Solution:

a. From Ohm's law: $I = \frac{V}{R}$

$$= \frac{12 \text{ volts}}{220 \text{ ohms}}$$

$$= 0.0545 \text{ amps.}$$

b. From Equation 2: $E_e = IVt$

$$= (0.0545 \tfrac{C}{s})(12 \tfrac{\text{joules}}{C})(30 \text{ s})$$

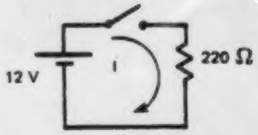
$$= 19.62 \text{ joules.}$$

From Equation 1: $Q = \frac{E}{V}$

$$= \frac{19.62 \text{ joules}}{12 \text{ joules/C}}$$

$$= 1.64 \text{ C}$$

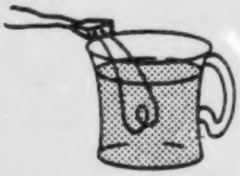
or, $Q = It = (0.0545 \text{ C/s})(30 \text{ s}) = 1.64 \text{ C}$.



1 amp = 1 C/s.
1 volt = 1 joule/C.

In practical applications of pure resistive loads in electrical circuits, actual temperature of the resistor depends on power delivered, mass and specific heat of the resistor and rate at which heat is conducted away by the environment. To make accurate measurements of the electrical equivalent of heat, it is necessary to limit investigation to systems in which heat energy can easily be distributed through a known mass of material with a known specific heat. This is most easily accomplished with liquid heating systems such as oil sump, aquarium and coffee pot heaters. If the mass of the liquid is much greater than the mass of the heating element used, heat contained in the heating element itself becomes insignificant. Example C is an application of Equation 3 to such a problem.

Since electrical energy is customarily measured in joules and heat energy is measured in calories, the relationship between electrical energy delivered in joules and heat energy gained in calories must be determined. This relationship is usually expressed as:



EXAMPLE C: CALCULATING HEAT ENERGY

Given: A small heating coil used to heat a cup of water.

Find: How much heat energy is necessary to raise the temperature of 200 g of water from 25° C to 80° C.

Solution:

$$E_h = m \times c \times \Delta t$$

$$= (200\ g)(1\ calorie/g\ °C)(80°\ C - 25°\ C)$$

$$= (200\ g)(1\ calorie/g\ °C)(55°\ C)$$

$$E_h = 11{,}000\ calories.$$

1 calorie = 4.184 joules

or

1 kilocalorie (thousand calories) = 4184 joules.

This relationship will be verified in the experimental portion of this module.

LABORATORY

MATERIALS

Graduated cylinder.
Centigrade thermometer.
a.c. voltmeter.
a.c. ammeter.
Stopwatch.
One-quart thermos bottle (wide mouth is best).
Water.
Electric heating element of known power rating. Note: This may be an oil sump heater, aquarium heater, the type of coil used in Example C or any similar heating element that operates on 115-V a.c. power.

PROCEDURE

1. Connect electric circuit as shown in Figure 2a and b; do not plug in circuit. Caution: Because this experiment requires high voltage, check circuit with instructor before plugging power cord into wall outlet.
2. Fill thermos bottle with cold water, about one inch from top as shown in Figure 3; insert heater and thermometer.
3. Remove heater and thermometer; pour water into graduated cylinder. Record its volume. Return water, heater and thermometer to thermos. Allow 5 minutes for water temperature to stabilize; record initial temperature of water.
4. Complete part A of data table to calculate approximate heating time necessary to cause a 10° C temperature rise in water.
5. Plug in power cord; close knife switch for appropriate time as measured with stopwatch. Record voltage and current measured. Stir water gently with thermometer. (Make sure heating element does not touch thermometer.)
6. After a 10° C rise in temperature has been attained, open switch; allow 30 seconds for temperature to stabilize; record final temperature of water.
7. Complete data table; prepare a brief analysis of probable errors in laboratory.

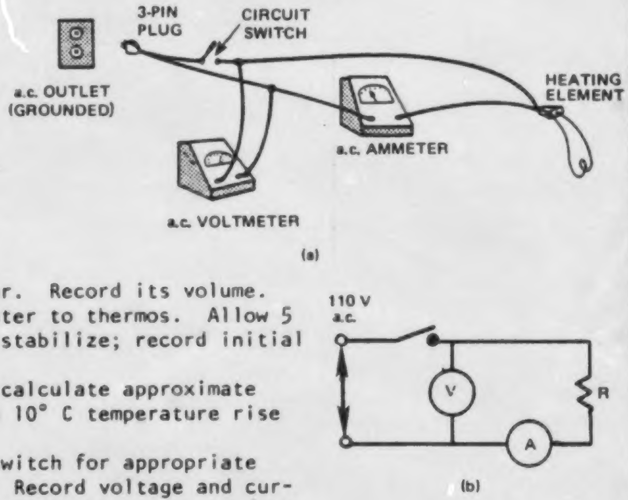
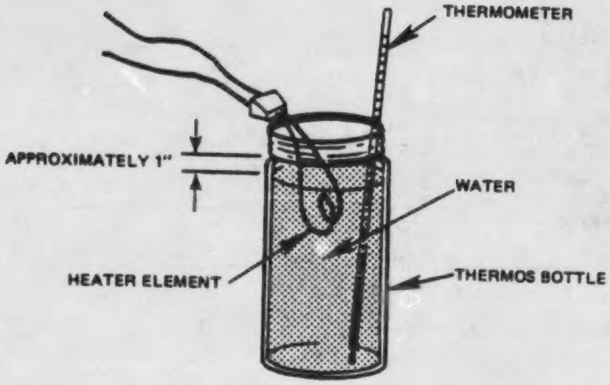


FIG. 2

FIG. 3


THERMOMETER
APPROXIMATELY 1"
WATER
HEATER ELEMENT
THERMOS BOTTLE

1 cm^3 of water has a mass of 1 g.

DATA TABLE

Part A

Volume of water: v = _____ cm^3.

Mass of water: m = _____ g.

Heat energy necessary to cause 10° C temperature rise:

E_h = _____ g x 1 calorie/g °C x 10° C.

= _____ calories x 4.184 joules/calorie.

E_h = _____ joules.

Rated power of heater: P = _____ watts.

Time necessary to deliver electrical energy:

t = electrical energy necessary/power of heater.

= _____ joules/ _____ joules/s.

t = _____ s.

Part B

Electrical energy.

Voltage: V = _____ volts.

Current: I = _____ amps.

Time: t = _____ s.

Electrical energy:

E_e = VIt.

= $\frac{\text{joules}}{\text{C}}$ x _____ $\frac{\text{C}}{\text{s}}$ x _____ s.

E_e = _____ joules.

Heat energy.

Beginning temperature: T_1 = _____° C.

Final temperature: T_2 = _____° C.

Change in temperature: $\Delta t = T_2 - T_1$

= _____° C - _____° C

= _____° C.

Heat energy.

$E_h = m \times c \times \Delta t$

= _____ g x (1 $\frac{\text{calorie}}{\text{g} \times {}^\circ\text{C}}$) x _____° C

= _____ calories.

Electrical equivalent of heat = E_e/E_h

= _____ joules/_____ calories

E_e/E_h = _____ joules/calorie.

Percent error = $\frac{\text{theoretical results - experimental results}}{\text{theoretical results}}$

x 100

= $\frac{\text{4.184 J/cal - \_\_\_\_\_ J/cal}}{\text{4.184 J/cal}}$ x 100

= _____ %.

Error analysis notes.

SELF CHECK

1. How much electrical energy is expended when a charge of 16 coulombs is moved through a potential difference of 22.5 volts?
2. How much electrical energy is expended when a current of 0.45 amps is driven by a potential of 115 volts for one minute?
3. If energy in problem 2 is converted to heat and transferred to 1 kg of water with an initial temperature of 2° C, what is the final temperature?
4. Explain in a short paragraph how electrical energy is converted to heat energy in an electrical conductor.
5. How much time is required for a 250-watt heating element to raise the temperature of 250 g of water by 25° C?
6. What is the resistance of a heating element if the element can raise 100 grams of water through 40 C° in 20 seconds when a current of 2 Amps is flowing through the element?

7T3

MECHANICAL EQUIVALENT OF HEAT

Drilling holes and sawing timbers cause tools to heat up when used. A technician needs to know how to calculate the heat that will be produced in any mechanical action. Precisely how mechanical energy and heat energy are related to each other needs to be known so that steps can be taken to make machinery more efficient.

In the experimental part of this module, tiny balls of lead will be dropped through a given height and, in the process, will convert gravitational potential energy into heat energy. Ratio of the quantities (mechanical energy expended/heat energy produced) yields the "mechanical equivalent of heat."

OBJECTIVES

To complete this module, the student should be able to:

- Define "mechanical equivalent of heat."
- Demonstrate a knowledge of the relationship between heat and mechanical energy by stating equivalent energies in calories, joules and British thermal units.
- Calculate several problems concerning increase in temperature of a falling object.
- Calculate several problems concerning transfer of potential to kinetic energy.

640

DISCUSSION

The term "mechanical equivalent of heat" implies that there is a relationship between mechanical energy and heat energy. The discovery of this relationship - that heat energy is just the mechanical energy of motion of atoms and molecules - was one of the great discoveries in physics. When this became clear, the separate sciences of mechanics and calorimetry (heat exchange) were unified.

When mechanical energy is converted to heat energy (or vice versa) the principle of conservation of energy applies. Therefore, a precise relationship is found to exist between energy measured in mechanical units and energy measured in heat units. Several such conversions are given in Table 1.

TABLE 1: CONVERSION TABLE

Heat Energy	Mechanical Energy
1 calorie	4.184 joules
1 kilocalorie	4184 joules
1 Btu (British thermal unit)	778 ft-lb or 1054 joules

Examples A, B and C illustrate the equivalence of mechanical and heat energies.

EXAMPLE A: TEMPERATURE INCREASE CALCULATION / AUTO BRAKES

Given: A 1600-kg car traveling 20 m/s is brought to a stop. Assume 40% of the heat developed goes into the brake drum. The specific heat of iron is 0.11 cal/g °C or 0.11 kcal/kg °C.

Find: Increase in temperature of the 5-kg brake drum.

Solution:

$$\text{Kinetic energy lost} = \frac{1}{2} mv^2 = \frac{1}{2} (1600 \text{ kg})(20 \text{ m/s})^2$$

$$= 320{,}000 \text{ joules.}$$

$$\text{Heat developed} = 320{,}000 \text{ J} \times \frac{1 \text{ kcal}}{4.184 \text{ J}} = 76.4 \text{ kcal.}$$

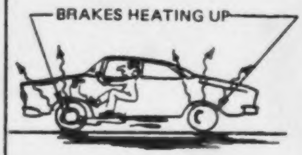
40% of this heat goes into the brake drum. The other 60% goes into heating the brake lining, tires and road.

$$0.40 \times 76.4 = 30.6 \text{ kcal.}$$

$$\text{Heat} = mc\Delta t.$$

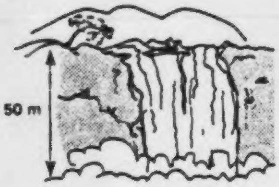
$$30.6 \text{ kcal} = (5 \text{ kg})\left(0.11 \frac{\text{kcal}}{\text{kg } ^\circ\text{C}}\right) \Delta t.$$

$$\text{Increase in temperature} = \Delta t = 55.6^\circ \text{ C.}$$



EXAMPLE B: TEMPERATURE INCREASE CALCULATION/WATERFALL

Note: Kinetic energy could be neglected.



Given: A 50-meter waterfall. Assume that water is traveling at 2 m/s as it goes over the fall, and after it hits the bottom its speed is zero. (Assume that all mechanical energy is converted to heat energy.)

Find: Amount of temperature increase of each gram of water as it falls.

Solution: Mechanical energy = kinetic energy + potential energy.

$$ME = \frac{1}{2} mv^2 + mgh$$

$$= \frac{1}{2} (0.001 \text{ kg})(2 \text{ m/s})^2 + (0.001 \text{ kg})(9.8 \text{ m/s}^2)(50 \text{ m})$$

$$= 0.002 \text{ Nm} + 0.490 \text{ Nm}$$

$ME = 0.492$ Nm or 0.492 J.

Heat developed $= 0.492 \text{ J} \times \frac{1 \text{ cal}}{4.184 \text{ J}} = 0.118$ cal.

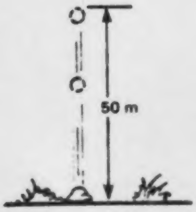
Heat $= mc\Delta t$.

$0.118 \text{ cal} = (1 \text{ g})(1 \text{ cal/g °C}) \Delta t$.

Increase in temperature $= \Delta t = 0.118°$ C.

Temperature does not increase very much because water has a high value of specific heat.

EXAMPLE C: TEMPERATURE INCREASE CALCULATION/LEADWEIGHT



Given: A 1-gram lead weight falls from rest 50 meters above the ground. Lead has a specific heat of 0.031 cal/g °C. Assume that all mechanical energy is converted to heat energy.

Find: Its increase in temperature.

Solution:

Potential energy $= mGh$

$$= (0.001 \text{ kg})(9.8 \text{ m/s}^2)(50 \text{ m}) = 0.49 \text{ J}.$$

Heat $= 0.49 \text{ J} \times \frac{1 \text{ cal}}{4.184 \text{ J}}$

$= 0.117$ cal

Heat $= mc\Delta t$.

$0.117 \text{ cal} = (1 \text{ g})(0.031 \text{ cal/g °C}) \Delta t$.

Increase in temperature $= \Delta t = 3.8°$ C. This temperature is much larger than that of Example B, showing that lead heats up much faster than water. It has a lower value of specific heat than water.

LABORATORY

MATERIALS

Cardboard mailing tube - about 100 cm long.
Lead shot - at least 500 grams.
Thermometer measuring to nearest tenth of a degree.
Two beakers - one larger than the other.
Small amount of ice.
Two cork stoppers for ends of cardboard mailing tube.
Triple-beam balance.
Meter stick.

PROCEDURES

In this experiment, tiny lead balls will be dropped through a given height, thereby converting mechanical energy into heat energy as in Example C. Instead of calculating increase in temperature of lead as in Example C, increase will be experimentally measured. Heat gained by the lead, in calories, will be determined by the formula:

$$\text{Heat energy} = mc\ (T_2 - T_1)$$

where

m = mass of lead shot in grams.
c = specific heat of lead = 0.031 cal/g °C.
T_1 = initial temperature in °C.
T_2 = final temperature in °C.

Loss in potential energy, in joules, of the lead shot is calculated from the formula:

$$\text{Potential energy} = mGh$$

where

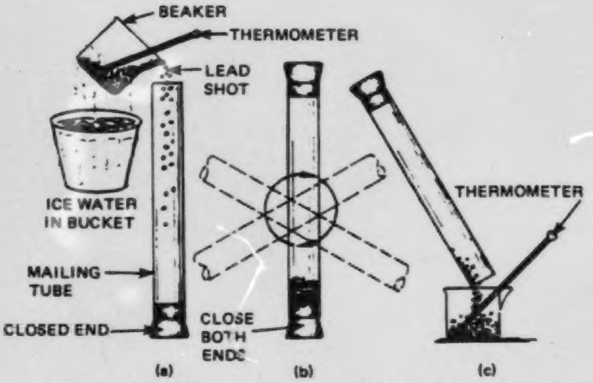
m = mass of lead shot in kg.
$G = 9.8\ m/s^2$.
h = height through which shot falls in meters.

The mechanical equivalent of heat is calculated by the formula:

$$\text{Mechanical equivalent of heat} = \frac{\text{potential energy in joules}}{\text{heat energy in calories}}.$$

The experimental value is compared with the correct value of 4.184 joules/calorie.

1. Record results in data table (use constant units).
2. Record room temperature.
3. Weigh a small beaker. Pour between 500 and 700 grams of lead shot into beaker. Weigh and subtract weight of beaker to get weight of lead shot.
4. Set beaker of lead shot in a larger beaker containing ice water. Stirring, cool shot down about 4 or 5 degrees below room temperature. Be careful to keep shot dry. Remove beaker of shot from ice water. Record initial temperature of shot.
5. Hold mailing tube in a vertical position. Insert cork in lower end. Pour lead shot into mailing tube; insert stopper into top end (Figure 1a).
6. Invert tube, allowing shot to fall to other end; repeat for a total of 98 times. Counting time shot was poured from beaker to tube (Figure 1a) and from tube to beaker (Figure 1c), this will make 100 times shot has fallen length of tube (Figure 1b).
7. Remove stopper from top end. Invert tube again, letting shot roll into beaker (Figure 1c).
8. Move thermometer gently around in shot; record highest temperature. This temperature should be a few degrees above room temperature. Check to make sure thermometer has reached its



(a) FILL MAILING TUBE (b) ROTATE TUBE AND SHOT 98 TIMES (c) EMPTY MAILING TUBE

FIG. 1

peak value. This step must be done quickly, before shot begins to cool down appreciably.

9. Calculate heat (in calories) absorbed by lead shot.
10. Measure distance shot fell down tube. Multiply this number by 100 to get total height through which shot fell.
11. Calculate potential energy in joules.
12. Calculate mechanical equivalent of heat.
13. Compare with correct value of 4.184 joules/calorie.

Note: A percent error of 40% or more in this experiment is not uncommon because of the many variables that were not accounted for - such as heating air and tube. The purpose of the experiment was not to achieve a low percent of error, but to show that mechanical energy really is converted into heat. A percentage error of more than 20% is not very convincing. Repeat experiment with a slightly different initial temperature to give better results.

DATA TABLE

Room temperature _____.

Beaker weight _____.

Beaker and shot weight _____.

Lead shot weight _____

Initial temperature of lead shot _____.

Final temperature of lead shot _____.

Calculated value of heat energy in calories _____.

Average distance lead shot fell (in one fall) _____.

Number of times tube was inverted _____.

Total height through which lead shot fell _____.

Calculated value of potential energy in joules _____.

Calculated value of mechanical equivalent of heat _____.

Correct value of mechanical equivalent of heat _____.

Percent error _____.

Note:

- a. Why is it advisable to start with the shot below room temperature and end up above room temperature?
- b. Mention some sources of error and actions that can be taken to correct errors and improve experimental results.

SELF CHECK

1. Define "mechanical equivalent of heat."
2. Demonstrate a knowledge of the relationship between heat and mechanical energy by stating equivalent energies in calories, joules and British thermal units.
3. Calculate increase in temperature of a lead object that falls 40 meters:
    - a. If object has a mass of 10 kg.
    - b. If object has a mass of 500 kg.
4. Considering answers to problem 3, calculate increase in temperature of a copper object of unknown mass that falls 6 m. Specific heat of copper is 0.093 cal/g °C.
5. A 30-gram lead bullet traveling 300 m/s is stopped by a wall. Initial temperature of bullet is 30° C. Calculate:
    - a. Kinetic energy in joules of bullet before striking wall.
    - b. Heat, in calories, that goes into bullet. Assume 90% goes into bullet and 10% goes into wall.
    - c. Final temperature of bullet. Hint: The bullet will not go above the melting temperature of lead (330° C) unless all of the lead melts.

    Is there enough heat to melt some or all of the bullet? If so, how much is melted? It takes 5.9 calories to melt one gram of lead.
6. A mechanic lifts a 375-lb motor using a pulley system. A pull on the chain a distance of 10 feet with a constant force of 50 lb lifts the motor 15 inches. Calculate:
    - a. Work done by mechanic in ft x lb.
    - b. Increase in potential energy in ft x lb.

    If answer to part a is greater than answer to part b, calculate amount of energy left over. Assuming the motor is at rest after it is moved, where did the energy go?
    - c. Efficiency using the following formula:

$$\% \text{ efficiency} = \frac{\text{work output}}{\text{work input}} \times 100\%.$$

7. From what height must a block of ice at 0°C be dropped if it is to completely melt by the time it reaches the ground? It takes 80 calories to melt 1 gram of ice. (Assume all the mechanical energy is converted into heat energy.)

APPENDIX

ADDITIONAL APPLICATION MODULES AVAILABLE FOR UTC PHYSICS

1F1, ATMOSPHERIC PRESSURE

In this application module air-pressure magnitude and measurements are described by use of manometers and mercury and aneroid barometers. Boyle's law apparatus is used to demonstrate through experiments the effects of air pressure. A table of various units for standard pressure and four worked examples are included. Procedures enable the student to construct a mercury barometer and to measure and calculate pressure and volume of confined air (Boyle's law).

1F3, HYDRAULIC JACK

The hydraulic jack is an example of a force magnifier in a closed hydraulic system employing Pascal's law. By use of valves, a fluid reservoir, and pressure, large loads can be moved short distances with relative ease. Calculation of the input/output pressures is illustrated. The laboratory demonstrates this principle by use if disposable syringes.

1E1, POTENTIAL DIFFERENCE AND CHARGE FLOW

This module defines potential difference and discusses five ways that it can be produced. Characteristics of and measurements of potential difference are discussed in the text and examined in the laboratory exercise. In the laboratory, the simple elements of an electrical circuit are constructed, and measurements are made using a voltmeter or voltohmmeter (VOM).

1E2, SERIES AND PARALLEL VOLTAGE SOURCES

Calculation and determination of emf's in linear and parallel circuits that have like or unlike poles connected are described. Examples of motor voltage calculation are shown. Laboratory procedures enable the student to connect several different circuits and to measure voltages with a VOM across designated components.

1E5, LEAD-ACID STORAGE BATTERY

Basic mechanisms of producing electrical energy in an acid battery are described. An automotive battery is examined in detail including the effect of changing to specific gravity and temperature and resulting electric potential output. Students are asked to construct, charge, and discharge a lead-acid storage cell.

1T3, THERMAL STRESS

Mathematical and practical relationship of thermal expansion is expressed and described. A table of linear coefficients of expansion of several materials is provided. A procedure that demonstrates thermal expansion of a steel wire is included.

2T2, HEAT ENERGY REQUIRED FOR MELTING

Examination is made of energy requirements necessary to cause a change from solid to liquid state (heat of fusion). A table of melting points and heats of fusion of common solids and liquids is included. Working examples are provided. A procedure whereby the student can measure and calculate the heat of fusion is provided.

2T3, HEAT ENERGY REQUIRED FOR BOILING

The temperature of heated water as a function of time is measured, and the effect of heat of vaporization noted. Procedures for measurement of temperature-versus-time during boiling water and the cooling effect of an evaporating liquid are given for student completion.

3M1, GRAPHICAL APPROACH TO LINEAR MOTION

The general subject of relationships between speed, position and time is called kinematics. In this module, only the description of motion along a straight line will be studied. Such motion is called rectilinear motion. A later module will concern present circular motion, which is somewhat more complicated. In the laboratory, the motion of an automobile is analyzed.

3M4, ACCELERATION OF A MASS BY CONSTANT FORCE

Describes the relationship of force, mass, and acceleration; and includes a discussion of displacement, initial velocity, final velocity, and time as parameters of motion. Construction of an Atwood's machine, a device used to determine the mass of acceleration of an object, also is discussed. In the laboratory, the student will use a pulley and other simple materials to construct an Atwood's machine and will determine the mass and acceleration of an object.

3F1, FLOW RATE MEASUREMENT WITH A VENTURI METER

Presents operating principles of a venturi flowmeter, a typical device used in industry for accurate flow-rate measurements. An experiment is included to give the student hands-on experience in use of a laboratory model of a venturi tube.

3T2, RATE OF HEAT CONDUCTION

Treats the rate of heat conduction through solid materials, taking into account physical quantities affecting rate, and relating them through a coefficient of thermal conductivity or an average coefficient of heat transmission. Use of insulation in reducing rate of heat conduction is emphasized. In the laboratory, the student will determine rate of heat conduction for several materials.

4M3, CONSERVATION OF MOMENTUM IN COLLISIONS

Momentum conservation is applied to elastic and inelastic collisions. In the laboratory the student will analyze both elastic and inelastic collisions in one dimension to verify conservation of momentum. An air track and two air carts are employed in the laboratory.

4F1, ROCKET ENGINES

General characteristics of rocket propulsion are discussed, and concepts of momentum and impulse are applied to performance of rocket engines. An experiment provides a hands-on test of the impulse produced by ignition of a solid-fuel model rocket engine.

5M1, STATIC AND SLIDING FRICTION

Resistance occurs in all energy systems. This module examines friction, the resistance in a mechanical energy system. Lubricating oils are used in automobile engines specifically to reduce friction between moving parts. At other times, it is more important to increase friction. Lining materials in automobile brakes, for example, are carefully selected to increase friction between moving parts. In the laboratory, (1) a sliding friction block is shown to exhibit force of static and kinetic friction; and (2) friction blocks are slid on an angled surface to determine coefficient of static friction at various angles of repose.

5F1, LAMINAR AND TURBULENT FLOW

Discusses laminar and turbulent flow and presents the Reynolds number as a ratio that describes flow quality. In the laboratory, the student will use a venturi tube, a manometer, and a needle valve to investigate laminar and turbulent flow of fluids that have different viscosities.

5F5, SHOCK ABSORBERS

A shock absorber is a mechanical device that provides resistance in a mechanical system. In most applications, this resistance is applied to reduce mechanical oscillations. In this module, the principles of operation and construction details of several shock absorbers are presented. In the laboratory, the student will measure resistance of an automobile shock absorber when a force is applied to it in both the compression and decompression modes.

5F6, DRAG-RESISTANCE TO MOTION THROUGH FLUIDS

When an object moves through a fluid, or when a fluid moves around an object, a force acts to impede the motion. In this module, the properties of this fluid drag force and calculations of its magnitude are presented. Calculations for the terminal velocity of an object moving through a fluid are discussed. In the laboratory, the student will construct a device with which to measure the drag force exerted on a sphere by a regulated stream of gas and will calculate the sphere's drag resistance.

5T2, HEAT FLOW THROUGH INSULATION

Insulation slows down (impedes, resists) flow of heat in the same manner that a resistor impedes flow of electrical current. Increased insulation can reduce greatly the heat losses in a structure and thereby substantially reduce heating and cooling costs. In the laboratory, the student will use heat lamps and oven thermometers to determine thermal resistance of several commercial insulating materials.

6M1, HORSEPOWER

The origin and use of the term "horsepower" are investigated and four worked examples involving horsepower are provided. In the laboratory, the student will measure the power of an electric motor or winch in terms of its horsepower and will determine the horsepower of an automobile from its weight and acceleration.

6M2, PRONY BRAKE

The prony brake is a device used to determine operating characteristics, such as horsepower, of motors, engines, and turbines. Manufacturers often check the hp of power sources with the prony brake to ensure that the tested output power is equal to that specified. In the laboratory, the student will use a prony brake to measure speed and torque characteristics of an electric motor, will prepare a graph of speed versus torque, and will prepare a graph of motor efficiency versus torque.

6E2, ELECTRIC HEATER

Presents the electric heater as an example of a purely resistive load. Calculations are provided for determination of a heater's power consumption and its conversion of electrical energy to heat energy. In the laboratory, the student will use a voltmeter and an ammeter to determine power consumed by an electric heater and its conversion of electrical energy to heat.

6E4, ELECTRICAL POWER CONSUMPTION

Home appliances use varying amounts of electrical power and add varying contributions to the monthly electric bill. This module examines relative levels of electrical power consumption for some common appliances, and discusses their costs of operation. In the laboratory, the student will measure electrical power consumption of some appliances with a wattmeter and a kilowatt-hour meter, and will determine relative costs of operation.

7M1, PILEDRIVER

Forging hammers and piledrivers convert gravitational potential energy to kinetic energy. In this module, the piledriver as an energy storage and delivery device is investigated. In the laboratory, the student will construct a piledriver and will investigate the effects of changes in the mass and height of the piledriver blow.

7M2, POTENTIAL ENERGY OF A SPRING

Stretching or compressing of a spring stores mechanical energy in the spring. This energy can be recovered as useful work by allowing the force present in the spring to cause motion toward the equilibrium position of the spring. This module discusses forces acting upon a spring and the storage of energy in it. In the laboratory, the student will determine a spring constant and will calculate the period of a spring-mass system.

7M5, KINETIC ENERGY AND HORSEPOWER OF AN AUTOMOBILE

In the automobile engine, chemical energy in the fuel is converted by combustion into heat energy and, in turn, into mechanical energy. Engine horsepower is the rate at which the engine can develop energy for work in propelling the automobile. This module presents a discussion of automobile engine horsepower and its measurement. In the laboratory, the student will weigh an automobile and time its acceleration on a drag strip to determine the horsepower of the engine.

7F1, ENERGY STORED IN LARGE TANKS

Presents development of Torricelli's theorem from Bernoulli's equation. Energy exchanges that occur in the steady flow of an incompressible, viscous fluid are presented, and an application of Torricelli's theorem to the energy stored in large tanks is included. In the laboratory, the student will use an aspirator bottle or a container with a hole near the bottom to demonstrate Torricelli's theorem.

7F4, ENERGY CONVERSION IN FLUID SYSTEMS

Demonstrates conversion of potential energy (capacity to do work by virtue of position) into kinetic energy (capacity to do work by virtue of motion) in a fluid system. In the laboratory, the student will use a water motor equipped with a prony brake to determine the energy in a fluid system.

7F6, SIPHONS

Physical principles that produce flow in a siphon are explained. Relationship between atmospheric pressure, liquid pressure, elevation, liquid velocity, and fluid resistance in a siphon are illustrated by application of Bernoulli's equation. In the laboratory, the student will use containers and tubing to illustrate the relationship between fluid elevation, flow in the siphon, and fluid resistance in the siphon.

7E1, ENERGY STORED IN A BATTERY

Describes the battery as a chemical energy storage device. Energy required to charge the battery is compared to energy available at discharge, and work done by the battery at various times of its discharge is discussed. In the laboratory, the student will use an ammeter and a voltmeter to measure the charging and discharging of a battery.

7T1, HEAT STORAGE SYSTEMS

Investigates the way in which various materials store heat. The heat pump is described as a mechanical device that transfers heat from one medium to another, and examples of the use of heat pumps are provided. In the laboratory, the student will compare the heat storage capacity of various materials.

7T4, THERMAL DRYING

The concept of kinetic energy of atoms and molecules is applied to an explanation of thermal drying of moist materials. The thermal nature of drying is explored in a discussion of molecular behavior of water vapor in accordance with the kinetic molecular theory. Evaporation of moisture from a moisture-containing solid is discussed as an important mechanism of drying. General characteristics of several types of industrial thermal dryers are presented. In the laboratory, the student will use a triple beam balance and a source of drying heat to determine moisture content of an apple and a piece of freshly cut wood.

7T5, MIXING LIQUIDS AT DIFFERENT TEMPERATURES

Substances possess internal kinetic energy in the form of the microscopic motion of their molecules. When liquids at different temperatures are mixed, molecular motion causes them to reach an equilibrium temperature. This module discusses internal kinetic energy of liquids and several methods of determining specific heat and final temperature of substances mixed at different temperatures. In the laboratory, the student will construct a simple calorimeter to determine specific heat of a liquid and final temperature of a mixture.